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A Hierarchical method to solve one machine multicriteria sequencing problem

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Abstract

The problem of minimizing a function of three criteria maximum earliness, total of square completion times, and total lateness in a hierarchical (lexicographical) method is proposed in this article. On one machine, n independent tasks (jobs) must planned. It is always available starting at time zero and can only do the mono task (job) at a time period. Processing for the task (job) $j(j = 1, 2, ..., nj)$ is necessary meantime the allotted positive implementation time p_{tj}. For the problem of three criteria maximization earliness, a total of square completion times, and total lateness in a hierarchy instance, the access of limitation that which is the desired sequence is held out. The Generalized Least Deviation Method (GLDM) and a robust technique for analyzing historical data to project future trends are analyzed.

Keywords. Sequencing with one machine, Hierarchical, Lexicographic format, Square completion times, Multicriteria.
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1. Introduction

In real-world situations, making decisions is often complicated by competing standards. Making decisions grows increasingly challenging as the number of constraints rises. Modeling and developing sequence techniques has always been a challenge for operations researchers. Several techniques and formulations have been developed for various kinds of problems [25]. Each duty, sometimes referred to as a task, consists of a basic sequencing challenge, and an execution time on one of the machines capable of carrying it out, Of course, it should be done in a way that guarantees the result at the end. Referred to as sequencing, is ideal, satisfies all side constraints, or minimizes the given objective function [13]. Sequencing theory was developed to overcome problems with, for instance, nurse sequencing [15].

The one machine example is taken into consideration in this work because it provides a useful laboratory for the development of concepts for heuristics and interactive methods that may be useful in broader models.

There are two methods for handling multi-criteria problems: the hierarchical approach and the simultaneous approach. The method based on hierarchy. One of the two criteria is the major criterion, while the other is the secondary criterion. The secret must be to minimize the first performance measurement while using the lowest second performance measurement value to defeat similarity in preference sequencing. The simultaneous approach considers two standards at the same time. This method usually generates all possible sequencings and selects the optimal one according to the values of the assembly goal function for both criteria. Most problems using multiple criteria sequencing are NP-hard [1]. Evolutionary algorithms (EAs) have emerged as a strength optimizing toolset to tackle sequencing issues [7, 8]. Erne [9] offered an integer programming model heuristic method for minimizing the weighted sum of total completion time, maximum tardiness, and maximum earliness for a sequencing problem with many criteria and sequencing-dependent setup time. Nelson et al. [24] provided many sequences for the three-criteria problems, flow time g, maximum tardiness Tmax, and a number of tardy jobs nT, using mean algorithms.

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Hoogeveen [12] offered a technique for reducing the growing measure of R regularity functions. For the multi-criteria problem $1/F(C_j, \sum T_j, L_{max})$ [3] provided an efficient approach for discovering the set of all efficient sequences. Research on multi-criteria decision-making problems is extensively covered in [6]. Approximate techniques and mathematical programming are employed to handle multi-criteria decision making problems [14]. Using a hierarchical approach, [4] presented a multi-criteria problem. The multi-criteria problem is solved using a modified branch and bound technique in [5].

Sequencing σ establishes the completion time $C_{tj}(\sigma)$ for each job j so that the jobs do not execute concurrently. The penalty function g_j calculates the freckle set back accomplish j at period C_{tj}. The totall square completing period $\sum C_{tj}^2$ and maximization costing g_{max}, where maximization costing means $g_j(C_{tj})$, is minimized hierarchically (C_{tj}), is the multi criteria problem in this article, where g_j stands for a cost function, either regular or irregular, routine implies that $g_j(C_{tj})$ does not disappear as C_{tj} rises, adapt Tt_{max}, $\sum L_{tj}, \sum C_{tj}^2$. If not, a measure said to be not regulator, like Et_{max}.

The fundamental planning issue can be portrayed as finding for each of the assignments, which are too called occupations, an execution interim on one of the machines that are able to execute it, such that all side limitations are met. Obviously, this ought to be wiped out such a way that the resulting solution, which is called a plan, is best conceivable, so , it minimizes the given objective work. A planning hypothesis has been created to illuminate issues happening in for occurrence nurture planning.

