Computational Methods for Differential Equations
http://cmde.tabrizu.ac.ir
Vol. 7, No. 3, 2019, pp. 370-382

Analytical approximate solution of leptospirosis epidemic model with standard incidence rate

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Abstract In this paper, we consider a mathematical model of leptospirosis disease which is an infectious disease. The model we are considering is a system of nonlinear ordinary differential equations and it is difficult to find exact solution. He's homotopy perturbation method is employed to compute an approximation to the solution of the system of nonlinear ordinary differential equations governing on the problem. The findings obtained by HPM are compared with nonstandard finite difference (NSFD) and Runge-Kutta fourth order (RK4) methods. Some plots are presented to show the reliability and simplicity of the method.

Keywords. Leptospirosis, Homotopy perturbation method, Epidemic model, Numerical simulations.

2010 Mathematics Subject Classification. 92D25, 49J15, 93D20.

Received: 21 January 2018 ; Accepted: 18 November 2018.
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1. Introduction

Mathematical modeling has become an important tool in analyzing the spread and control of infectious diseases [12, 13]. These models help us to understand different factors like the transmission and recovery rates and predict how the diseases will spread over a period of time. In the past decades, leptospirosis infection has arisen as a globally important contagious disease. This type of disease occurred in the urban regions of developed and industrialized countries and in the rural areas as well across the globe. Individual belonging to very crowded area especially in a city and not using clean water are usually infected with this disease. Sewer cleaners, rice planters, agriculture labor and workers that cleans the canals can easily contact with this infection. There are two main reasons which are responsible for significant mortality rate due to leptospirosis: delays in diagnosis of the disease and pathogenicity of some leptospiral rinsing.

Numerous models have been investigated which represents the dynamics of both human and vector populations of SIR type as described in [3–5]. In order to study the dynamical aspects of leptospirosis disease, Pongsuumpun et al. in [14] proposed a very simple mathematical model. In their work, they studied the dynamics of both rats and human populations as the time evolve. The human population was further stratified into two groups; adults and juveniles. A deterministic model for the dynamics of leptospirosis disease was proposed by Triampo et al. [17]. They consider a case study of leptospirosis in Thailand and present some numerical results. Zaman [18] considered the real data of [17] and studied the transmission dynamics and the role of optimal control theory of leptospirosis.

Since, most of the mathematical models raised from biological problems are nonlinear by nature and it is difficult to find the analytical solution of such problems. Therefore, it is a great challenge for mathematicians and researchers to find such numerical and perturbation methods which give the best approximation to the solution of such nonlinear problems. Convergence and accuracy are the key concepts while developing and implementing a numerical scheme otherwise results will be inappropriate. As far as the analytical perturbation methods are concerned, a parameter (negligibly small) needs to be exerted in the equation. Exertion and production of such parameter is a difficult task in these methods. Recent research provided powerful methods like artificial parameter method in which this small parameter is absent.

An approximate solution of nonlinear differential equations can be effectively obtained using the well-known Homotopy Analysis Method (HAM). The method is used with perturbation methods in recent decades. The basic and fundamental scheme of the method was first introduced by Liao and He. The method uses a free parameter whose appropriate selection yields fast convergence of the algorithm. At initial stage, He in [7] introduced HPM and applies the procedure to some interesting problems. Ali et al. used optimal Homotopy Analysis Method (OHAM) in order to obtain the solution of multi-point boundary value problem. The methods mentioned above are free from the choice of small parameter and have all the advantages of perturbation methods.

This work is an extension of [8–11] by considering HPM applied to leptospirosis epidemic model. We will compare the results obtained by HPM with Runge-Kutta fourth order (RK4) method. The motivations of this method are: the method can be applied both to linear and nonlinear problems with no discretization or linearization. Numerous problems of nonlinear nature can be solved accurately and effectively using HPM because of its rapid convergence [1, 2, 6, 15, 16].

The rest of manuscript is as follow. In section 2 we included the basic concept of HPM. The model is formulated and solved by HPM in section 3. Sample numerical result and discussion is given in section 4. Conclusion is presented at the end of the paper.

