Computational Methods for Differential Equations
http://cmde.tabrizu.ac.ir
Vol. 8, No. 4, 2020, pp. 827-839
DOI:10.22034/cmde.2019.33123.1534

Approximate analytic compacton solutions of the $K(p, p)$ equation by reduced differential transform method

Turgut Ak*
Armutlu Vocational School,
Yalova University,
77500 Yalova, Turkey.
E-mail: akturgut@yahoo.com

Sharanjeet Dhawan
Department of Mathematics,
Central University of Haryana,
123029 Haryana, India.
E-mail: dhawan311@gmail.com

Abstract In the present work, we focus on solutions of $K(p, p)$ equation which are solitons with compact support called compactons. Such a study of compact solitary waves will help us understanding solitons at a deeper level. One of the interesting feature, they govern is quasi elastic collision and gaining the same coherent shape again after scattering. Numerical scheme used to study the compacton solutions of $K(p, p)$ equation is based on reduced differential transform method. Both one dimensional differential transform method and two dimensional reduced differential transform method have been used. Test problems under consideration show the efficient working of the proposed scheme.

Keywords. Reduced differential transform method, $K(p, p)$ equation, compacton.
2010 Mathematics Subject Classification. 74H10, 35C07.

1. Introduction

Being capable of describing a variety of phenomenon, study of nonlinear evolution equations has always played an important role. So far, authors have developed some numerical schemes for solving various types of nonlinear PDEs [1, 2, 7, 8, 12, 13]. In earlier studies, a lot of attention was paid towards the soliton solutions. In the present work, we focus on $K(p, p)$ equation containing the nonlinear dispersion term which gives rise to solitons with compact support called compactons. Their study is important as they play a very crucial role in pattern formation and occurrence of nonlinear structures in various physical systems. Their relevance can be found in various aspects such as fluid dynamics [4], monochromatic short surface wind waves [6], shear waves in plates [5].

Initially, Rosenau et al. studied a class of equations possessing solutions as compact solitary waves with exciting properties as they maintain their coherence after multiple

Received: 30 April 2019 ; Accepted: 21 December 2019.
∗ corresponding author.

collisions [9, 10]. The energy which was lost during the multiple collisions, came again into existence as compactons and anticompactons. Like compactons, anticompactons are also same travelling wave travelling in opposite direction with negative amplitude. These waves exhibit both elastic and nearly elastic collisions that are similar to the soliton interactions associated with completely integrable partial differential equations PDEs supporting an infinite number of conservation laws.

In this work, we have used a numerical scheme based on reduced differential transform method. After introductory section, some details of the governing mathematical model can be found in Section 2 followed by basic definitions in Section 3. Section 4 contains working of the proposed scheme. For experimental studies, two test problems have been considered for varying values of p under different cases. The errors for $N-$approximate compacton solutions are presented through tables whereas errors for $4-$approximate solution can be seen in graphs. Finally, conclusion is drawn and list of references studied for the successful completion of this work is presented at the last.

2. Mathematical modelling

Problem under consideration is $K(p, p)$ equation is given by

$$u_t - c_0 u_x + (u^p)_x + (u^p)_{xxx} = 0 \ , \ 1 < p \leq 3 \tag{2.1}$$

where $u(x, t)$ is the wave amplitude, x is the spatial coordinate, t is time, and c_0 is a constant velocity.

3. Basic definitions

Some important definitions related to differential transformation are introduced as follows.

3.1. **One dimensional differential transform method.** The transformation of the k-th derivative of a function in one variable is as follows:

If $u(t) \in R$ can be expressed as a Taylor series about fixed point t_0, then $u(t)$ can be represented as

$$u(t) = \sum_{k=0}^{\infty} \frac{u^{(k)}(t_0)}{k!}(t - t_0)^k. \tag{3.1}$$

If $u_n(t) = \sum_{k=0}^{n} \frac{u^{(k)}(t_0)}{k!}(t - t_0)^k$, is the n-partial sums of a Taylor series equation (3.1), then

$$u_n(t) = \sum_{k=0}^{n} \frac{u^{(k)}(t_0)}{k!}(t - t_0)^k + R_n(t), \tag{3.2}$$

where $u_n(t)$ is called the n-th Taylor polynomial for $u(t)$ about t_0 and $R_n(t)$ is remainder term. If $U(k)$ is defined as

$$U(k) = \frac{1}{k!}\left[\frac{d^k u(t)}{dt^k}\right]_{t=t_0}, \tag{3.3}$$

where $k = 0, 1, 2, ...$ then, Eq. (3.1) reduces

$$u(t) = \sum_{k=0}^{\infty} U(k)(t - t_0)^k, \tag{3.4}$$

and the n-partial sums of a Taylor series equation (3.2) reduces to

$$u_n(t) = \sum_{k=0}^{n} U(k)(t - t_0)^k + R_n(t). \tag{3.5}$$

The $U(k)$ defined in Eq. (3.5) is called the differential transform of function $u(t)$. For simplicity, we assume $t_0 = 0$. Then, Eq. (3.5) reduces to

$$u_n(t) = \sum_{k=0}^{n} U(k)t^k + R_n(t). \tag{3.6}$$

From the above definition, it is clear that the roots of differential transform method lie in Taylor series expansion.