There are two approaches for the multicriteria issues; the various leveled and the concurrent approaches. Within the various leveled approach, one of the two criteria is considered as the essential basis and the other one as the auxiliary criterion. The problem is to play down the essential basis whereas breaking ties in favor of the schedule has the least auxiliary measure esteem. Within the synchronous approach, two criteria are considered at the same time. This approach regularly produces all effective plans and chooses the one that yields the finest composite objective work esteem of the two criteria. Most multicriteria planning issues are NP-hard in nature. A long time, as an effective optimization device, developmental calculations (EAs) have been presented to illuminate the arrange planning issues.

Within the generation division, planning can be in a more extensive point of view characterized as a preparation of organizing, controlling, and optimizing work or workloads with respect to finding the ideal plan for a particular structure and generation framework conditions, the planning is considered a complex combinatorial optimization issue, generally demonstrated of NP-hard sort. Correct optimization strategies are primarily utilized as they were for the frameworks which have a particular topology where exceptionally solid disentangling presumptions must be utilized, so they are not as well pertinent in a real-world situation for more complex frameworks. In that case, surmised optimization strategies and metaheuristics based on stochastic nearby look approach, machine learning procedures, particularly manufactured neural systems (ANN), fluffy rationale strategies, and master frameworks, are at the center of investigation intrigued to discover ideal or near-optimal arrangements rather than correct scientific optimization models.

In differentiate to other strategies, dispatching rules (we moreover utilize the term need rules all through the taking after content) speak to the profitable viable and overwhelming approach of the shop floor control within the complex industry environment, such as, e.g., in semiconductor fabricating for fathoming complex planning issues in real-time. Need rules are well known since they are characterized by the effortlessness of usage, palatable execution, and a significantly diminished computational prerequisite. By the by, the choice of appropriate dispatching rules isn't a unimportant assignment and depends on the significant key execution pointers.

The impacts created by the chosen need to run the show are for the most part troublesome to clarify by expository strategies, in this way the recreation is utilized exceptionally frequently to assess the plan effectiveness within the complex planning issue. As an outline, within the recreation consider, Vinod and Sridharan [26] assessed the execution measures based on stream time and lateness of occupations for the distinctive combinations of due-date task strategies and seven planning choice rules connected in a dynamic job shop framework. Xanthopoulos et al. [27] compared seventeen dispatching rules within the consider centered on stochastic energetic planning issues with sequence-dependent setups. Execution measures were cruel work-in-progress, cruel cycle time, cruel lateness, and a division of late employments.

Authors of [22] explained the generalized fifth-order KdV like equation with prime number $p = 3$ via a generalized bilinear differential operator. N-lump was investigated to the variable-coefficient Caudrey-Dodd-Gibbon-Kotera-Sawada equation [21]. Applications of $tan(\phi/2)$-expansion method for the Biswas-Milovic equation [18], the Gerdjikov-Ivanov model [20], the Kundu-Eckhaus equation [19] and the fifth-order integrable equations [16] were studied. Lump solutions were analyzed to the fractional generalized CBS-BK equation [28] and the (3+1)-D Burger system [10]. The approximations of one-dimensional hyperbolic equation with non-local integral conditions were constructed by reduced differential transform method [23]. The generalized Hirota bilinear strategy by the number prime was used to the (2+1)-dimensional generalized fifth-order KdV like equation [22]. The traveling wave solutions and analytical treatment of the simplified MCH equation and the combined KdV-mKdV equations were studied [2].

The structure of this paper is given as under: This paper is formed because the section 2 contains the exponent taking fundamental ideas and related results which are thoroughly crucial to know the novelty of this paper. In section 3, we investigate $1//Lex(Et_{max}, \sum_{j=1}^{n}(C_{t_j})^2, \sum_{j=1}^{n} L_{tj})$ problem. Generalized Least Deviation Method Description is discussed in section 4. In addition, soft computing results is provided in section 5. Finally, we approach some kind of results and conclusion in the sixth section.

2. Exponent taking fundamental ideas

Tasks (jobs) $j(j = 1, 2, ..., n)$ have been performed on a one machine in this study using the notation.

N_j: tasks collection.

n: the number of tasks (jobs) in given sequencing.

