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Srivastava-Luo-Raina M-transform involving the incomplete I-functions

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Abstract

In this article, the Srivastava-Luo-Raina M-transform is applied to establish the image formula for the multiplication of a family of polynomials and the incomplete I-functions. Additionally, we discovered the image formulations for a few significant and valuable cases of incomplete I-functions. The conclusions drawn in this study are general in nature, and by assigning specific values to the parameters involved in the primary conclusions, numerous previously discovered and few undiscovered conclusions can be achieved.

Keywords. Incomplete gamma function, Incomplete I-functions, Mellin-Barnes type contour integral.

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1. Introduction and Preliminaries

Jangid et al. [11] have newly investigated new categories of incomplete I-functions, which are described in the following way (see [7, 8, 12, 16]):

$$
\begin{aligned}
{}^{\gamma}I^{m,\,n}_{p,\,q}(z) &= {}^{\gamma}I^{m,\,n}_{p,\,q}\left[z \,\middle|\, \begin{matrix} (\mathrm{u}_1,\mathcal{U}_1;\varpi_1:t),(\mathrm{u}_2,\mathcal{U}_2;\varpi_2),\cdots,(\mathrm{u}_p,\mathcal{U}_p;\varpi_p) \\ (\mathrm{e}_1,\mathcal{E}_1;\kappa_1),\cdots,(\mathrm{e}_q,\mathcal{E}_q;\kappa_q) \end{matrix} \right] \\
&= {}^{\gamma}I^{m,\,n}_{p,\,q}\left[z \,\middle|\, \begin{matrix} (\mathrm{u}_1,\mathcal{U}_1;\varpi_1:t),(\mathrm{u}_j,\mathcal{U}_j;\varpi_j)_{2,\,p} \\ (\mathrm{e}_j,\mathcal{E}_j;\kappa_j)_{1,\,q} \end{matrix} \right] = \frac{1}{2\pi i}\int_{\pounds} \psi(l,t)\, z^{l}\, dl,
\end{aligned} \tag{1.1}
$$

and

$$
\begin{aligned}
{}^{\Gamma}I^{m,\,n}_{p,\,q}(z) &= {}^{\Gamma}I^{m,\,n}_{p,\,q}\left[z \,\middle|\, \begin{matrix} (\mathrm{u}_1,\mathcal{U}_1;\varpi_1:t),(\mathrm{u}_2,\mathcal{U}_2;\varpi_2),\cdots,(\mathrm{u}_p,\mathcal{U}_p;\varpi_p) \\ (\mathrm{e}_1,\mathcal{E}_1;\kappa_1),\cdots,(\mathrm{e}_q,\mathcal{E}_q;\kappa_q) \end{matrix} \right] \\
&= {}^{\Gamma}I^{m,\,n}_{p,\,q}\left[z \,\middle|\, \begin{matrix} (\mathrm{u}_1,\mathcal{U}_1;\varpi_1:t),(\mathrm{u}_j,\mathcal{U}_j;\varpi_j)_{2,\,p} \\ (\mathrm{e}_j,\mathcal{E}_j;\kappa_j)_{1,\,q} \end{matrix} \right] = \frac{1}{2\pi i}\int_{\pounds} \Psi(l,t)\, z^{l}\, dl,
\end{aligned} \tag{1.2}
$$

for all $z \neq 0$, where

$$
\psi(l,t) = \frac{\{\gamma(1-\mathrm{u}_1+\mathcal{U}_1 l,t)\}^{\varpi_1} \prod\limits_{k=1}^{m}\{\Gamma(\mathrm{e}_k-\mathcal{E}_k l)\}^{\kappa_k} \prod\limits_{k=2}^{n}\{\Gamma(1-\mathrm{u}_k+\mathcal{U}_k l)\}^{\varpi_k}}{\prod\limits_{k=n+1}^{p}\{\Gamma(\mathrm{u}_k-\mathcal{U}_k l)\}^{\varpi_k} \prod\limits_{k=m+1}^{q}\{\Gamma(1-\mathrm{e}_k+\mathcal{E}_k l)\}^{\kappa_k}}, \tag{1.3}
$$

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and

$$\Psi(l,t)=\frac{\{\Gamma(1-\mathrm{u}_1+\mathcal{U}_1 l,t)\}^{\varpi_1}\prod_{k=1}^{m}\{\Gamma(\mathrm{e}_k-\mathcal{E}_k l)\}^{\kappa_k}\prod_{k=2}^{n}\{\Gamma(1-\mathrm{u}_k+\mathcal{U}_k l)\}^{\varpi_k}}{\prod_{k=n+1}^{p}\{\Gamma(\mathrm{u}_k-\mathcal{U}_k l)\}^{\varpi_k}\prod_{k=m+1}^{q}\{\Gamma(1-\mathrm{e}_k+\mathcal{E}_k l)\}^{\kappa_k}}. \tag{1.4}$$

The incomplete I-functions ${}^{\gamma}I_{p,\,q}^{m,\,n}(z)$ and ${}^{\Gamma}I_{p,\,q}^{m,\,n}(z)$ exist for all $t \geq 0$ under the same condition and contour as mentioned in Rathie [17]. The incomplete Gamma functions $\gamma(w,y)$ and $\Gamma(w,y)$ are defined as follows:

$$\gamma(w,y)=\int_0^y v^{w-1}e^{-v}dv, \qquad (\Re(w)>0;\, y\geq 0)\,, \tag{1.5}$$

and

$$\Gamma(w,y)=\int_y^\infty v^{w-1}e^{-v}dv, \qquad (\Re(w)>0;\, y\geq 0)\,, \tag{1.6}$$

acknowledged as the lower and upper Gamma functions respectively. It is important to point out that the incomplete I-function changed to the I-function for $t=0$ studied in [17]. Additionally, if $\varpi_1=1$ the incomplete I-functions holds the decomposition formula given below:

$${}^{\gamma}I_{p,\,q}^{m,\,n}(z)+{}^{\Gamma}I_{p,\,q}^{m,\,n}(z)=I_{p,\,q}^{m,\,n}(z), \qquad (\varpi_1=1)\,. \tag{1.7}$$