TABLE 1. The fundamental operations of one-dimensional DTM

Original Function	Transformed Function
$w(t) = u(t) \pm v(t)$	$W(k) = U(k) \pm V(k)$
$w(t) = \frac{d^m u(t)}{dt^m}$	$W(k) = \frac{(k+m)!}{k!} U(k+m)$
$w(t) = u(t)\, v(t)$	$W(k) = U(k) * V(k) = \sum_{r=0}^{k} U(r)\, V(k-r)$
$w(x) = x^m$	$W(k) = \delta(k-m) = \begin{cases} 1 & k = m \\ 0 & otherwise \end{cases}$
$w(t) = e^{\lambda t}$	$W(k) = \frac{\lambda^k}{k!}$
$w(t) = \sin(\alpha t + \beta)$	$W(k) = \frac{\alpha^k}{k!} \sin\left(\frac{k\pi}{2} + \beta\right)$
$w(t) = \cos(\alpha t + \beta)$	$W(k) = \frac{\alpha^k}{k!} \cos\left(\frac{k\pi}{2} + \beta\right)$

3.2. **Two dimensional reduced differential transform method.** Let us express the function $w(x,t)$ as $w(x,t) = f(x)g(t)$. Using properties of one dimensional DTM, it can be represented as

$$\begin{aligned} w(x,t) &= \sum_{i=0}^{\infty} F(i)x^i \sum_{j=0}^{\infty} G(j)t^j \\ &= \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} W(i,j)x^i t^j, \end{aligned} \tag{3.7}$$

where $W(i,j) = F(i)G(j)$ is called the spectrum of $w(x,t)$. If $w(x,t)$ is analytical function in the given domain, then the spectrum function

$$W_k(x) = \frac{1}{k!} \left[\frac{\partial^k}{\partial t^k} w(x,t) \right]_{t=t_0}, \tag{3.8}$$

is reduced transformed function of $w(x,t)$. Also, the differential inverse transform of $W_k(x)$ is defined as

$$w(x,t) = \sum_{k=0}^{\infty} W_k(x)(t-t_0)^k. \tag{3.9}$$

From (3.8) and (3.9), we get

$$w(x,t) = \sum_{k=0}^{\infty} \frac{1}{k!}\left[\frac{\partial^k}{\partial t^k} w(x,t)\right]_{t=t_0} (t-t_0)^k. \tag{3.10}$$

Therefore, it can be observed that the concept of reduced differential transform method is derived from the two dimensional differential transform method.

TABLE 2. The fundamental operations of two-dimensional RDTM

Original Function	Transformed Function
$w(x,t) = u(x,t) \pm v(x,t)$	$W_k(x) = U_k(x) \pm V_k(x)$
$w(x,t) = \frac{\partial}{\partial x} u(x,t)$	$W_k(x) = \frac{d}{dx} U_k(x)$
$w(x,t) = \frac{\partial}{\partial t} u(x,t)$	$W_k(x) = (k+1)\, U_{k+1}(x)$
$w(x,t) = \frac{\partial^{r+s}}{\partial x^r \partial t^s} u(x,t)$	$W_k(x) = \frac{(k+s)!}{k!} \frac{d^r}{dx^r} U_{k+s}(x)$
$w(x,t) = u(x,t)\, v(x,t)$	$W_k(x) = \sum_{r=0}^{k} U_r(x)\, V_{k-r}(x)$
$w(x,t) = u(x,t)\, v(x,t)\, z(x,t)$	$W_k(x) = \sum_{r=0}^{k} \sum_{s=0}^{k-r} U_r(x)\, V_s(x)\, Z_{k-r-s}(x)$
$w(x,t) = x^m t^n$	$W_k(x) = x^m \delta(k-n) = \begin{cases} x^m & k=n \\ 0 & otherwise \end{cases}$

4. NUMERICAL APPLICATIONS

In this section, two test problems are considered to illustrate the performance of the method for $p = 2$ and $p = 3$. The accuracy of the method is measured by using the absolute error norms. Here, we use compacton solutions of Eq. (2.1) which is given by [3, 11, 13]

$$u_c(x,t) = \alpha^{\gamma} \cos^{2\gamma}\left[\beta\left(x - x_0 - (c-c_0)t\right)\right], \tag{4.1}$$

where c is the compacton velocity, x_0 is the position of its maximum at $t = 0$ and

$$\alpha = \frac{2cp}{p+1}\ ,\ \beta = \frac{p-1}{2p}\ ,\ \gamma = \frac{1}{p-1}. \tag{4.2}$$