P_{t_j}: operationally time for task (job) j.

D_{t_j}: the period where the task (job) j has to perfectly completing.

$\overline{Dt_j}$: baseline for task (job) j.

Ct_j: the completing period of task (job) j.

C^2t_j: the square completing period of task (job) j.

$Ct_1 = pt_1$

$Ct_j = Ct_{(j-1)} + pt_j, j = 2, \cdots, n.$

$s_j = Dt_j - pt_j$: the slacked period of task (job) j.

$Lt_j = Ct_j - Dt_j$: the lately of task (job) j.

$Et_j = max0, Dt_j - Ct_j$: the earliness (premature) of task (job) j.

$\sum C^2t_j$: totally completing period.

$E_{tmax} = max_j Et_j$: maximization earliness (premature).

$\sum Lt_j$: totally lately tasks.

SPTO= Sequencing the tasks (jobs) in non-decreasing order of processing time, the Rule of the least processing time is used.
EDDO= The earliest due date order rule is applied by sequencing the jobs in non-decreasing order of their due dates.

Theorem 2.1. *[17] The following minimizes the $1/g_{max}$ problem: If there are any unassigned jobs, allocate the one with the lowest cost to the final unassigned spot on the timetable.*

Theorem 2.2. *[11] The $1/Et_{max}$ problem is resolved by executing the jobs in a non-decreasing order of $Dt_j - pt_j$ in accordance with the minimum slack time (MSTO) criteria.*

Definition 2.3. [12] Minimization in a hierarchy: The order of relevance for the performance criteria $g_1, g_2, ..., g_k$ is indexed in decreasing order. First, g_1 is reduced, next, g_2 is reduced. provided that the sequencing has a minimum g_1 value, if required, g_1 and g_2 must have values that are equal to those found in the previous stage in order for g_3 to be minimized.

3. The $1/Lex(Et_{max}, \sum_{j=1}^{n}(C_{t_j})^2, \sum_{j=1}^{n} L_{tj})$ problem

This problem (issue) can be defined as follows:

$$\begin{cases} Min \sum_{j=1}^{n} L_{tj}, \\ S.t., \\ Et_{max} = Et^*, Et^* = Et_{max}(MSTO), \\ \sum_{j=1}^{n}(C_{t_j})^2 \leq Ct^*, Ct^* \in [\sum_{j=1}^{n}(C_{t_j})^2(SPTO), \sum_{j=1}^{n}(C_{t_j})^2(MSTO)]. \end{cases} \tag{3.1}$$

Given that Et_{max} is the most crucial function in this problem (3.1) and should be at its best, next way EtCtLt algorithm provides desired solution (outcome).

Algorithm 1 Algorithm (EtCtLt).

Move 1: Solving $\underline{Et_{\max}}$ problem for finding Et^*.
Move 2: Ascertain $\overline{Dt_j} = Dt_j + Et^*$ for all $j \in N_j$, where $N_j = \{1, 2, \ldots, n\}$.
Move 3: Let $h = \sum_{j=1}^{n} p_{tj}$.
Move 4: Find the task (job) $j^* \in N_j$ that verifies $\overline{Dt_{j^*}} \leq h$ (choose the task with the minimum processing time if there is a tie, and if there is still a tie, select the task with the earliest baseline).
Move 5: Set $h = h - p_{tj^*}$, $r = r + 1$, $N_j = N_j \setminus \{j^*\}$, and update the sequence $\sigma = (\sigma, \sigma(r))$. If $N_j = \emptyset$, go to Move 6; otherwise, go to Move 4.
Move 6: In the sequence σ, compute $Et_{\max}$, $\sum_{j=1}^{n}(Ct_j)^2$, and $\sum_{j=1}^{n} L_{tj}$.