A general class of polynomials was studied by the Srivastava [19], described in the following way:

$$S_V^U[t]=\sum_{R=0}^{[V/U]}\frac{(-V)_{UR}}{R!}A_{V,R}\,t^R, \tag{1.8}$$

where $U\in\mathbb{Z}^+$ and $A_{V,\,R}$ are real or complex numbers arbitrary constant. The notations $[k]$ indicates the Floor function and $(\kappa)_\mu$ indicate the Pochhammer symbol described by:

$$(\kappa)_0=1 \qquad and \qquad (\kappa)_\mu=\frac{\Gamma(\kappa+\mu)}{\Gamma(\kappa)}, \qquad (\mu\in\mathbb{C}),$$

in the form of the Gamma function. Srivastava et al. [18] introduced the following Srivastava-Luo-Raina M-transform:

$$M_{\lambda,\mu}[g(x)](\mathfrak{p},\mathfrak{q})=\int_0^\infty\frac{e^{-\mathfrak{p}x}g(\mathfrak{q}x)}{(x^\mu+\mathfrak{q}^\mu)^\lambda}dx, \tag{1.9}$$

provided that $\lambda\in\mathbb{C},\Re(\lambda)\geq 0,\mu\in\mathbb{N},\ \mathfrak{p}\in\mathbb{C}$, and $\mathfrak{q}\in\mathbb{R}^+$ are the transform variable. The Srivastava-Luo-Raina M-transform is closely linked to the well-known integral transforms Laplace, Natural and Sumudu, as stated in equation (1.9): The Laplace transform [12] is defined by:

$$L[g(x)](\mathfrak{p})=\int_0^\infty e^{-\mathfrak{p}x}g(x)\,dx, \qquad \Re(\mathfrak{p})>0. \tag{1.10}$$

From equation (1.9) and (1.10), we have the following Laplace-$M_{\lambda,\mu}[g(x)](\mathfrak{p},\mathfrak{q})$ transfrom duality relation [1]:

$$L[g(x)](\mathfrak{p})=M_{0,\mu}[g(x)](\mathfrak{p},1), \qquad \Re(\mathfrak{p})>0 \tag{1.11}$$

and

$$M_{\lambda,\mu}[g(x)](\mathfrak{p},\mathfrak{q})=L\left(\frac{g(\mathfrak{q}x)}{(x^\mu+\mathfrak{q}^\mu)^\lambda}\right)(\mathfrak{p})=\frac{1}{\mathfrak{q}}L\left[\frac{g(x)}{\left((\frac{x}{\mathfrak{q}})^\mu+\mathfrak{q}^\mu\right)^\lambda}\right]\left(\frac{\mathfrak{p}}{\mathfrak{q}}\right), \qquad \mathfrak{p},\mathfrak{q}>0. \tag{1.12}$$

Put $\lambda=0$ in (1.9), we obtained the Natural transform [6, 15] and it is described as:

$$N[g(x)](\mathfrak{p},\mathfrak{q})=\int_0^\infty e^{-\mathfrak{p}x}g(\mathfrak{q}x)dx, \qquad \mathfrak{p},\mathfrak{q}>0. \tag{1.13}$$

From equation (1.9) and (1.13), we have the following Natural- $M_{\lambda,\mu}[g(x)](\mathfrak{p},\mathfrak{q})$ transform duality relation [1]:

$$N[g(x)](\mathfrak{p},\mathfrak{q}) = M_{0,\mu}[g(x)](\mathfrak{p},\mathfrak{q}), \qquad \mathfrak{p},\mathfrak{q} > 0, \tag{1.14}$$

and

$$M_{\lambda,\mu}[g(x)](\mathfrak{p},\mathfrak{q}) = N\left[\frac{g(x)}{\left((\frac{x}{\mathfrak{q}})^{\mu} + \mathfrak{q}^{\mu}\right)^{\lambda}}\right](\mathfrak{p},\mathfrak{q}), \qquad \mathfrak{p},\mathfrak{q} > 0. \tag{1.15}$$

Sumudu transform [10] is described by:

$$S[g(x)](\mathfrak{q}) = \int_0^{\infty} e^{-x} g(\mathfrak{q}x)dx, \qquad \mathfrak{q} > 0. \tag{1.16}$$

From equation (1.9) and (1.16), we have the following Sumudu- $M_{\lambda,\mu}[g(x)](\mathfrak{p},\mathfrak{q})$ transform duality relation [1]:

$$S[g(x)](\mathfrak{q}) = M_{0,\mu}[g(x)](0,\mathfrak{q}), \qquad \mathfrak{q} > 0, \tag{1.17}$$

and

$$M_{\lambda,\mu}[g(x)](\mathfrak{p},\mathfrak{q}) = \frac{1}{\mathfrak{p}} S\left[\frac{g(x)}{\left((\frac{x}{\mathfrak{q}})^{\mu} + \mathfrak{q}^{\mu}\right)^{\lambda}}\right]\left(\frac{\mathfrak{q}}{\mathfrak{p}}\right), \qquad \mathfrak{p},\mathfrak{q} > 0. \tag{1.18}$$

Another special case of the integral transform (1.9) when $\mathfrak{p} = 0$ corresponds to the classical Stieltjes transform, which was investigated by Srivastava [20]. Convergence condition of the Srivastava-Luo-Raina M-transform (1.9) is given in Theorem 2.3 of [18].

Lemma 1.1. *For $\lambda, \mathfrak{p} \in \mathbb{C}, \mathfrak{q} \in \mathbb{R}^+, \Re(\lambda) \geq 0, \mu \in \mathbb{N}$, and $a, \nu > 0$, then we have the following assertion:*

$$M_{\lambda,\mu}[t^{\nu-1}](\mathfrak{p},\mathfrak{q}) = \frac{\mathfrak{q}^{\nu-\lambda\mu-1}\,\mathfrak{p}^{-\nu}}{\mu\Gamma(\lambda)} H^{2,\,1}_{1,\,2}\left[\mathfrak{p}\mathfrak{q} \;\middle|\; \begin{matrix} (1, \frac{1}{\mu}), \\ (\nu, 1), (\lambda, \frac{1}{\mu}) \end{matrix}\right], \tag{1.19}$$

$$M_{\lambda,\mu}[e^{-at}](\mathfrak{p},\mathfrak{q}) = \frac{\mathfrak{q}^{-\lambda\mu}}{\mu(\mathfrak{p}+a\mathfrak{q})\Gamma(\lambda)} H^{2,\,1}_{1,\,2}\left[\mathfrak{q}(\mathfrak{p}+a\mathfrak{q}) \;\middle|\; \begin{matrix} (1, \frac{1}{\mu}), \\ (1, 1), (\lambda, \frac{1}{\mu}) \end{matrix}\right], \tag{1.20}$$

and

$$M_{\lambda,\mu}[t^{\nu-1}\, e^{-at}](\mathfrak{p},\mathfrak{q}) = \frac{\mathfrak{q}^{\nu-\lambda\mu-1}}{\mu(\mathfrak{p}+a\mathfrak{q})^{\nu}\,\Gamma(\lambda)} H^{2,\,1}_{1,\,2}\left[\mathfrak{q}(\mathfrak{p}+a\mathfrak{q}) \;\middle|\; \begin{matrix} (1, \frac{1}{\mu}), \\ (\nu, 1), (\lambda, \frac{1}{\mu}) \end{matrix}\right]. \tag{1.21}$$

2. Main Results

In this section, we establish the Srivastava-Luo-Raina M-transform associated with the multiplication of family of polynomials and the incomplete I-function.