2. Analysis of Homotopy Perturbation Method (HPM)

To illustrate the basic idea of HPM, consider the general nonlinear differential equation

$$A(\mu) - f(r) = 0, \quad r \in \Omega, \tag{2.1}$$

with the boundary condition,

$$\beta(\mu, \frac{\delta\mu}{\delta n}) = 0, \quad r \in \Gamma, \tag{2.2}$$

where A is a general differential operator, β is a boundary operator, $f(r)$ a known analytic function, Γ is the boundary of the domain Ω. The operator A is divided into linear part L and nonlinear part N. Therefore, equation (2.1) can be written as,

$$L(u) + N(u) - f(r) = 0. \tag{2.3}$$

By using the homotopy technique, one can construct a homotopy

$$\upsilon(r, p) : \Omega \times [0, 1] \longrightarrow R \tag{2.4}$$

which satisfies

$$H(\upsilon, p) = (1 - p)[L(\upsilon) - L(\mu_0)] + p[A(\upsilon) - f(r)] = 0, \tag{2.5}$$

or

$$H(\upsilon, p) = L(\upsilon) - L(\mu_0) + pL(\upsilon_0) + p[N(\upsilon) - f(r)] = 0, \tag{2.6}$$

where $p \in [0, 1]$ is an embedding parameter and μ_0 is the initial approximation of given equation that satisfies the boundary conditions. Clearly, we have

$$H(\upsilon, 0) = L(\upsilon) - L(\mu_0) = 0, \tag{2.7}$$

$$H(\upsilon, 1) = A(\upsilon) - f(r) = 0. \tag{2.8}$$

The changing process of p from zero to one is just that of $\upsilon(r, p)$ changing from $\mu_0(r)$ to $\mu(r)$. This is called deformation, and also $L(\upsilon) - L(\mu_0)$ and $A(\upsilon) - f(r)$ are called homotopic in topology. If the embedding parameter p $(0 \leq p \leq 1)$ is considered as a small parameter, applying the classical perturbation technique, we can naturally assume that the solution of the equation can be given as a power series in p,

$$\upsilon = \upsilon_0 + p\upsilon_1 + p^2\upsilon_2 + p^3\upsilon_3 + \ldots\ldots\ldots \tag{2.9}$$

Setting $p = 1$ results in the approximate solution as

$$\upsilon = \lim_{p\to 1} \upsilon = \upsilon_0 + \upsilon_1 + \upsilon_2 + \upsilon_3 + ... \tag{2.10}$$

3. Mathematical formulation

In this section, we extend the model presented in [6] by taking into account the interaction of susceptible human with infected vector and disease related death rate in both infected human and vectors. To understand the basic properties of the epidemic model, we first formulate the model in detail and define the parameter involve in the model. To this end, we assume that $S_h(t)$ represents number of susceptible human at time t; $I_h(t)$ represents number of human in the population, which is infected from the leptospirosis disease at time t; $R_h(t)$ represents number of human in the population which is recovered at time t; we denote the total population size by N_h, with $N_h(t) = S_h(t) + I_h(t) + R_h(t)$. For vector population, let $S_v(t)$ is susceptible vector and $I_v(t)$ is infectious vector at time t. The total population size of vector population is denoted by N_v with $N_v(t) = S_v(t) + I_v(t)$.

$$\begin{aligned}
\tfrac{dS_h}{dt} &= b_1 N_h - \mu_h S_h - \tfrac{\beta_2 S_h I_v}{N_v} - \tfrac{\beta_1 S_h I_h}{N_h} + \lambda_h R_h,\\
\tfrac{dI_h}{dt} &= \tfrac{\beta_2 S_h I_v}{N_v} + \tfrac{\beta_1 S_h I_h}{N_h} - \mu_h I_h - \delta_h I_h - \gamma_h I_h,\\
\tfrac{dR_h}{dt} &= \gamma_h I_h - \mu_h R_h - \lambda_h R_h,\\
\tfrac{dS_v}{dt} &= b_2 N_v - \gamma_v S_v - \tfrac{\beta_3 S_v I_h}{N_h},\\
\tfrac{dI_v}{dt} &= \tfrac{\beta_3 S_v I_h}{N_h} - \gamma_v I_v - \delta_v I_v,
\end{aligned} \tag{3.1}$$

with initial conditions

$$S_h(0) \geq 0,\ I_h(0) \geq 0,\ R_h(0) \geq 0,\ S_v(0) \geq 0,\ I_v(0) \geq 0. \tag{3.2}$$