Example 3.1. Considering the problem (3.1) with the following inputs.

j	1	2	3	4	5
P_{t_j}	6	3	7	10	10
D_{t_j}	7	15	17	11	10

$E^* = 0, h = 36.$
$\overline{D_{t_1}} = 7, \quad \overline{D_{t_2}} = 15, \quad \overline{D_{t_3}} = 17, \quad \overline{D_{t_4}} = 11, \quad \overline{D_{t_5}} = 10$

r	h	t^*
$r1$	36	$2j$
$r2$	33	$1j$
$r3$	27	$3j$
$r4$	20	$5j$
$r5$	10	$4j$

Sequencing $(2j, 1j, 3j, 5j, 4j)$ getting $(Et_{max}, \sum_{j=1}^{n}(C_{tj})^2, \sum_{j=1}^{n} L_{tj}) = (12, 2318, 31)$ based on an algorithm (EtCtLt).

4. Laplace transform

Laplace transform $F(s)$ is given as

$$F(s) = L(f(x)) = \int_0^\infty f(x)e^{-sx}dx, \tag{4.1}$$

a function $f(x)$ determined for $0 < x < l$ and at least for those s for which the integral converges. On the interval $[0, 1)$, $f(x)$ be a continuous function which is of exponential order, that is, for some and $x > 0$

$$\sup \frac{|f(x)|}{e^{cx}} < \infty. \tag{4.2}$$

In this case the Laplace transform (4.1) exists for all $s > c$. The Laplace transforms of Caputo fraction derivatives for $m - 1 < \alpha \leq m, m \in N$

$$L[D^\alpha f(x)] = \frac{s^m F(s) - s^{m-1}F(0) - s^{m-2}F'(0) - ... - f^{(m-1)}(0)}{s^{m-\alpha}}. \tag{4.3}$$

We can change FDEs into algebraic equations. Then, we can achive the unknown Laplace function $F(s)$ by solving this algebraic equations.

4.1. **Inverse Laplace transform.**

The Inverse Laplace Transform of $F(s)$ is given as:

$$f(x) = L^{-1}[F(s)] = \frac{1}{2\pi i} \lim_{T\to\infty} \int_{T-i\sigma}^{T+i\sigma} e^{sx}F(s)ds, \tag{4.4}$$

where σ is large enough that $F(s)$ is determined for the real part of $s \geq \sigma$. Also, we define special functions Mittag-Leffler functions and the generalized Mittag-Leffler functions which used in fractional calculus.

For $\alpha, \beta > 0$ and $z \in C$

$$E_\alpha(z) = \sum_{n=0}^\infty \frac{z^n}{\Gamma(n\alpha + 1)}, \tag{4.5}$$

$$E_{\alpha,\beta}(z) = \sum_{n=0}^\infty \frac{z^n}{\Gamma(n\alpha + \beta)}. \tag{4.6}$$

Theorem 4.1. *(Schouten-Van der Pol Theorem): Consider a function $f(x)$ which has the Laplace transform $F(s)$ which is analytic in the half-plane $Re(s) > s_0$. We can use this knowledge to find $g(x)$ whose Laplace transform $G(s)$ equals $F(\varphi(s))$, where $\varphi(s)$ is also analytic for $Re(s) > s_0$. This means that if*

$$G(s) = F(\varphi(s)) = \int_0^\infty f(\tau)\exp(-\varphi(s)\tau)d\tau = Lf(\tau); s \to \varphi(s),$$

and

$$g(x) = \frac{1}{2\pi i}\int_{c-i\sigma}^{c+i\sigma} e^{sx}F(\varphi(s))ds = L^{-1}[F(\varphi(s))],$$

then

$$g(x) = \int_0^\infty f(\tau)\left(\frac{1}{2\pi i}\int_{c-i\sigma}^{c+i\sigma} \exp(-\varphi(s)\tau)e^{sx}ds\right)d\tau$$

$$= \int_0^\infty f(\tau)\left(L^{-1}[\exp(-\varphi(s)\tau)]\right)d\tau.$$

Theorem 4.2. *Let $F(s)$ be analytic function in some half -plane. The condition $F(s) = O(|s|^k)$, k an integer more large or equal zero, in this half-plane (with the possible exception of a circular disk centered at the origin) is necessary and sufficient to assert that $F(s)$ can not be represented as L-transform of some distribution T of D_0'. We now present some interesting applications of the above two theorems.*