Theorem 2.1. *If $\lambda, \mathfrak{p} \in \mathbb{C}$, $\Re(\lambda), \xi_1 \geq 0, \mu \in \mathbb{N}$, $\mathfrak{q} \in \mathbb{R}^+$, and with the condition presented in (1.2), then the result is as follows.*

$$M_{\lambda,\mu}\left[{}^{\Gamma}I^{m,\,n}_{p,\,q}\left[zx^{\xi_1} \;\middle|\; \begin{matrix} (\mathrm{u}_1,\mathcal{U}_1;\varpi_1:t),(\mathrm{u}_j,\mathcal{U}_j;\varpi_j)_{2,\,p} \\ (\mathrm{e}_j,\mathcal{E}_j;\kappa_j)_{1,\,q} \end{matrix}\right]\right](\mathfrak{p},\mathfrak{q}) = \frac{\mathfrak{q}^{-\mu\lambda}}{\mathfrak{p}\mu}\frac{1}{2\pi i}\int_{\Delta} B\left(\lambda - \frac{s}{\mu}, \frac{s}{\mu}\right)$$
$$\times (\mathfrak{p}\mathfrak{q})^{s}\left({}^{\Gamma}I^{m,\,n+1}_{p+1,\,q}\left[z\left(\frac{\mathfrak{q}}{\mathfrak{p}}\right)^{\xi_1} \;\middle|\; \begin{matrix} (\mathrm{u}_1,\mathcal{U}_1;\varpi_1:t),(s,\xi_1;1),(\mathrm{u}_j,\mathcal{U}_j;\varpi_j)_{2,\,p} \\ (\mathrm{e}_j,\mathcal{E}_j;\kappa_j)_{1,\,q} \end{matrix}\right]\right) ds. \tag{2.1}$$

Proof. The LHS of equation (2.1) is:

$$L_{11} = M_{\lambda,\mu}\left[{}^{\Gamma}I_{p,\,q}^{m,\,n}\left[zx^{\xi_1}\,\middle|\,\begin{matrix}(\mathrm{u}_1,\mathcal{U}_1;\varpi_1:t),(\mathrm{u}_j,\mathcal{U}_j;\varpi_j)_{2,\,p}\\(\mathrm{e}_j,\mathcal{E}_j;\kappa_j)_{1,\,q}\end{matrix}\right]\right](\mathfrak{p},\mathfrak{q}). \tag{2.2}$$

Replace the incomplete I-function by (1.2) and using the definition of Srivastava-Luo-Raina M-transform defined in (1.9), we get:

$$L_{11} = \int_0^\infty \frac{e^{-\mathfrak{p}x}}{(x^\mu+\mathfrak{q}^\mu)^\lambda}\left[\frac{1}{2\pi i}\int_{\pounds}\Psi(l,t)\,(\mathfrak{q}zx^{\xi_1})^l\,dl\right]dx. \tag{2.3}$$

Change the integration order and with the help of (1.19) of Lemma 1.1, we obtain:

$$\begin{aligned}&\frac{1}{2\pi i}\int_{\pounds}\Psi(l,t)\,(z)^l\frac{\mathfrak{q}^{\xi_1 l+1-\mu\lambda}\mathfrak{p}^{-\xi_1 l-1}}{\mu\Gamma(\lambda)}\;H_{1,\,2}^{2,\,1}\left[\mathfrak{pq}\,\middle|\,\begin{matrix}(1,\frac{1}{\mu}),\\(\xi_1 l+1,1),(\lambda,\frac{1}{\mu})\end{matrix}\right]dl\\&=\frac{\mathfrak{q}^{-\mu\lambda}}{\mathfrak{p}\;\mu}\frac{1}{2\pi i}\int_{\pounds}\Psi(l,t)\;\left[z\left(\frac{\mathfrak{q}}{\mathfrak{p}}\right)^{\xi_1}\right]^l\frac{1}{2\pi i}\int_{\Delta}\frac{\Gamma(\xi_1 l+1-s)\Gamma\left(\lambda-\frac{s}{\mu}\right)\Gamma\left(\frac{s}{\mu}\right)}{\Gamma(\lambda)}(\mathfrak{pq})^s ds\,dl.\end{aligned} \tag{2.4}$$

Change the order of integration and after some adjustment of terms, we achieve the intended outcomes. □

Theorem 2.2. *If $\lambda,\mathfrak{p}\in\mathbb{C}$, $\Re(\lambda),\xi_1\geq 0,\mu\in\mathbb{N}$, $\mathfrak{q}\in\mathbb{R}^+$, and with the condition presented in (1.2), then the result is as follows.*

$$\begin{aligned}&M_{\lambda,\mu}\left[{}^{\gamma}I_{p,\,q}^{m,\,n}\left[zx^{\xi_1}\,\middle|\,\begin{matrix}(\mathrm{u}_1,\mathcal{U}_1;\varpi_1:t),(\mathrm{u}_j,\mathcal{U}_j;\varpi_j)_{2,\,p}\\(\mathrm{e}_j,\mathcal{E}_j;\kappa_j)_{1,\,q}\end{matrix}\right]\right](\mathfrak{p},\mathfrak{q})=\frac{\mathfrak{q}^{-\mu\,\lambda}}{\mathfrak{p}\mu}\frac{1}{2\pi i}\int_{\Delta}B\left(\lambda-\frac{s}{\mu},\frac{s}{\mu}\right)\\&\times(\mathfrak{pq})^s\left({}^{\gamma}I_{p+1,\,q}^{m,\,n+1}\left[z\left(\frac{\mathfrak{q}}{\mathfrak{p}}\right)^{\xi_1}\,\middle|\,\begin{matrix}(\mathrm{u}_1,\mathcal{U}_1;\varpi_1:t),(s,\xi_1;1),(\mathrm{u}_j,\mathcal{U}_j;\varpi_j)_{2,\,p}\\(\mathrm{e}_j,\mathcal{E}_j;\kappa_j)_{1,\,q}\end{matrix}\right]\right)ds.\end{aligned} \tag{2.5}$$

Theorem 2.2 is proved in the same way as Theorem 2.1 with the same conditions.