Here b_1 is the recruitment rate of human population, susceptible human can be infected by two ways of transmission, β_1 which represents the direct transmission from infected human and β_2 is the rate of transmission from infected vector. μ_h is the natural mortality rate of human, λ_h is the recovery rate of human. In this work, we assumed that disease may be fatal to some infectious host, so δ_h represents the disease related death rate of infected individuals. The rate of recovery from the infection is shown by γ_h. b_2 is the recruitment rate of vector population. γ_v is the natural mortality rate of vector population. The infectious vector die due to disease at a rate of δ_v, β_3 represents the disease carrying of susceptible vector per host per unit time.

Now, we apply the homotopy perturbation technique to our model (3.1). To do this, first we define the operator $L = \frac{d}{dt}$. The homotopy of above system is

$$\begin{aligned}
LS_h(t) - LS_{h0}(t) &= p\left[b_1 N_h - \mu_h S_h - \tfrac{\beta_2 S_h I_v}{N_v} - \tfrac{\beta_1 S_h I_h}{N_h} + \lambda_h R_h - LS_{h0}(t)\right],\\
LI_h(t) - LI_{h0}(t) &= p\left[\tfrac{\beta_2 S_h I_v}{N_v} + \tfrac{\beta_1 S_h I_h}{N_h} - \mu_h I_h - \delta_h I_h - \gamma_h I_h - LI_{h0}(t)\right],\\
LR_h(t) - LR_{h0}(t) &= p\left[\gamma_h I_h - \mu_h R_h - \lambda_h R_h - LR_{h0}(t)\right],\\
LS_v(t) - LS_{v0}(t) &= p\left[b_2 N_v - \gamma_v S_v - \tfrac{\beta_3 S_v I_h}{N_h} - LS_{v0}(t)\right],\\
LI_v(t) - LI_{v0}(t) &= p\left[\tfrac{\beta_3 S_v I_h}{N_h} - \gamma_v I_v - \delta_v I_v - LI_{v0}(t)\right].
\end{aligned} \tag{3.3}$$

We assume that the solution of the system (3.3) is in the form,

$$\begin{aligned}
S_h(t) &= S_{h0} + pS_{h1} + p^2 S_{h2} + \ldots\ldots\\
I_h(t) &= I_{h0} + pI_{h1} + p^2 I_{h2} + \ldots\ldots\\
R_h(t) &= R_{h0} + pR_{h1} + p^2 R_{h2} + \ldots\ldots\\
S_v(t) &= S_{v0} + pS_{v1} + p^2 S_{v2} + \ldots\ldots\\
I_v(t) &= I_{v0} + pI_{v1} + p^2 I_{v2} + \ldots\ldots
\end{aligned} \tag{3.4}$$

Considering (3.4) in (3.3), and comparing the same coefficient, we obtain,

$$\begin{aligned}
LS_{h_1} &= b_1 N_h - \mu_h Sh_0 - \tfrac{\beta_2 S_{h0} I_{v0}}{N_v} - \tfrac{\beta_1 S_{h0} I_{h0}}{N_h} + \lambda_h R_{h0} - LS_{h0},\\
LI_{h_1} &= \tfrac{\beta_2 S_{h0} I_{v0}}{N_v} + \tfrac{\beta_1 S_{h0} I_{h0}}{N_h} - \mu_h I_{h0} - \delta_h I_{h0} - \gamma_h I_{h0} - LI_{h0},\\
LR_{h_1} &= \gamma_h I_{h0} - \mu_h R_{h0} - \lambda_h R_{h0} - LR_{h0},\\
LS_{v_1} &= b_2 N_v - \gamma_v S_{v0} - \tfrac{\beta_3 S_{v0} I_{h0}}{N_h} - LS_{v0},\\
LI_{v_1} &= \tfrac{\beta_3 S_{v0} I_{h0}}{N_h} - \gamma_v I_{vo} - \delta_v I_{v0} - LI_{v0},
\end{aligned} \tag{3.5}$$