Lemma 4.3. *The following relationships hold true*

1-

$$L^{-1}\{e^{-t\sqrt{4s^{2\alpha}-5}}\} = L^{-1}\{e^{-2t\sqrt{s^{2\alpha}-\frac{5}{4}}}\} = \int_0^\infty e^{\frac{5}{4}\tau}\frac{t}{\sqrt{\pi\tau^3}}e^{-\frac{t^2}{\tau}}\left(\sum_{n=0}^{\infty}\frac{(-\tau)^n x^{-2\alpha n-1}}{n!\Gamma(-2\alpha n)}\right)d\tau$$

$$= \frac{t}{\sqrt{\pi}}\sum_{n=0}^{\infty}\frac{(-1)^n x^{-2\alpha n}}{n!\Gamma(-2\alpha n)}\int_0^\infty \tau^{n-\frac{3}{2}}e^{\frac{5}{4}\tau-\frac{t^2}{\tau}}d\tau := F(x,t).$$

2-

$$L^{-1}\{\frac{e^{-t\sqrt{4s^{2\alpha}-5}}}{s-1}\} = \int_0^x e^{x-u}F(u,t)du.$$

3-

$$L^{-1}\{\frac{s^{2\alpha-2}e^{-t\sqrt{4s^{2\alpha}-5}}}{1-s^{2\alpha}}\} = -\int_0^x (x-u)E_{2\alpha,2}((x-u)^{2\alpha})F(u,t)du.$$

where E_α is Mittag-Leffler function.

Proof. 1-To calculate the inverse

$$L^{-1}\{e^{-t\sqrt{4s^{2\alpha}-5}}\} = L^{-1}\{e^{-2t\sqrt{s^{2\alpha}-\frac{5}{4}}}\}.$$

We solve the following problem

$$L^{-1}\{e^{-2t\sqrt{s-\frac{5}{4}}}\}.$$

We know

$$L^{-1}\{e^{-a\sqrt{s}}\} = \frac{a}{2\sqrt{\pi x^3}}e^{-\frac{a^2}{4x}}\},$$

then

$$L^{-1}\{e^{-2t\sqrt{s}}\} = \frac{2t}{2\sqrt{\pi x^3}}e^{-\frac{4t^2}{4x}} = \frac{t}{\sqrt{\pi x^3}}e^{-\frac{t^2}{x}}\}.$$

So, by applying the shifting theorem of Laplace transforms

$$L^{-1}\{e^{-2t\sqrt{s-\frac{5}{4}}}\} = e^{\frac{5}{4}x}\frac{t}{\sqrt{\pi x^3}}e^{-\frac{t^2}{x}}.$$

Now we must obtain the inversion

$$L^{-1}\{e^{-t\sqrt{4s^{2\alpha}-5}}\} = L^{-1}\{e^{-2t\sqrt{s^{2\alpha}-\frac{5}{4}}}\}.$$

for this work, we can apply Theorem 4.1.

By setting

$$F(s) = e^{-2t\sqrt{s-\frac{5}{4}}}, \quad f(\tau) = e^{\frac{5}{4}\tau}\frac{t}{\sqrt{\pi\tau^3}}e^{-\frac{t^2}{\tau}}.$$

And $\varphi(s) = s^{2\alpha}$, we get

$$L^{-1}\{e^{-t\sqrt{4s^{2\alpha}-5}}\} = L^{-1}\{e^{-2t\sqrt{s^{2\alpha}-\frac{5}{4}}}\} = \int_0^\infty e^{\frac{5}{4}\tau}\frac{t}{\sqrt{\pi\tau^3}}e^{-\frac{t^2}{\tau}}\left(L^{-1}\{e^{-\tau s^{2\alpha}}\}\right)d\tau.$$