Theorem 2.3. *If $\lambda,\mathfrak{p}\in\mathbb{C}$, $\Re(\lambda)$, ξ_1, $\xi_2\geq 0,\mu\in\mathbb{N}$, $\mathfrak{q}\in\mathbb{R}^+$, $U\in\mathbb{Z}^+$, $A_{V,\,R}$ are real or complex numbers arbitrary constant and with the condition presented in (1.2), then the result is as follows.*

$$\begin{aligned}&M_{\lambda,\mu}\left[{}^{\Gamma}I_{p,\,q}^{m,\,n}\left[zx^{\xi_1}\,\middle|\,\begin{matrix}(\mathrm{u}_1,\mathcal{U}_1;\varpi_1:t),(\mathrm{u}_j,\mathcal{U}_j;\varpi_j)_{2,\,p}\\(\mathrm{e}_j,\mathcal{E}_j;\kappa_j)_{1,\,q}\end{matrix}\right]\times S_V^U[tx^{\xi_2}]\right](\mathfrak{p},\mathfrak{q})\\&=\frac{\mathfrak{q}^{-\mu\,\lambda}}{\mathfrak{p}\mu}\sum_{R=0}^{[V/U]}\frac{(-V)_{UR}}{R!}A_{V,R}\left[t\left(\frac{\mathfrak{q}}{\mathfrak{p}}\right)^{\xi_2}\right]^R\frac{1}{2\pi i}\int_{\Delta}B\left(\lambda-\frac{s}{\mu},\frac{s}{\mu}\right)(\mathfrak{pq})^s\\&\times\left({}^{\Gamma}I_{p+1,\,q}^{m,\,n+1}\left[z\left(\frac{\mathfrak{q}}{\mathfrak{p}}\right)^{\xi_1}\,\middle|\,\begin{matrix}(\mathrm{u}_1,\mathcal{U}_1;\varpi_1:t),(s-\xi_2 R,\xi_1;1),(\mathrm{u}_j,\mathcal{U}_j;\varpi_j)_{2,\,p}\\(\mathrm{e}_j,\mathcal{E}_j;\kappa_j)_{1,\,q}\end{matrix}\right]\right)ds.\end{aligned} \tag{2.6}$$

Proof. The LHS of equation (2.6) is:

$$L_9 = M_{\lambda,\mu}\left[{}^{\Gamma}I_{p,\,q}^{m,\,n}\left[zx^{\xi_1}\,\middle|\,\begin{matrix}(\mathrm{u}_1,\mathcal{U}_1;\varpi_1:t),(\mathrm{u}_j,\mathcal{U}_j;\varpi_j)_{2,\,p}\\(\mathrm{e}_j,\mathcal{E}_j;\kappa_j)_{1,\,q}\end{matrix}\right]\times S_V^U[tx^{\xi_2}]\right](\mathfrak{p},\mathfrak{q}). \tag{2.7}$$

Replace the incomplete I-function and Srivastava polynomial by (1.2) and (1.8) respectively and using the definition of Srivastava-Luo-Raina M-transform defined in (1.9), we get:

$$L_9 = \int_0^\infty \frac{e^{-\mathfrak{p}x}}{(x^\mu+\mathfrak{p}^\mu)^\lambda}\left[\frac{1}{2\pi i}\int_{\pounds}\Psi(l,t)\,(\mathfrak{q}zx^{\xi_1})^l\,dl\sum_{R=0}^{[V/U]}\frac{(-V)_{UR}}{R!}A_{V,R}\,(t\mathfrak{q}x^{\xi_2})^R\right]dx. \tag{2.8}$$

Change the integration order and with the help of (1.19) of Lemma 1.1, we obtain:

$$L_9 = \sum_{R=0}^{[V/U]} \frac{(-V)_{UR}}{R!} A_{V,R}\,(t)^R \times \frac{1}{2\pi i}\int_{\pounds} \Psi(l,t)\,(z)^l \times \frac{\mathfrak{q}^{\xi_1 l+\xi_2 R+1-\mu\lambda-1}\mathfrak{p}^{-\xi_1 l-\xi_2 R-1}}{\mu\Gamma(\lambda)}$$
$$\times H^{2,\,1}_{1,\,2}\left[\mathfrak{pq}\left|\begin{array}{c}(1,\frac{1}{\mu}),\\ (\xi_1 l+\xi_2 R+1,1),(\lambda,\frac{1}{\mu})\end{array}\right.\right] dl$$
$$= \frac{\mathfrak{q}^{-\mu\lambda}}{\mu\mathfrak{p}} \sum_{R=0}^{[V/U]} \frac{(-V)_{UR}}{R!} A_{V,R}\left[t\left(\frac{\mathfrak{q}}{\mathfrak{p}}\right)^{\xi_2}\right]^R \frac{1}{2\pi i}\int_{\pounds} \Psi(l,t)\left[z\left(\frac{\mathfrak{q}}{\mathfrak{p}}\right)^{\xi_2}\right]^l$$
$$\times \frac{1}{2\pi i}\int_{\Delta} \frac{\Gamma(\xi_1 l+\xi_2 R+1-s)\Gamma\left(\lambda-\frac{s}{\mu}\right)\Gamma\left(\frac{s}{\mu}\right)}{\Gamma(\lambda)} (\mathfrak{pq})^s ds\, dl. \tag{2.9}$$

Change the order of integration and after some adjustment of terms, we achieve the intended outcomes. □