and

$$\begin{aligned}
LS_{h2} &= \mu_h S_{h1} - \tfrac{\beta_2 S_{h1} I_{v0}}{N_v} - \tfrac{\beta_1 S_{h0} I_{h1}}{N_h} - \tfrac{\beta_1 S_{h1} I_{h0}}{N_h} - \tfrac{\beta_2 S_{h0} I_{v1}}{N_v},\\
LI_{h2} &= \tfrac{\beta_2 S_{h1} I_{v0}}{N_v} + \tfrac{\beta_2 S_{h0} I_{v1}}{N_v} + \tfrac{\beta_1 S_{h0} I_{h1}}{N_h} + \tfrac{\beta_1 S_{h1} I_{h0}}{N_h} - \mu_h I_{h1} - \delta_h I_{h1} - \gamma_h I_{h1},\\
LR_{h2} &= \gamma_h I_{h1} - \mu_h R_{h1} - \lambda_h R_{h1},\\
LS_{v2} &= -\gamma_v S_{v1} - \tfrac{\beta_3 S_{v0} I_{h1}}{N_h} - \tfrac{\beta_3 S_{v1} I_{h0}}{N_h},\\
LI_{v2} &= \tfrac{\beta_3 S_{v1} I_{h0}}{N_h} + \tfrac{\beta_3 S_{v0} I_{h1}}{N_h} - \gamma_v I_{v1} - \delta_v I_{v1}.
\end{aligned} \tag{3.6}$$

In order to obtain the solution of the zeroth order problem, we consider the following cases.

Zeroth Order Problem or $\mathbf{P}^0$

$$S_{h0} = 130,\ I_{h0} = 90,\ R_{h0} = 70,\ S_{v0} = 150,\ I_{v0} = 60,\ N_h = 290,\ N_v = 210. \tag{3.7}$$

First Order Problem or $\mathbf{P}^1$

$$\begin{aligned}
S_{h_1} &= (b_1 N_h - \mu_h Sh_0 - \tfrac{\beta_2 S_{h0} I_{v0}}{N_v} - \tfrac{\beta_1 S_{h0} I_{h0}}{N_h} + \lambda_h R_{h0})t,\\
I_{h_1} &= (\tfrac{\beta_2 S_{h0} I_{v0}}{N_v} + \tfrac{\beta_1 S_{h0} I_{h0}}{N_h} - \mu_h I_{h0} - \delta_h I_{h0} - \gamma_h I_{h0})t,\\
R_{h_1} &= (\gamma_h I_{h0} - \mu_h R_{h0} - \lambda_h R_{h0})t,\\
S_{v_1} &= (b_2 N_v - \gamma_v S_{v0} - \tfrac{\beta_3 S_{v0} I_{h0}}{N_h})t,\\
I_{v_1} &= (\tfrac{\beta_3 S_{v0} I_{h0}}{N_h} - \gamma_v I_{vo} - \delta_v I_{v0})t.
\end{aligned} \tag{3.8}$$