4.1. **Case I: (p=2).** For $p = 2$, Eq. (2.1) turns into

$$u_t - c_0 u_x + \left(u^2\right)_x + \left(u^2\right)_{xxx} = 0. \tag{4.3}$$

The compacton solution of Eq. (4.3)

$$u_c(x,t) = \frac{4c}{3} \cos^2\left[\frac{1}{4}\left(x - x_0 - (c-c_0)t\right)\right]. \tag{4.4}$$

We consider Eq. (4.3) subject to initial condition

$$u(x,0) = \frac{4c}{3} \cos^2\left[\frac{1}{4}\left(x - x_0\right)\right]. \tag{4.5}$$

Therefore, by applying the reduced differential transform method on Eq. (4.3), for $k = 0, 1, 2, \ldots$ we get following recursive equation

$$\begin{aligned}(k+1)\,U_{k+1}(x) - c_0\frac{d}{dx}U_k(x) + 2\sum_{r=0}^{k}U_r(x)\,\frac{d}{dx}U_{k-r}(x)\\ +6\sum_{r=0}^{k}\frac{d}{dx}U_r(x)\,\frac{d^2}{dx^2}U_{k-r}(x)\\ +2\sum_{r=0}^{k}U_r(x)\,\frac{d^3}{dx^3}U_{k-r}(x) = 0,\end{aligned} \tag{4.6}$$

and their initial value is obtained from initial condition (4.5) as follow

$$U_0(x) = \frac{4c}{3}\cos^2\left[\frac{1}{4}(x-x_0)\right], \tag{4.7}$$

then, by utilizing the initial condition in recursive equation (4.6) for $k = 0, 1, 2, 3$ the first five terms of $U_{k+1}(x)$ obtained as follow

$$U_1(x) = \frac{c}{3}\sin\left(\frac{x}{2}\right)(c-c_0), \tag{4.8}$$

$$U_2(x) = -\frac{c}{12}\cos\left(\frac{x}{2}\right)(c-c_0)^2, \tag{4.9}$$

$$U_3(x) = -\frac{c}{72}\sin\left(\frac{x}{2}\right)(c-c_0)^3, \tag{4.10}$$

$$U_4(x) = \frac{c}{576}\cos\left(\frac{x}{2}\right)(c-c_0)^4. \tag{4.11}$$

Similarly, we can obtain other components using the recurrence relation (4.6). By substituting the these quantities in inverse differential transform, the approximate solution of Eq. (4.3) in the Poisson series form is:

$$U_4(x,t) = U_0(x) + U_1(x)\,t + U_2(x)\,t^2 + U_3(x)\,t^3 + U_4(x)\,t^4, \tag{4.12}$$

$$\begin{aligned}U_4(x,t) = \frac{c}{576}\Big[768\cos^2\left(\frac{x}{2}\right) + 192t\sin\left(\frac{x}{2}\right)(c-c_0)\\ -48t^2\cos\left(\frac{x}{2}\right)(c-c_0)^2 - 8t^3\sin\left(\frac{x}{2}\right)(c-c_0)^3\\ +t^4\cos\left(\frac{x}{2}\right)(c-c_0)^4\Big],\end{aligned} \tag{4.13}$$

which exactly is the first five terms of the Poisson series of the exact solution (4.4). For $p = 2$, the obtained $4-$approximate compacton solutions are drawn in Figure 1 as $3D$ and $2D$, respectively.

Table 3 shows that as the number of opened terms in the series increases, the approach gets better slightly and the amount of errors decreases at $t = 1$. For $N-$approximate compacton solutions, this situation is illustrated in Figure 2 in detail.

On the other hand, Table 4 displays that as the number of opened terms in the series increases and time progresses, the absolute error norms increase at $x = 0$. As seen from the Figure 3, absolute errors for 4−compacton solution increase as time progresses.

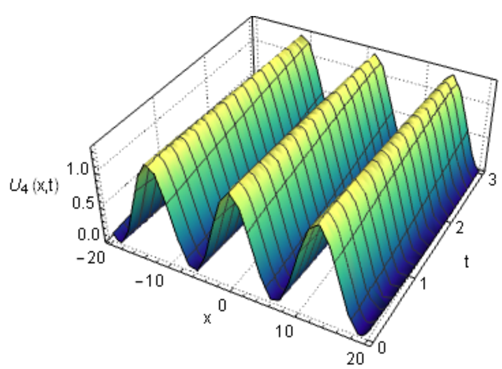
TABLE 3. The errors for $N-$approximate compacton solutions of Eq. (4.3) with $c = 1$, $c_0 = 0.5$, $p = 2$ at $t = 1$