Theorem 2.4. *If $\lambda,\mathfrak{p}\in\mathbb{C}$, $\Re(\lambda)$, ξ_1, $\xi_2\geq 0, \mu\in\mathbb{N}$, $\mathfrak{q}\in\mathbb{R}^+$, $U\in\mathbb{Z}^+$, $A_{V,\,R}$ are real or complex numbers arbitrary constant and with the condition presented in (1.2), then the result is as follows.*

$$M_{\lambda,\mu}\left[{}^{\gamma}I^{m,\,n}_{p,\,q}\left[zx^{\xi_1}\left|\begin{array}{c}(\mathrm{u}_1,\mathcal{U}_1;\varpi_1:t),(\mathrm{u}_j,\mathcal{U}_j;\varpi_j)_{2,\,p}\\ (\mathrm{e}_j,\mathcal{E}_j;\kappa_j)_{1,\,q}\end{array}\right.\right]\times S^U_V[tx^{\xi_2}]\right](\mathfrak{p},\mathfrak{q})$$
$$= \frac{\mathfrak{q}^{-\mu\,\lambda}}{\mathfrak{p}\mu}\sum_{R=0}^{[V/U]}\frac{(-V)_{UR}}{R!}A_{V,R}\left[t\left(\frac{\mathfrak{q}}{\mathfrak{p}}\right)^{\xi_2}\right]^R\frac{1}{2\pi i}\int_{\Delta} B\left(\lambda-\frac{s}{\mu},\frac{s}{\mu}\right)(\mathfrak{pq})^s$$
$$\times\left({}^{\gamma}I^{m,\,n+1}_{p+1,\,q}\left[z\left(\frac{\mathfrak{q}}{\mathfrak{p}}\right)^{\xi_1}\left|\begin{array}{c}(\mathrm{u}_1,\mathcal{U}_1;\varpi_1:t),(s-\xi_2 R,\xi_1;1),(\mathrm{u}_j,\mathcal{U}_j;\varpi_j)_{2,\,p}\\ (\mathrm{e}_j,\mathcal{E}_j;\kappa_j)_{1,\,q}\end{array}\right.\right]\right) ds. \tag{2.10}$$

Theorem 2.4 is proved in the same way as Theorem 2.3 with the same conditions.

Theorem 2.5. *If $\lambda,\mathfrak{p}\in\mathbb{C},\Re(\lambda)$, $a\geq 0;\mu\in\mathbb{N}$, $\mathfrak{q}\in\mathbb{R}^+$, and with the condition presented in (1.2), then the result is as follows.*

$$M_{\lambda,\mu}\left[{}^{\Gamma}I^{m,\,n}_{p,\,q}\left[z\,e^{\frac{-ax}{l}}\left|\begin{array}{c}(\mathrm{u}_1,\mathcal{U}_1;\varpi_1:t),(\mathrm{u}_j,\mathcal{U}_j;\varpi_j)_{2,\,p}\\ (\mathrm{e}_j,\mathcal{E}_j;\kappa_j)_{1,\,q}\end{array}\right.\right]\right](\mathfrak{p},\mathfrak{q})$$
$$= \frac{\mathfrak{q}^{-\mu\,\lambda}}{\mu(\mathfrak{p}+a\mathfrak{q})}\,\frac{1}{2\pi i}\int_{\Delta} B\left(\lambda-\frac{s}{\mu},\frac{s}{\mu}\right)[\mathfrak{q}(\mathfrak{p}+a\mathfrak{q})]^s$$
$$\times\left({}^{\Gamma}I^{m,\,n+1}_{p+1,\,q}\left[z\left|\begin{array}{c}(\mathrm{u}_1,\mathcal{U}_1;\varpi_1:t),(s,0;1),(\mathrm{u}_j,\mathcal{U}_j;\varpi_j)_{2,\,p}\\ (\mathrm{e}_j,\mathcal{E}_j;\kappa_j)_{1,\,q}\end{array}\right.\right]\right) ds. \tag{2.11}$$

Proof. The LHS of equation (2.11) is:

$$L_7 = M_{\lambda,\mu}\left[{}^{\Gamma}I^{m,\,n}_{p,\,q}\left[z\,e^{-\frac{ax}{l}}\left|\begin{array}{c}(\mathrm{u}_1,\mathcal{U}_1;\varpi_1:t),(\mathrm{u}_j,\mathcal{U}_j;\varpi_j)_{2,\,p}\\ (\mathrm{e}_j,\mathcal{E}_j;\kappa_j)_{1,\,q}\end{array}\right.\right]\right](\mathfrak{p},\mathfrak{q}). \tag{2.12}$$

Replace the incomplete I-function by (1.2) and using the definition of Srivastava-Luo-Raina M-transform defined in (1.9), we get:

$$L_7 = \int_0^{\infty}\frac{e^{-\mathfrak{p}x}}{(x^{\mu}+\mathfrak{q}^{\mu})^{\lambda}}\left[\frac{1}{2\pi i}\int_{\pounds}\Psi(l,t)\,(z\,e^{-\frac{a\mathfrak{q}x}{l}})^l\,dl\right]dx. \tag{2.13}$$

Change the integration order and with the help of (1.20) in Lemma 1.1, we obtain:

$$L_7 = \frac{1}{2\pi i}\int_{\pounds} \Psi(l,t)\,(z)^l \times \frac{\mathfrak{q}^{-\mu\lambda}}{\mu(\mathfrak{p}+a\mathfrak{q})\Gamma(\lambda)} \times H^{2,\,1}_{1,\,2}\left[\mathfrak{q}[\mathfrak{p}+a\mathfrak{q}] \,\middle|\, \begin{matrix}(1,\frac{1}{\mu}),\\ (1,1),(\lambda,\frac{1}{\mu})\end{matrix}\right] dl$$

$$= \frac{\mathfrak{q}^{-\mu\lambda}}{\mu(\mathfrak{p}+a\mathfrak{q})} \times \frac{1}{2\pi i}\int_{\pounds} \Psi(l,t)\, z^l \times \frac{1}{2\pi i}\int_{\Delta} \frac{\Gamma(1-s)\Gamma\left(\lambda-\frac{s}{\mu}\right)\Gamma\left(\frac{s}{\mu}\right)}{\Gamma(\lambda)}[\mathfrak{q}(\mathfrak{p}+a\mathfrak{q})]^s ds\, dl. \tag{2.14}$$

Change the order of integration and after some adjustment of terms, we achieve the intended outcomes. □