Second Order Problem or $\mathbf{P}^2$

$$\begin{aligned}
S_{h2} = \Big[\mu_h(b_1 N_h - \mu_h Sh_0 - \tfrac{\beta_2 S_{h0} I_{v0}}{N_v} - \tfrac{\beta_1 S_{h0} I_{h0}}{N_h} + \lambda_h R_{h0}) - \tfrac{\beta_2}{N_v}(b_1 N_h - \mu_h Sh_0\\
- \tfrac{\beta_2 S_{h0} I_{v0}}{N_v} - \tfrac{\beta_1 S_{h0} I_{h0}}{N_h} + \lambda_h R_{h0}) I_{v0} - \tfrac{\beta_1 S_{h0}}{N_v}(\tfrac{\beta_2 S_{h0} I_{v0}}{N_v} + \tfrac{\beta_1 S_{h0} I_{h0}}{N_h}\\
- \mu_h I_{h0} - \delta_h I_{h0} - \gamma_h I_{h0}) - \tfrac{\beta_1 I_{h0}}{N_h}(b_1 N_h - \mu_h Sh_0 - \tfrac{\beta_2 S_{h0} I_{v0}}{N_v} - \tfrac{\beta_1 S_{h0} I_{h0}}{N_h} + \lambda_h R_{h0})\\
- \tfrac{\beta_2 S_{h0}}{N_v}(\tfrac{\beta_3 S_{v0} I_{h0}}{N_h} - \gamma_v I_{vo} - \delta_v I_{v0})\Big] t^2/2,
\end{aligned} \tag{3.9}$$

$$I_{h2} = \Bigg[\frac{\beta_2}{N_v}(b_1N_h-\mu_hSh_0-\frac{\beta_2S_{h0}I_{v0}}{N_v}-\frac{\beta_1S_{h0}I_{h0}}{N_h}+\lambda_hR_{h0})I_{v0}+\frac{\beta_1S_{h0}}{N_v}(\frac{\beta_2S_{h0}I_{v0}}{N_v}$$
$$+\frac{\beta_1S_{h0}I_{h0}}{N_h}-\mu_hI_{h0}-\delta_hI_{h0}-\gamma_hI_{h0})+\frac{\beta_1I_{h0}}{N_h}(b_1N_h-\mu_hSh_0-\frac{\beta_2S_{h0}I_{v0}}{N_v}-\frac{\beta_1S_{h0}I_{h0}}{N_h}$$
$$+\lambda_hR_{h0})+\frac{\beta_2S_{h0}}{N_v}(\frac{\beta_3S_{v0}I_{h0}}{N_h}-\gamma_vI_{vo}-\delta_vI_{v0})+(\mu_h+\delta_h+\gamma_h)$$
$$(\frac{\beta_2S_{h0}I_{v0}}{N_v}+\frac{\beta_1S_{h0}I_{h0}}{N_h}-\mu_hI_{h0}-\delta_hI_{h0}-\gamma_hI_{h0})\Bigg]t^2/2, \quad (3.10)$$

$$R_{h2} = \Bigg[\gamma_h(\frac{\beta_2S_{h0}I_{v0}}{N_v}+\frac{\beta_1S_{h0}I_{h0}}{N_h}-\mu_hI_{h0}-\delta_hI_{h0}-\gamma_hI_{h0})$$
$$-(\mu_h+\lambda_h)(\gamma_hI_{h0}-\mu_hR_{h0}-\lambda_hR_{h0})\Bigg]t^2/2, \quad (3.11)$$

$$S_{v2} = \Bigg[-\gamma_v(b_2N_v-\gamma_vS_{v0}-\frac{\beta_3S_{v0}I_{h0}}{N_h})-\frac{\beta_3S_{v0}}{N_h}(\frac{\beta_2S_{h0}I_{v0}}{N_v}+\frac{\beta_1S_{h0}I_{h0}}{N_h}$$
$$-\mu_hI_{h0}-\delta_hI_{h0}-\gamma_hI_{h0})-\frac{\beta_3I_{h0}}{N_h}(b_2N_v-\gamma_vS_{v0}-\frac{\beta_3S_{v0}I_{h0}}{N_h})\Bigg]t^2/2, \quad (3.12)$$

$$I_{v2} = \Bigg[\frac{\beta_3S_{v0}}{N_h}(\frac{\beta_2S_{h0}I_{v0}}{N_v}+\frac{\beta_1S_{h0}I_{h0}}{N_h}-\mu_hI_{h0}-\delta_hI_{h0}-\gamma_hI_{h0})+\frac{\beta_3I_{h0}}{N_h}(b_2N_v$$
$$-\gamma_vS_{v0}-\frac{\beta_3S_{v0}I_{h0}}{N_h})-(\gamma_v+\delta_v)(\frac{\beta_3S_{v0}I_{h0}}{N_h}-\gamma_vI_{vo}-\delta_vI_{v0})\Bigg]t^2/2. \quad (3.13)$$