x	$\|u(x,t) - U_1(x,t)\|$	$\|u(x,t) - U_2(x,t)\|$	$\|u(x,t) - U_3(x,t)\|$	$\|u(x,t) - U_4(x,t)\|$
−20	1.644827×10^{-2}	1.032390×10^{-3}	8.790851×10^{-5}	3.136581×10^{-6}
−15	5.560644×10^{-3}	1.660925×10^{-3}	3.245266×10^{-5}	5.159681×10^{-6}
−10	7.538518×10^{-3}	1.628889×10^{-3}	3.591003×10^{-5}	5.130710×10^{-6}
−5	1.763952×10^{-2}	9.490233×10^{-4}	8.999084×10^{-5}	3.061190×10^{-6}
0	2.072505×10^{-2}	1.082811×10^{-4}	1.082811×10^{-4}	2.258040×10^{-7}
5	1.556797×10^{-2}	1.122521×10^{-3}	8.350665×10^{-5}	3.422993×10^{-6}
10	4.219309×10^{-3}	1.690320×10^{-3}	2.552050×10^{-5}	5.258814×10^{-6}
15	8.807426×10^{-3}	1.585857×10^{-3}	4.261548×10^{-5}	5.003138×10^{-6}
20	1.833134×10^{-2}	8.506784×10^{-4}	9.380274×10^{-5}	2.757649×10^{-6}

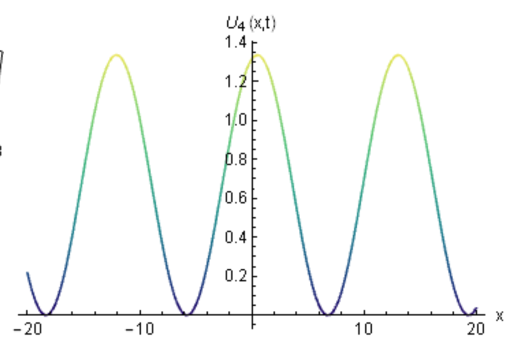
TABLE 4. The errors for $N-$approximate compacton solutions of Eq. (4.3) with $c = 1$, $c_0 = 0.5$, $p = 2$ at $x = 0$

t	$\|u(x,t) - U_1(x,t)\|$	$\|u(x,t) - U_2(x,t)\|$	$\|u(x,t) - U_3(x,t)\|$	$\|u(x,t) - U_4(x,t)\|$
0.0	0.0000000000000	0.0000000000000	0.0000000000000	0.0000000000000
0.5	5.201555×10^{-3}	6.778153×10^{-6}	6.778153×10^{-6}	3.531142×10^{-9}
1.0	2.072505×10^{-2}	1.082811×10^{-4}	1.082811×10^{-4}	2.258040×10^{-7}
1.5	4.632825×10^{-2}	5.467479×10^{-4}	5.467479×10^{-4}	2.568465×10^{-6}
2.0	8.161163×10^{-2}	1.721708×10^{-3}	1.721708×10^{-3}	1.440318×10^{-5}
2.5	1.260246×10^{-1}	4.183746×10^{-3}	4.183746×10^{-3}	5.480618×10^{-5}
3.0	1.788741×10^{-1}	8.625913×10^{-3}	8.625913×10^{-3}	1.631499×10^{-4}

FIGURE 1. 4−approximate solution for compacton solution of Eq. (4.3)

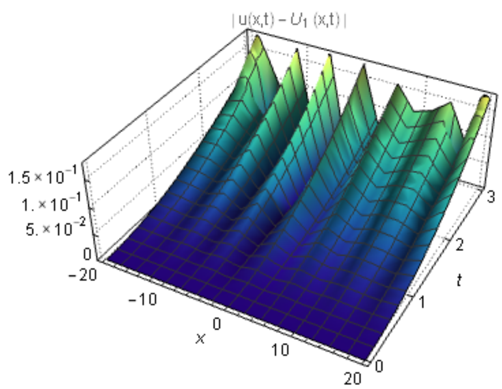


(a) 3D

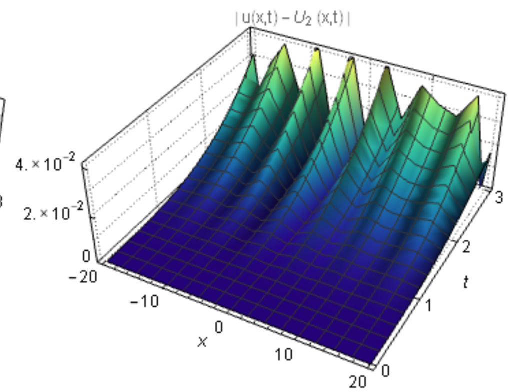


(b) 2D

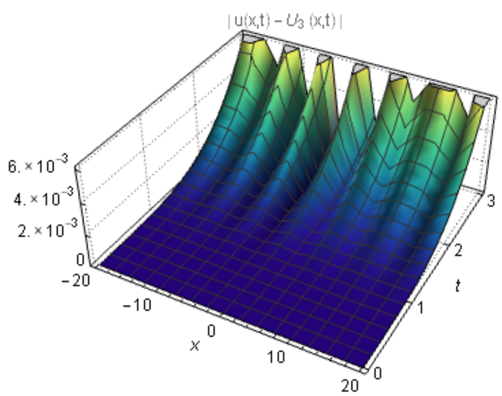
Figure 2. Comparison of absolute errors for $N-$compacton solution of Eq. (4.3)