Theorem 2.6. *If $\lambda,\mathfrak{p}\in\mathbb{C},\Re(\lambda)$, $a\geq 0,\mu\in\mathbb{N}$, $\mathfrak{q}\in\mathbb{R}^+$, and with the condition presented in (1.2), then the result is as follows.*

$$M_{\lambda,\mu}\left[{}^{\gamma}I^{m,\,n}_{p,\,q}\left[z\,e^{\frac{-ax}{l}} \,\middle|\, \begin{matrix}(\mathrm{u}_1,\mathcal{U}_1;\varpi_1:t),(\mathrm{u}_j,\mathcal{U}_j;\varpi_j)_{2,\,p}\\ (\mathrm{e}_j,\mathcal{E}_j;\kappa_j)_{1,\,q}\end{matrix}\right]\right](\mathfrak{p},\mathfrak{q})$$
$$= \frac{\mathfrak{q}^{-\mu\,\lambda}}{\mu(\mathfrak{p}+a\mathfrak{p})}\,\frac{1}{2\pi i}\int_{\Delta} B\left(\lambda-\frac{s}{\mu},\frac{s}{\mu}\right)[\mathfrak{q}(\mathfrak{p}+a\mathfrak{q})]^s$$
$$\times\left({}^{\gamma}I^{m,\,n+1}_{p+1,\,q}\left[z \,\middle|\, \begin{matrix}(\mathrm{u}_1,\mathcal{U}_1;\varpi_1:t),(s,0;1),(\mathrm{u}_j,\mathcal{U}_j;\varpi_j)_{2,\,p}\\ (\mathrm{e}_j,\mathcal{E}_j;\kappa_j)_{1,\,q}\end{matrix}\right]\right) ds. \tag{2.15}$$

Theorem 2.6 is proved in the same way as Theorem 2.5 with the same conditions.

Theorem 2.7. *If $\lambda,\mathfrak{p}\in\mathbb{C},\Re(\lambda),a,\,b\geq 0,\mu\in\mathbb{N}$, $\mathfrak{q},U\in\mathbb{R}^+$, and $A_{V,\,R}$ are real or complex numbers arbitrary constant and with the condition presented in (1.2), then the result is as follows.*

$$M_{\lambda,\mu}\left[{}^{\Gamma}I^{m,\,n}_{p,\,q}\left[z\,e^{\frac{-ax}{l}} \,\middle|\, \begin{matrix}(\mathrm{u}_1,\mathcal{U}_1;\varpi_1:t),(\mathrm{u}_j,\mathcal{U}_j;\varpi_j)_{2,\,p}\\ (\mathrm{e}_j,\mathcal{E}_j;\kappa_j)_{1,\,q}\end{matrix}\right]\times S^U_V[t\,e^{\frac{-bx}{R}}]\right](\mathfrak{p},\mathfrak{q})$$
$$= \frac{\mathfrak{q}^{-\mu\,\lambda}}{\mu(\mathfrak{p}+(a+b)\mathfrak{q})}\,S^U_V[t]\,\frac{1}{2\pi i}\int_{\Delta} B\left(\lambda-\frac{s}{\mu},\frac{s}{\mu}\right)[\mathfrak{q}(\mathfrak{p}+(a+b)\mathfrak{q})]^s$$
$$\times\left({}^{\Gamma}I^{m,\,n+1}_{p+1,\,q}\left[z \,\middle|\, \begin{matrix}(\mathrm{u}_1,\mathcal{U}_1;\varpi_1:t),(s,0;1),(\mathrm{u}_j,\mathcal{U}_j;\varpi_j)_{2,\,p}\\ (\mathrm{e}_j,\mathcal{E}_j;\kappa_j)_{1,\,q}\end{matrix}\right]\right) ds. \tag{2.16}$$

Proof. The LHS of equation (2.16) is:

$$L_5 = M_{\lambda,\mu}\left[{}^{\Gamma}I^{m,\,n}_{p,\,q}\left[z\,e^{-\frac{ax}{l}} \,\middle|\, \begin{matrix}(\mathrm{u}_1,\mathcal{U}_1;\varpi_1:t),(\mathrm{u}_j,\mathcal{U}_j;\varpi_j)_{2,\,p}\\ (\mathrm{e}_j,\mathcal{E}_j;\kappa_j)_{1,\,q}\end{matrix}\right] S^U_V[t\,e^{\frac{-bx}{R}}]\right](\mathfrak{p},\mathfrak{q}). \tag{2.17}$$

Replace the incomplete I-function and Srivastava polynomial by (1.2) and (1.8) respectively and using the definition of Srivastava-Luo-Raina M-transform defined in (1.9), we get:

$$L_5 = \int_0^{\infty}\frac{e^{-\mathfrak{p}x}}{(x^{\mu}+\mathfrak{q}^{\mu})^{\lambda}}\left[\frac{1}{2\pi i}\int_{\pounds}\Psi(l,t)\,(z\,e^{-\frac{a\mathfrak{q}x}{l}})^l\,dl\sum_{R=0}^{[V/U]}\frac{(-V)_{UR}}{R!}A_{V,R}\,(t\,e^{\frac{-b\mathfrak{q}x}{R}})^R\right]dx. \tag{2.18}$$

Change the integration order and with the help of (1.20) in Lemma 1.1, we obtain:

$$L_5 = \sum_{R=0}^{[V/U]} \frac{(-V)_{UR}}{R!} A_{V,R}\,(t)^R \times \frac{1}{2\pi i}\int_{\pounds} \Psi(l,t)\,(z)^l \times \frac{\mathfrak{q}^{-\mu\lambda}}{\mu(\mathfrak{p}+(a+b)\mathfrak{q})\Gamma(\lambda)}$$
$$\times H_{1,\,2}^{2,\,1}\left[\mathfrak{q}[\mathfrak{p}+(a+b)\mathfrak{q}] \;\middle|\; \begin{matrix}(1,\frac{1}{\mu}),\\ (1,1),(\lambda,\frac{1}{\mu})\end{matrix}\right] dl$$
$$= \frac{\mathfrak{q}^{-\mu\lambda}}{\mu(\mathfrak{p}+(a+b)\mathfrak{q})} \sum_{R=0}^{[V/U]} \frac{(-V)_{UR}}{R!} A_{V,R}\,(t)^R \times \frac{1}{2\pi i}\int_{\pounds} \Psi(l,t)\,z^l$$
$$\times \frac{1}{2\pi i}\int_{\Delta} \frac{\Gamma(1-s)\Gamma\left(\lambda-\frac{s}{\mu}\right)\Gamma\left(\frac{s}{\mu}\right)}{\Gamma(\lambda)}[\mathfrak{q}(\mathfrak{p}+(a+b)\mathfrak{q})]^s ds\, dl. \tag{2.19}$$

Change the order of integration and after some adjustment of terms, we achieve the intended outcomes. □