To find the solution we consider $p = 1$ in the system (3.4), we get

$$\begin{aligned}
S_h(t)&=S_{h0}+S_{h1}+S_{h2}+.....\\
I_h(t)&=I_{h0}+I_{h1}+2I_{h2}+.....\\
R_h(t)&=R_{h0}+R_{h1}+2R_{h2}+.....\\
S_v(t)&=S_{V0}+S_{V1}+2S_{V2}+.....\\
I_v(t)&=I_{V0}+I_{V1}+2I_{V2}+.....
\end{aligned} \quad (3.14)$$

$$S_h(t) = S_{h0} + (b_1N_h - \mu_h Sh_0 - \frac{\beta_2 S_{h0}I_{v0}}{N_v} - \frac{\beta_1 S_{h0}I_{h0}}{N_h} + \lambda_h R_{h0})t$$
$$+ \Big[\mu_h(b_1N_h - \mu_h Sh_0 - \frac{\beta_2 S_{h0}I_{v0}}{N_v} - \frac{\beta_1 S_{h0}I_{h0}}{N_h} + \lambda_h R_{h0})$$
$$- \frac{\beta_2}{N_v}(b_1N_h - \mu_h Sh_0 - \frac{\beta_2 S_{h0}I_{v0}}{N_v} - \frac{\beta_1 S_{h0}I_{h0}}{N_h} + \lambda_h R_{h0})I_{v0} - \frac{\beta_1 S_{h0}}{N_v}(\frac{\beta_2 S_{h0}I_{v0}}{N_v}$$
$$+ \frac{\beta_1 S_{h0}I_{h0}}{N_h} - \mu_h I_{h0} - \delta_h I_{h0} - \gamma_h I_{h0}) - \frac{\beta_1 I_{h0}}{N_h}(b_1N_h - \mu_h Sh_0 - \frac{\beta_2 S_{h0}I_{v0}}{N_v}$$
$$- \frac{\beta_1 S_{h0}I_{h0}}{N_h} + \lambda_h R_{h0}) - \frac{\beta_2 S_{h0}}{N_v}(\frac{\beta_3 S_{v0}I_{h0}}{N_h} - \gamma_v I_{vo} - \delta_v I_{v0})\Big]t^2/2, \quad (3.15)$$

$$I_h(t) = I_{h0} + (\frac{\beta_2 S_{h0}I_{v0}}{N_v} + \frac{\beta_1 S_{h0}I_{h0}}{N_h} - \mu_h I_{h0} - \delta_h I_{h0} - \gamma_h I_{h0})t$$
$$+ \Big[\frac{\beta_2}{N_v}(b_1N_h - \mu_h Sh_0 - \frac{\beta_2 S_{h0}I_{v0}}{N_v} - \frac{\beta_1 S_{h0}I_{h0}}{N_h} + \lambda_h R_{h0})I_{v0} + \frac{\beta_1 S_{h0}}{N_v}(\frac{\beta_2 S_{h0}I_{v0}}{N_v}$$
$$+ \frac{\beta_1 S_{h0}I_{h0}}{N_h} - \mu_h I_{h0} - \delta_h I_{h0} - \gamma_h I_{h0}) + \frac{\beta_1 I_{h0}}{N_h}(b_1N_h - \mu_h Sh_0 - \frac{\beta_2 S_{h0}I_{v0}}{N_v}$$
$$- \frac{\beta_1 S_{h0}I_{h0}}{N_h} + \lambda_h R_{h0}) + \frac{\beta_2 S_{h0}}{N_v}(\frac{\beta_3 S_{v0}I_{h0}}{N_h} - \gamma_v I_{vo} - \delta_v I_{v0}) + (\mu_h + \delta_h + \gamma_h)$$
$$(\frac{\beta_2 S_{h0}I_{v0}}{N_v} + \frac{\beta_1 S_{h0}I_{h0}}{N_h} - \mu_h I_{h0} - \delta_h I_{h0} - \gamma_h I_{h0})\Big]t^2/2, \quad (3.16)$$