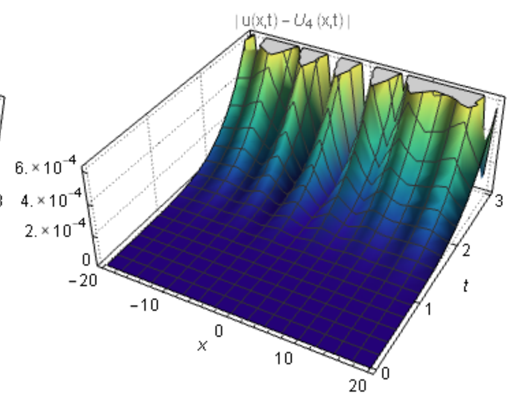
(a) 1−Absolute error



(b) 2−Absolute error



(c) 3−Absolute error



(d) 4−Absolute error

4.2. **Case II: (p=3).** For $p = 3$, Eq. (2.1) turns into

$$u_t - c_0 u_x + \left(u^3\right)_x + \left(u^3\right)_{xxx} = 0. \tag{4.14}$$

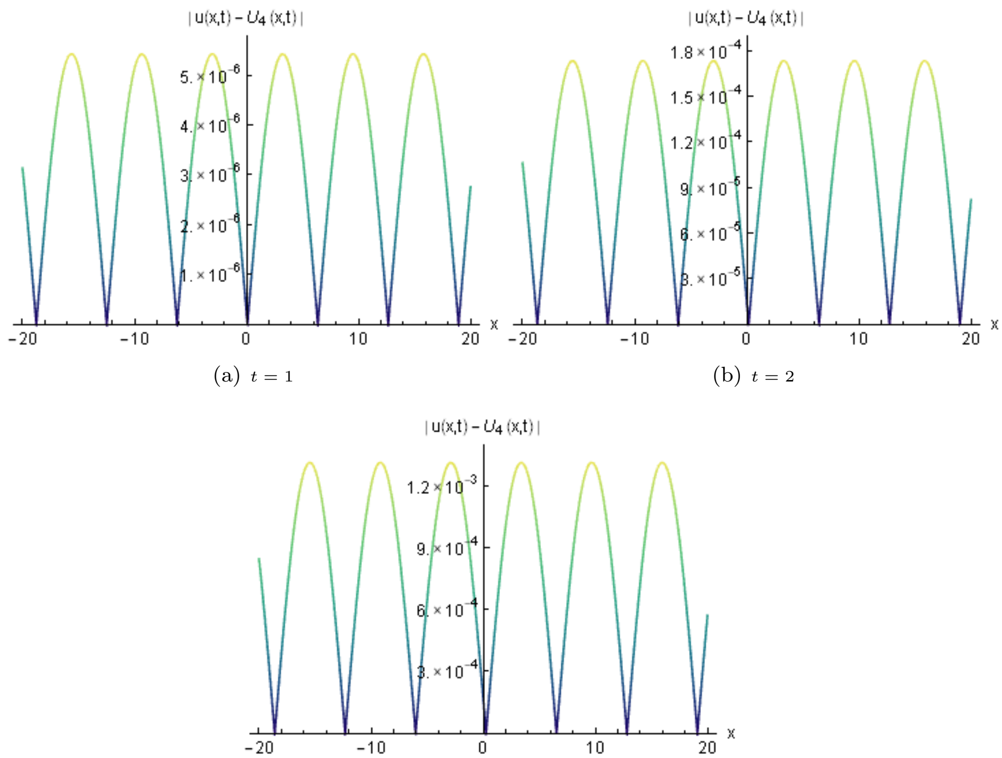
The compacton solution of Eq. (4.14)

$$u_c\left(x,t\right) = \frac{3c}{2}\cos\left[\frac{1}{3}\left(x - x_0 - (c - c_0)t\right)\right]. \tag{4.15}$$

We consider Eq. (4.14) subject to initial condition

$$u\left(x,0\right) = \frac{3c}{2}\cos\left[\frac{1}{3}\left(x - x_0\right)\right]. \tag{4.16}$$

FIGURE 3. Absolute errors for 4−compacton solution of Eq. (4.3) with respect to time



(a) $t=1$

(b) $t=2$

(c) $t=3$

Therefore, by applying the reduced differential transform method on Eq. (4.14), for $k = 0, 1, 2, \ldots$ we get following recursive equation

$$
\begin{aligned}
(k+1)\,U_{k+1}(x) - c_0 \frac{d}{dx}U_k(x) &+ 3\sum_{r=0}^{k}\sum_{s=0}^{k-r} U_r(x)\,U_s(x)\,\frac{d}{dx}U_{k-r-s}(x) \\
&+6\sum_{r=0}^{k}\sum_{s=0}^{k-r} \frac{d}{dx}U_r(x)\,\frac{d}{dx}U_s(x)\,\frac{d}{dx}U_{k-r-s}(x) \\
&+18\sum_{r=0}^{k}\sum_{s=0}^{k-r} U_r(x)\,\frac{d}{dx}U_s(x)\,\frac{d^2}{dx^2}U_{k-r-s}(x) \\
&+3\sum_{r=0}^{k}\sum_{s=0}^{k-r} U_r(x)\,U_s(x)\,\frac{d^3}{dx^3}U_{k-r-s}(x) = 0,
\end{aligned}
\tag{4.17}
$$