To obtain $L^{-1}\{e^{-\tau s^{2\alpha}}\}$, we can apply Theorem 4.2 by considering $F(s) = \exp(-s^{\alpha})$, where $\alpha > 1$. Set $s = re^{i\theta}$ with $r = |s| > 0, -\pi < \theta < \pi$. Then $F(s) = \exp(-r^{\alpha}e^{i\alpha\theta})$ and $|F(s)| = \exp(-r^{\alpha}\cos(\alpha\theta))$. Since $\alpha > 1$ we can choose $0 < \theta < \frac{\pi}{2}$ such that $\cos(\alpha\theta) < 0$. This shows that $|F(s)|$ cannot be bounded by $K|s|^k$ in any half-plane $Rs > a$. Therefore by Theorem 4.2, $\exp(-s^{\alpha})$ is not a Laplace transform of a function. Thus for $0 < \alpha < \frac{1}{2}$, we have

$$L^{-1}\{e^{-\tau s^{2\alpha}}\} = L^{-1}\left\{\sum_{n=0}^{\infty}\frac{(-\tau)^n}{n!s^{-2\alpha n}}\right\} = \sum_{n=0}^{\infty}\frac{(-\tau)^n}{n!}\left[\frac{1}{s^{-2\alpha n}}\right] = \sum_{n=0}^{\infty}\frac{(-\tau)^n x^{-2\alpha n-1}}{n!\Gamma(-2\alpha n)}.$$

Therefore

$$\begin{aligned}L^{-1}\{e^{-t\sqrt{4s^{2\alpha}-5}}\} = L^{-1}\{e^{-2t\sqrt{s^{2\alpha}-\frac{5}{4}}}\} &= \int_0^{\infty} e^{\frac{5}{4}\tau}\frac{t}{\sqrt{\pi\tau^3}}e^{-\frac{t^2}{\tau}}\left(\sum_{n=0}^{\infty}\frac{(-\tau)^n x^{-2\alpha n-1}}{n!\Gamma(-2\alpha n)}\right)d\tau \\ &= \frac{t}{\sqrt{\pi}}\sum_{n=0}^{\infty}\frac{(-1)^n x^{-2\alpha n}}{n!\Gamma(-2\alpha n)}\int_0^{\infty}\tau^{n-\frac{3}{2}}e^{\frac{5}{4}\tau-\frac{t^2}{\tau}}d\tau := F(x,t).\end{aligned}$$

2- For obtaining

$$L^{-1}\{\frac{e^{-t\sqrt{4s^{2\alpha}-5}}}{s-1}\},$$

we can use the convolution theorem in Laplace transform as

$$L^{-1}\left\{\frac{e^{-t\sqrt{4s^{2\alpha}-5}}}{s-1}\right\} = L^{-1}\left\{\frac{1}{s-1} * L^{-1}\{e^{-t\sqrt{4s^{2\alpha}-5}}\}\right\} = e^x * F(x,t) = \int_0^x e^{x-u}F(u,t)du.$$

3- For solving

$$L^{-1}\left\{\frac{s^{2\alpha-2}e^{-t\sqrt{4s^{2\alpha}-5}}}{1-s^{2\alpha}}\right\}.$$

Since

$$L^{-1}\{\frac{s^{2\alpha-2}}{1-s^{2\alpha}}\} = -xE_{2\alpha,2}(x^{2\alpha}),$$

then, using the convolution theorem gives

$$L^{-1}\left\{\frac{s^{2\alpha-2}e^{-t\sqrt{4s^{2\alpha}-5}}}{1-s^{2\alpha}}\right\} = -\int_0^x (x-u)E_{2\alpha,2}((x-u)^{2\alpha})F(u,t)du.$$

□

5. Application of fractional Telegraph equation

In this section, the presented method in the paper is applied some linear fractional differential equations and it gives an exact solution. Take the following fractional Telegraph equation as

$$\frac{\partial^{2\alpha}u}{\partial x^{2\alpha}} = \frac{\partial^2 u}{\partial t^2} + \frac{\partial u}{\partial t} + u, \quad t \geq 0, \quad 0 < \alpha \leq 1, \tag{5.1}$$

$$u(0,t) = e^{-t}, \quad u(x,0) = e^{x}, \quad u_x(0,t) = e^{-t}, \quad t \geq 0, \quad 0 < x < 1. \tag{5.2}$$

As mentioned above, taking Laplace transform of both sides of (5.1) gives,

$$L\left[\frac{\partial^{2\alpha}u}{\partial x^{2\alpha}}\right] = s^{2\alpha}u(s,t) - s^{2\alpha-1}u_x(0,t) - s^{2\alpha-2}u(0,t). \tag{5.3}$$