$$R_h(t) = R_{h0} + (\gamma_h I_{h0} - \mu_h R_{h0} - \lambda_h R_{h0})t + \Big[\gamma_h(\frac{\beta_2 S_{h0}I_{v0}}{N_v} + \frac{\beta_1 S_{h0}I_{h0}}{N_h}$$
$$- \mu_h I_{h0} - \delta_h I_{h0} - \gamma_h I_{h0}) - (\mu_h + \lambda_h)(\gamma_h I_{h0} - \mu_h R_{h0} - \lambda_h R_{h0})\Big]t^2/2, \quad (3.17)$$

$$S_v(t) = S_{v0} + (b_2N_v - \gamma_v S_{v0} - \frac{\beta_3 S_{v0}I_{h0}}{N_h})t + \Big[-\gamma_v(b_2N_v - \gamma_v S_{v0} -$$
$$\frac{\beta_3 S_{v0}I_{h0}}{N_h}) - \frac{\beta_3 S_{v0}}{N_h}(\frac{\beta_2 S_{h0}I_{v0}}{N_v} + \frac{\beta_1 S_{h0}I_{h0}}{N_h} - \mu_h I_{h0} - \delta_h I_{h0} - \gamma_h I_{h0})$$
$$- \frac{\beta_3 I_{h0}}{N_h}(b_2N_v - \gamma_v S_{v0} - \frac{\beta_3 S_{v0}I_{h0}}{N_h})\Big]t^2/2, \quad (3.18)$$

$$I_v(t) = I_{v0} + (\frac{\beta_3 S_{v0} I_{h0}}{N_h} - \gamma_v I_{vo} - \delta_v I_{v0})t + \Big[\frac{\beta_3 S_{v0}}{N_h}(\frac{\beta_2 S_{h0} I_{v0}}{N_v} + \frac{\beta_1 S_{h0} I_{h0}}{N_h}$$
$$- \mu_h I_{h0} - \delta_h I_{h0} - \gamma_h I_{h0}) + \frac{\beta_3 I_{h0}}{N_h}(b_2 N_v - \gamma_v S_{v0} - \frac{\beta_3 S_{v0} I_{h0}}{N_h})$$
$$- (\gamma_v + \delta_v)(\frac{\beta_3 S_{v0} I_{h0}}{N_h} - \gamma_v I_{vo} - \delta_v I_{v0})\Big] t^2/2. \quad (3.19)$$

TABLE 1. Description of parameter and its value.

Notation	Description of Parameters	Values
μ_h	A natural death rate of a human	0.019
δ_h	Disease death rate of a human	0.725
λ_h	The rate at which the individuals become susceptible again	1.438
β_1	Direct transmission between susceptible human and infected human	1.058
β_2	Transmission between susceptible human and infected vector	0.173
β_3	Transmission between susceptible vector and infected human	0.984
δ_v	Disease death rate of Vector	0.954
γ_h	A recovery rate of infection of human	0.198
b_2	Birth rate for vector population	0.485
γ_v	Natural death rate of vector	0.755
b_1	Recruitment rate of human population	0.012

4. NUMERICAL RESULTS AND DISCUSSION

In this section, we find the numerical simulation of the model (3.1) by using HPM, and the results are compared with others standards methods NSFD and RK4. The results obtained from HPM have good agreement with NSFD and RK4. The values of the parameters used in the numerical simulations are presented in Table 1. Figure 1 represents the population of susceptible vector. Figure 2 represents the infected vector and Figure 3 represents the population of susceptible individuals. Figure 4 represents the infected human and Figure 5 represents the population of recovered human.

FIGURE 1. The plot represents the population of susceptible vector in the model.