and their initial value is obtained from initial condition (4.16) as follow

$$U_0(x) = \frac{3c}{2}\cos\left[\frac{1}{3}(x - x_0)\right], \tag{4.18}$$

then by utilizing the initial condition in recursive equation (4.17) for $k = 0, 1, 2, 3$ the first five terms of $U_{k+1}(x)$ obtained as follow

$$U_1(x) = \frac{c}{4}\sin\left(\frac{x}{3}\right)\left(3c^2 - 2c_0\right), \tag{4.19}$$

$$U_2(x) = -\frac{c}{48}\cos\left(\frac{x}{3}\right)\left(3c^2 - 2c_0\right)^2, \tag{4.20}$$

$$U_3(x) = -\frac{c}{864}\sin\left(\frac{x}{3}\right)\left(3c^2 - 2c_0\right)^3, \tag{4.21}$$

$$U_4(x) = \frac{c}{20736}\cos\left(\frac{x}{3}\right)\left(3c^2 - 2c_0\right)^4, \tag{4.22}$$

Similarly, we can obtain other components using the recurrence relation (4.17). By substituting the these quantities in inverse differential transform, the approximate solution of Eq. (4.14) in the Poisson series form is:

$$U_4(x,t) = U_0(x) + U_1(x)\,t + U_2(x)\,t^2 + U_3(x)\,t^3 + U_4(x)\,t^4, \tag{4.23}$$

$$\begin{aligned} U_4(x,t) = \frac{c}{20736}\Big[&31104\cos\left(\frac{x}{3}\right) + 5184t\sin\left(\frac{x}{3}\right)\left(3c^2 - 2c_0\right) \\ &-432t^2\cos\left(\frac{x}{3}\right)\left(3c^2 - 2c_0\right)^2 - 24t^3\sin\left(\frac{x}{3}\right)\left(3c^2 - 2c_0\right)^3 \\ &+t^4\cos\left(\frac{x}{3}\right)\left(3c^2 - 2c_0\right)^4\Big], \end{aligned} \tag{4.24}$$

which exactly is the first five terms of the Poisson series of the exact solution (4.15). For $p = 3$, the obtained $4-$approximate compacton solutions are plotted in Figure 4 as $3D$ and $2D$, respectively.

As seen from Table 5, as the degree of approximation increases, the approach gets better and the absolute errors decrease at $t = 1$. $N-$approximate compacton solutions are shown in Figure 5 for different values of N.

It can be seen from the Table 6, as time progresses, the absolute error norms increase at $x = 0$. Figure 6 demonstrates that the absolute errors for $4-$compacton solution increase as time progresses.

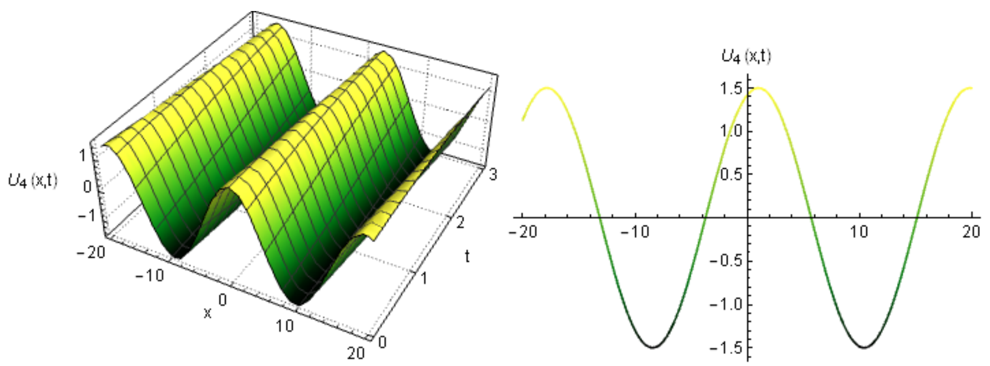
TABLE 5. The errors for $N-$approximate compacton solutions of Eq. (4.14) with $c = 1$, $c_0 = 0.5$, $p = 3$ at $t = 1$