Utilizing given the initial condition Eq. (5.1) gives

$$L\left[D_{xx}^{2\alpha}\right] = s^{2\alpha}u(s,t) - s^{2\alpha-1}e^{-t} - s^{2\alpha-2}e^{-t}.$$

But

$$L\left[\frac{\partial^2 u}{\partial t^2}\right] = \frac{\partial^2}{\partial t^2}[Lu(x,t)] = \frac{\partial^2 u(s,t)}{\partial t^2}.$$

Consequently, we have

$$s^{2\alpha}u(s,t) - s^{2\alpha-1}e^{-t} - s^{2\alpha-2}e^{-t} = \frac{\partial^2 u(s,t)}{\partial t^2} + \frac{\partial u(s,t)}{\partial t} + u(s,t).$$

Consequently, we have:

$$\frac{\partial^2 u(s,t)}{\partial t^2} + \frac{\partial u(s,t)}{\partial t} + u(s,t) - s^{2\alpha}u(s,t) = -s^{2\alpha-1}e^{-t} - s^{2\alpha-2}e^{-t}.$$

Hence

$$\frac{\partial^2 u(s,t)}{\partial t^2} + \frac{\partial u(s,t)}{\partial t} + (1 - s^{2\alpha})u(s,t) = -s^{2\alpha-2}(1+s)e^{-t}.$$

By solving the above equation as homogenous we get to characteristic equation

$$M^2 + M + (1 - s^{2\alpha}) = 0.$$

Hence, we have Hence

$$M = \frac{-1 \pm \sqrt{-1 - 4(1 - s^{2\alpha})}}{2} = \frac{-1 \pm \sqrt{-5 + 4s^{2\alpha}}}{2},$$

$$u_h = e^{-\frac{1}{2}t}\left[A(s)e^{t\sqrt{4s^{2\alpha}-5}} + B(s)e^{-t\sqrt{4s^{2\alpha}-5}}\right],$$

and

$$u_p = \frac{-1}{D_t^2 + D_t + (1 - s^{2\alpha})}[s^{2\alpha-2}(1+s)e^{-t}] = \frac{-e^{-t}}{1 - s^{2\alpha}}s^{2\alpha-2}(1+s).$$

Since the solution must bounded consequently we have

$$B(s) - \frac{1}{1 - s^{2\alpha}}s^{2\alpha-2}(1+s) = \frac{1}{s-1},$$

or

$$B(s) = \frac{1}{1 - s^{2\alpha}}s^{2\alpha-2}(1+s) + \frac{1}{s-1},$$

So that, we get

$$u(s,t) = e^{-\frac{1}{2}t}\left[\frac{1}{1-s^{2\alpha}}s^{2\alpha-2}(1+s)+\frac{1}{s-1}\right]e^{-t\sqrt{4s^{2\alpha}-5}} - \frac{e^{-t}}{1-s^{2\alpha}}s^{2\alpha-2}(1+s),$$

which includes that

$$u(x,t) = e^{-\frac{1}{2}t}\left(\int_0^x e^{x-u}F(u,t)du - 2\int_0^x (x-u)E_{2\alpha,\alpha}(x-u)^{2\alpha}F(u,t)du\right),$$

6. Conclusion

For the problem (issue) of multi-criteria sequencing $1/(Et_{max}, \sum_{j=1}^{n}(C_{t_j})^2, \sum_{j=1}^{n} L_{tj})$, for the hierarchical (lexicographical) scenario, an approach to find the best solution (outcome) is put forth. It is hoped that this paper's contribution would encourage more study in the zone of multi-measuring ruling take on, particularly three hierarchically ranked criteria. Experimentation with the following machine sequencing problem will be a future research topic: $1/Lex(Et_{max}, \sum_{j=1}^{n} L_{tj}, \sum_{j=1}^{n} C_{t_j})$. In addition, we have rigorously analyzed time series data from various domains, including environmental and epidemiological fields, employing the Generalized Least Deviation Method to identify the optimal model order for forecasting. Our results demonstrate that the complexity required for a predictive model is highly contingent on the dataset's characteristics, such as the nature of the data, its underlying dynamics, and the presence of non-linear patterns, rather than solely on the quantity of data available.

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