Theorem 2.8. *If $\lambda, \mathfrak{p} \in \mathbb{C}, \Re(\lambda), a,\, b \geq 0, \mu \in \mathbb{N}$, $\mathfrak{q}$, $U \in \mathbb{R}^+$, and $A_{V,\,R}$ are real or complex numbers arbitrary constant and with the condition presented in (1.2), then the result is as follows.*

$$M_{\lambda,\mu}\left[{}^{\gamma}I_{p,\,q}^{m,\,n}\left[z\,e^{\frac{-ax}{l}} \;\middle|\; \begin{matrix}(\mathrm{u}_1,\mathcal{U}_1;\varpi_1:t),(\mathrm{u}_j,\mathcal{U}_j;\varpi_j)_{2,\,p}\\ (\mathrm{e}_j,\mathcal{E}_j;\kappa_j)_{1,\,q}\end{matrix}\right] \times S_V^U[t\,e^{\frac{-bx}{R}}]\right](\mathfrak{p},\mathfrak{q})$$
$$= \frac{\mathfrak{q}^{-\mu\,\lambda}}{\mu(\mathfrak{p}+(a+b)\mathfrak{q})}\; S_V^U[t]\;\frac{1}{2\pi i}\int_{\Delta} B\left(\lambda-\frac{s}{\mu},\frac{s}{\mu}\right)[\mathfrak{q}(\mathfrak{p}+(a+b)\mathfrak{q})]^s$$
$$\times \left({}^{\gamma}I_{p+1,\,q}^{m,\,n+1}\left[z \;\middle|\; \begin{matrix}(\mathrm{u}_1,\mathcal{U}_1;\varpi_1:t),(s,0;1),(\mathrm{u}_j,\mathcal{U}_j;\varpi_j)_{2,\,p}\\ (\mathrm{e}_j,\mathcal{E}_j;\kappa_j)_{1,\,q}\end{matrix}\right]\right) ds. \tag{2.20}$$

Theorem 2.8 is proved in the same way as Theorem 2.7 with the same conditions.

Theorem 2.9. *If $\lambda, \mathfrak{p} \in \mathbb{C}, \Re(\lambda)$, $\xi_1 \geq 0, \mu \in \mathbb{N}$, $\mathfrak{q} \in \mathbb{R}^+$, and with the condition presented in (1.2), then the result is as follows.*

$$M_{\lambda,\mu}\left[{}^{\Gamma}I_{p,\,q}^{m,\,n}\left[z\,x^{\xi_1}\,e^{-\frac{ax}{l}} \;\middle|\; \begin{matrix}(\mathrm{u}_1,\mathcal{U}_1;\varpi_1:t),(\mathrm{u}_j,\mathcal{U}_j;\varpi_j)_{2,\,p}\\ (\mathrm{e}_j,\mathcal{E}_j;\kappa_j)_{1,\,q}\end{matrix}\right]\right](\mathfrak{p},\mathfrak{q})$$
$$= \frac{\mathfrak{q}^{-\mu\,\lambda}}{\mu(\mathfrak{p}+a\mathfrak{q})}\frac{1}{2\pi i}\int_{\Delta} B\left(\lambda-\frac{s}{\mu},\frac{s}{\mu}\right)[\mathfrak{q}(\mathfrak{p}+a\mathfrak{q})]^s$$
$$\times \left({}^{\Gamma}I_{p+1,\,q}^{m,\,n+1}\left[z\left(\frac{\mathfrak{q}}{\mathfrak{p}+a\mathfrak{q}}\right)^{\xi_1} \;\middle|\; \begin{matrix}(\mathrm{u}_1,\mathcal{U}_1;\varpi_1:t),(s,\xi_1;1),(\mathrm{u}_j,\mathcal{U}_j;\varpi_j)_{2,\,p}\\ (\mathrm{e}_j,\mathcal{E}_j;\kappa_j)_{1,\,q}\end{matrix}\right]\right) ds. \tag{2.21}$$

Proof. The LHS of equation (2.21) is:

$$L_2 = M_{\lambda,\mu}\left[{}^{\Gamma}I_{p,\,q}^{m,\,n}\left[z\,x^{\xi_1}\,e^{-\frac{ax}{l}} \;\middle|\; \begin{matrix}(\mathrm{u}_1,\mathcal{U}_1;\varpi_1:t),(\mathrm{u}_j,\mathcal{U}_j;\varpi_j)_{2,\,p}\\ (\mathrm{e}_j,\mathcal{E}_j;\kappa_j)_{1,\,q}\end{matrix}\right]\right](\mathfrak{p},\mathfrak{q}). \tag{2.22}$$

Replace the incomplete I-function by (1.2) and using the definition of Srivastava-Luo-Raina M-transform defined in (1.9), we get:

$$L_2 = \int_0^{\infty} \frac{e^{-\mathfrak{p}x}}{(x^{\mu}+\mathfrak{q}^{\mu})^{\lambda}}\left[\frac{1}{2\pi i}\int_{\pounds} \Psi(l,t)\,(\mathfrak{q}z\,x^{\xi_1}\,e^{-\frac{a\mathfrak{q}x}{l}})^l\,dl\right]dx. \tag{2.23}$$

Change the integration order and with the help of (1.21) in Lemma 1.1, we obtain:

$$\frac{1}{2\pi i}\int_{\pounds} \Psi(l,t)\,(z)^l\,\frac{\mathfrak{q}^{\xi_1 l+1-\mu\lambda-1}}{\mu(\mathfrak{p}+a\mathfrak{q})^{\xi_1 l+1}\Gamma(\lambda)} \times H^{2,\,1}_{1,\,2}\left[\mathfrak{q}[\mathfrak{p}+a\mathfrak{q}]\,\middle|\, \begin{matrix}(1,\frac{1}{\mu}),\\ (\xi_1 l+1,1),(\lambda,\frac{1}{\mu})\end{matrix}\right] dl$$
$$= \frac{\mathfrak{q}^{-\mu\lambda}}{\mu(\mathfrak{p}+a\mathfrak{q})} \times \frac{1}{2\pi i}\int_{\pounds} \Psi(l,t)\left(z\left(\frac{\mathfrak{q}}{\mathfrak{p}+a\mathfrak{q}}\right)^{\xi_1}\right)^l$$
$$\times \frac{1}{2\pi i}\int_{\Delta}\frac{\Gamma(\xi_1 l+1-s)\Gamma\left(\lambda-\frac{s}{\mu}\right)\Gamma\left(\frac{s}{\mu}\right)}{\Gamma(\lambda)}[\mathfrak{q}(\mathfrak{p}+a\mathfrak{q})]^s ds\, dl. \tag{2.24}$$