x	$\|u(x,t) - U_1(x,t)\|$	$\|u(x,t) - U_2(x,t)\|$	$\|u(x,t) - U_3(x,t)\|$	$\|u(x,t) - U_4(x,t)\|$
-20	7.469477×10^{-2}	1.519754×10^{-1}	1.485111×10^{-1}	1.477955×10^{-1}
-15	2.467354×10^{-1}	2.230968×10^{-1}	2.142179×10^{-1}	2.144368×10^{-1}
-10	2.745801×10^{-2}	1.092642×10^{-1}	1.074997×10^{-1}	1.067422×10^{-1}
-5	2.519921×10^{-1}	2.440151×10^{-1}	2.347984×10^{-1}	2.348723×10^{-1}
0	2.078515×10^{-2}	6.254818×10^{-2}	6.254818×10^{-2}	6.177658×10^{-2}
5	2.480129×10^{-1}	2.559898×10^{-1}	2.467731×10^{-1}	2.466992×10^{-1}
10	6.826649×10^{-2}	1.353967×10^{-2}	1.530419×10^{-2}	1.454673×10^{-2}
15	2.349434×10^{-1}	2.585819×10^{-1}	2.497030×10^{-1}	2.494841×10^{-1}
20	1.132457×10^{-1}	3.596509×10^{-2}	3.250073×10^{-2}	3.321629×10^{-2}

TABLE 6. The errors for $N-$approximate compacton solutions of Eq. (4.14) with $c = 1$, $c_0 = 0.5$, $p = 3$ at $x = 0$

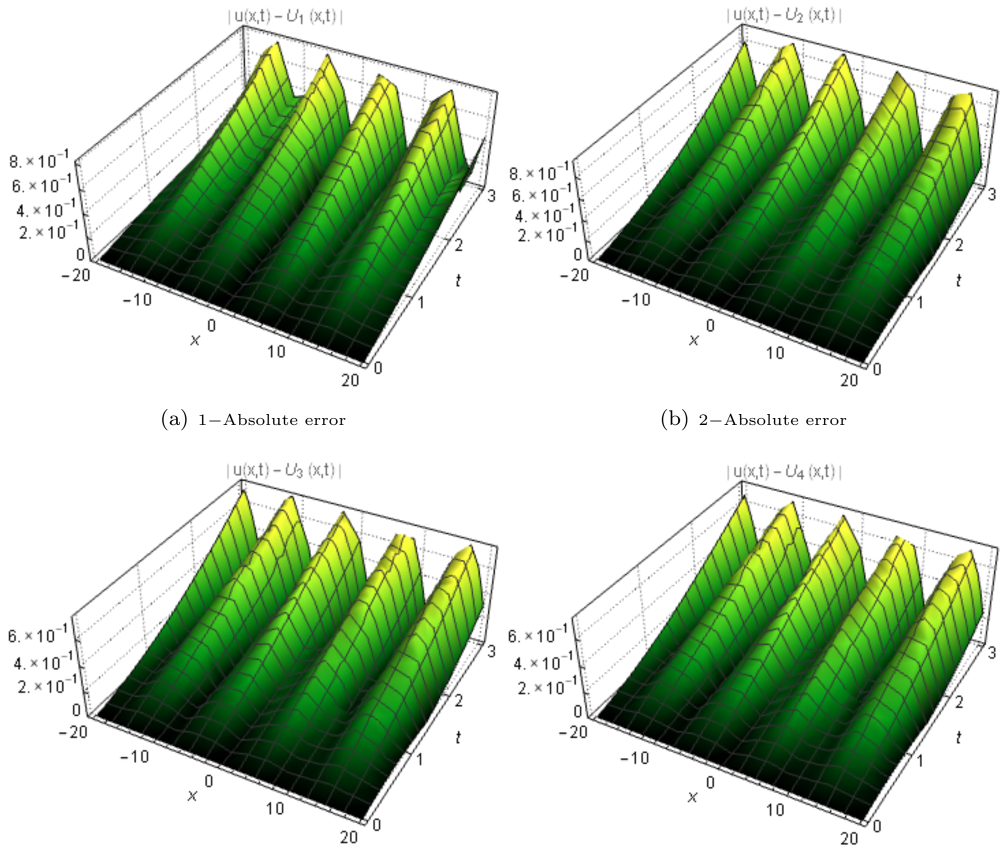
t	$\|u(x,t) - U_1(x,t)\|$	$\|u(x,t) - U_2(x,t)\|$	$\|u(x,t) - U_3(x,t)\|$	$\|u(x,t) - U_4(x,t)\|$
0.0	0.000000000000	0.000000000000	0.000000000000	0.000000000000
0.5	5.205320×10^{-3}	1.562801×10^{-2}	1.562801×10^{-2}	1.557979×10^{-2}
1.0	2.078515×10^{-2}	6.254818×10^{-2}	6.254818×10^{-2}	6.177658×10^{-2}
1.5	4.663137×10^{-2}	1.408686×10^{-1}	1.408686×10^{-1}	1.369624×10^{-1}
2.0	8.256458×10^{-2}	2.507688×10^{-1}	2.507688×10^{-1}	2.384231×10^{-1}
2.5	1.283354×10^{-1}	3.924979×10^{-1}	3.924979×10^{-1}	3.623571×10^{-1}
3.0	1.836262×10^{-1}	5.663738×10^{-1}	5.663738×10^{-1}	5.038738×10^{-1}

FIGURE 4. 4−approximate solution for compacton solution of Eq. (4.14)