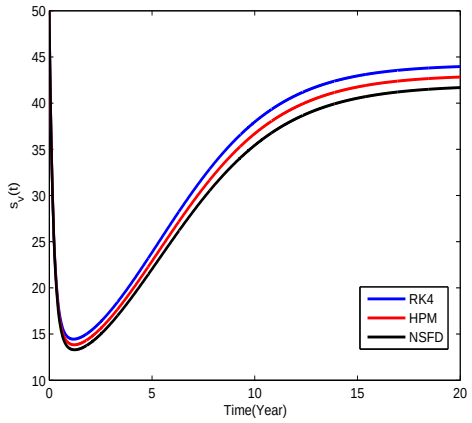


FIGURE 2. The plot shows the population of infected vector in the model.

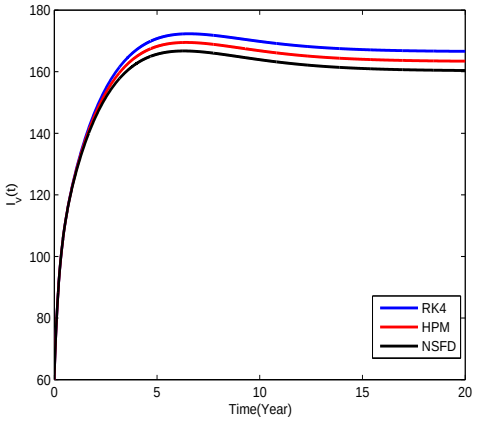


FIGURE 3. The plot represents the population of susceptible human in the model.

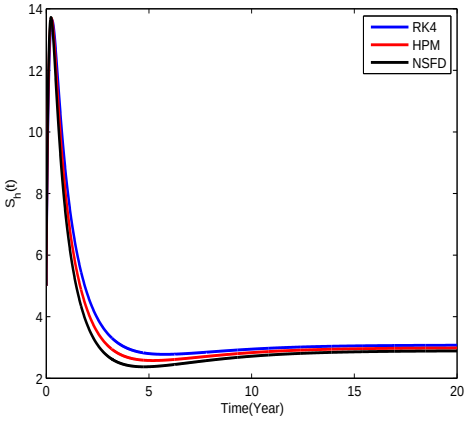


FIGURE 4. The plot represents the population of infected human in the model.

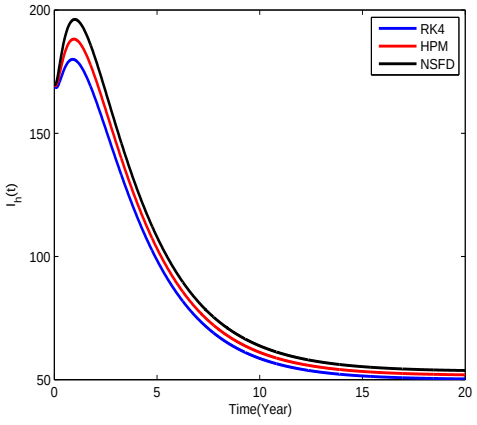
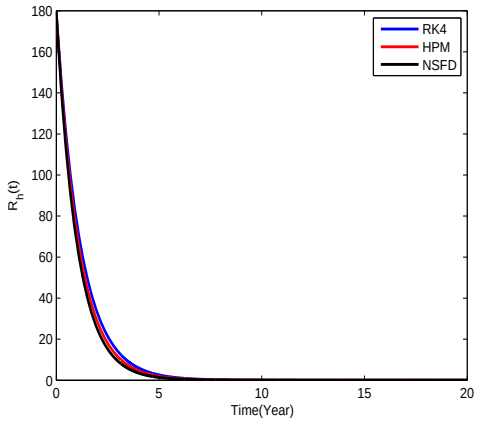


FIGURE 5. The plot shows the population of recovered human in the model.



5. CONCLUSION

In this article, the solution of leptospirosis epidemic model is accomplished. We used a semi analytical approach for the solution of proposed model, that is Homotopy Perturbation Method. We obtained the solution by Homotopy Perturbation Method and compared the results with Runge-Kutta fourth order and NSFD method, which shows that Homotopy Perturbation Method is a powerful technique for the solution of such type of nonlinear epidemic models.

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