(a) 3D (b) 2D

Figure 5. Comparison of absolute errors for $N-$compacton solution of Eq. (4.14) at $t = 1$



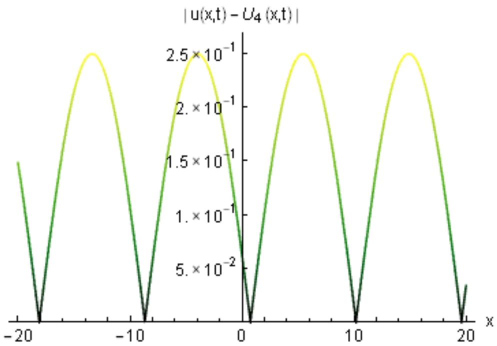
(a) 1−Absolute error

(b) 2−Absolute error

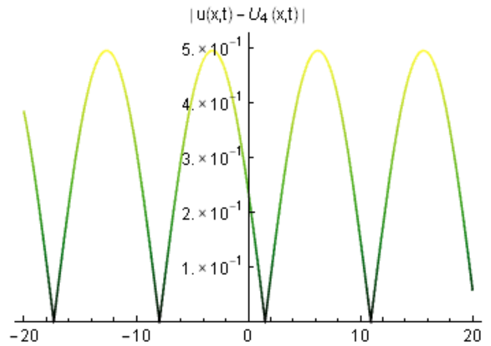
(c) 3−Absolute error

(d) 4−Absolute error

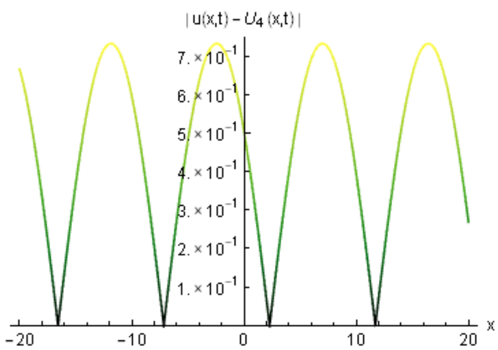
FIGURE 6. Absolute errors for 4−compacton solution of Eq. (4.14) with respect to time



(a) $t = 1$



(b) $t = 2$



(c) $t = 3$

5. Conclusion

A numerical approach based on reduced differential transform method has been proposed and implemented for $K(p,p)$ equation which includes both linear and nonlinear terms. The results so obtained guarantee the successful implementation of the scheme. Finally, the errors for $N-$approximate compacton solutions for different parameters can be seen in tabular results. Comparison of absolute errors for $4-$compacton, $N-$compacton solution can be seen via figures. All the results have been found satisfactory.

References

[1] T. Ak and S. Dhawan, *A practical and powerful approach to potential KdV and Benjamin equations*, Beni-Suef University Journal of Basic and Applied Sciences, *6*(4) (2017), 383-390.

[2] T. Ak, *An application of Galerkin method to generalized Benjamin-Bona-Mahony-Burgers equation*, Adıyaman University Journal of Science, *8*(2) (2018), 53-69.

[3] M. S. Ismail and T. R. Taha, *A numerical study of compactons*, Mathematics and Computers in Simulation, *47*(6) (1998), 519-530.

[4] Y. S. Kivshar, *Intrinsic localized modes as solitons with a compact support*, Physical Review E, *48*(1) (1993), R43(R).

[5] A. S. Kovalev, A. P. Mayer, E. S. Sokolovaa and C. Eckl, *Solitons in elastic plates*, Low Temperature Physics, *28*(10) (2002), 780-788.

[6] M. A. Manna, *Nonlinear asymptotic short-wave models in fluid dynamics*, Journal of Physics A: Mathematical and General, *34*(21) (2001), 4475.

[7] B. Mihaila, A. Cardenas, F. Cooper and A. Saxena, *Stability and dynamical properties of Rosenau-Hyman compactons using Padé approximants*, Physical Review E, *81*(5) (2010), 056708.

[8] S. P. Popov, *Compactons and Riemann waves of an extended modified Korteweg-de Vries equation with nonlinear dispersion*, Computational Mathematics and Mathematical Physics, *58*(3) (2018), 459-472.

[9] P. Rosenau and J. M. Hyman, *Compactons: Solitons with finite wavelength*, Physical Review Letters, *70*(5) (1993), 564.

[10] P. Rosenau, *Nonlinear dispersion and compact structures*, Physical Review Letters, *73*(13) (1994), 1737.

[11] P. Rosenau, *On a class of nonlinear dispersive-dissipative interactions*, Physica D: Nonlinear Phenomena, *123*(1-4) (1998), 525-546.

[12] F. Rus and F. R. Villatoro, *Self-similar radiation from numerical Rosenau-Hyman compactons*, Journal of Computational Physics, *227* (2007), 440-454.

[13] F. Rus and F. R. Villatoro, *Padé numerical method for the Rosenau-Hyman compacton equation*, Mathematics and Computers in Simulation, *76*(1-3) (2007), 188-192.