Change the order of integration and after some adjustment of terms, we achieve the intended outcomes. □

Theorem 2.10. *If $\lambda, \mathfrak{p} \in \mathbb{C}, \Re(\lambda)$, $\xi_1 \geq 0, \mu \in \mathbb{N}$, $\mathfrak{q} \in \mathbb{R}^+$, and with the condition presented in (1.2), then the result is as follows.*

$$M_{\lambda,\mu}\left[{}^{\gamma}I^{m,\,n}_{p,\,q}\left[z\,x^{\xi_1}\,e^{-\frac{ax}{l}}\,\middle|\,\begin{matrix}(\mathrm{u}_1,\mathcal{U}_1;\varpi_1:t),(\mathrm{u}_j,\mathcal{U}_j;\varpi_j)_{2,\,p}\\ (\mathrm{e}_j,\mathcal{E}_j;\kappa_j)_{1,\,q}\end{matrix}\right]\right](\mathfrak{p},\mathfrak{q})$$
$$= \frac{\mathfrak{q}^{-\mu\,\lambda}}{\mu(\mathfrak{p}+a\mathfrak{q})}\frac{1}{2\pi i}\int_{\Delta} B\left(\lambda-\frac{s}{\mu},\frac{s}{\mu}\right)[\mathfrak{q}(\mathfrak{p}+a\mathfrak{q})]^s$$
$$\times\left({}^{\gamma}I^{m,\,n+1}_{p+1,\,q}\left[z\left(\frac{\mathfrak{q}}{\mathfrak{p}+a\mathfrak{q}}\right)^{\xi_1}\,\middle|\,\begin{matrix}(\mathrm{u}_1,\mathcal{U}_1;\varpi_1:t),(s,\xi_1;1),(\mathrm{u}_j,\mathcal{U}_j;\varpi_j)_{2,\,p}\\ (\mathrm{e}_j,\mathcal{E}_j;\kappa_j)_{1,\,q}\end{matrix}\right]\right)ds. \tag{2.25}$$

Theorem 2.10 is proved in the same way as Theorem 2.9 with the same conditions.

Theorem 2.11. *If $\lambda, \mathfrak{p} \in \mathbb{C}, \Re(\lambda), \xi_1, \xi_2 \geq 0, \mu \in \mathbb{N}$, $\mathfrak{q}, U \in \mathbb{R}^+$, and $A_{V,\,R}$ are real or complex numbers arbitrary constant and with the condition presented in (1.2), then the result is as follows.*

$$M_{\lambda,\mu}\left[{}^{\Gamma}I^{m,\,n}_{p,\,q}\left[z\,x^{\xi_1}\,e^{-\frac{ax}{l}}\,\middle|\,\begin{matrix}(\mathrm{u}_1,\mathcal{U}_1;\varpi_1:t),(\mathrm{u}_j,\mathcal{U}_j;\varpi_j)_{2,\,p}\\ (\mathrm{e}_j,\mathcal{E}_j;\kappa_j)_{1,\,q}\end{matrix}\right]\times S^U_V[t\,x^{\xi_2}\,e^{\frac{-bx}{R}}]\right](\mathfrak{p},\mathfrak{q})$$
$$= \frac{\mathfrak{q}^{-\mu\,\lambda}}{\mu(\mathfrak{p}+(a+b)\mathfrak{q})}\sum_{R=0}^{[V/U]}\frac{(-V)_{UR}}{R!}A_{V,R}\left(t\left(\frac{\mathfrak{q}}{\mathfrak{p}+(a+b)\mathfrak{q}}\right)^{\xi_2}\right)^R$$
$$\times\frac{1}{2\pi i}\int_{\Delta} B\left(\lambda-\frac{s}{\mu},\frac{s}{\mu}\right)[\mathfrak{q}(\mathfrak{p}+(a+b)\mathfrak{q})]^s$$
$$\times\left({}^{\Gamma}I^{m,\,n+1}_{p+1,\,q}\left[z\left(\frac{\mathfrak{q}}{\mathfrak{p}+(a+b)\mathfrak{q}}\right)^{\xi_1}\,\middle|\,\begin{matrix}(\mathrm{u}_1,\mathcal{U}_1;\varpi_1:t),(s-\xi_2 R,\xi_1;1),(\mathrm{u}_j,\mathcal{U}_j;\varpi_j)_{2,\,p}\\ (\mathrm{e}_j,\mathcal{E}_j;\kappa_j)_{1,\,q}\end{matrix}\right]\right)ds. \tag{2.26}$$

Proof. The LHS of equation (2.26) is:

$$L_1 = M_{\lambda,\mu}\left[{}^{\Gamma}I^{m,\,n}_{p,\,q}\left[z\,x^{\xi_1}\,e^{-\frac{ax}{l}}\,\middle|\,\begin{matrix}(\mathrm{u}_1,\mathcal{U}_1;\varpi_1:t),(\mathrm{u}_j,\mathcal{U}_j;\varpi_j)_{2,\,p}\\ (\mathrm{e}_j,\mathcal{E}_j;\kappa_j)_{1,\,q}\end{matrix}\right]S^U_V[t\,x^{\xi_2}\,e^{\frac{-bx}{R}}]\right](\mathfrak{p},\mathfrak{q}). \tag{2.27}$$

Replace the incomplete I-function and Srivastava polynomial by (1.2) and (1.8) respectively and using the definition of Srivastava-Luo-Raina M-transform defined in (1.9), we get:

$$L_1 = \int_0^\infty \frac{e^{-\mathfrak{p}x}}{(x^\mu+\mathfrak{q}^\mu)^\lambda}\left[\frac{1}{2\pi i}\int_{\pounds}\Psi(l,t)\,(\mathfrak{q}z\,x^{\xi_1}\,e^{-\frac{a\mathfrak{q}x}{l}})^l\,dl\sum_{R=0}^{[V/U]}\frac{(-V)_{UR}}{R!}A_{V,R}\,(\mathfrak{q}\,t\,x^{\xi_2}\,e^{\frac{-b\mathfrak{q}x}{R}})^R\right]dx. \tag{2.28}$$

Change the integration order and with the help of (1.21) in Lemma 1.1, we obtain:

$$L_1 = \sum_{R=0}^{[V/U]} \frac{(-V)_{UR}}{R!} A_{V,R}\,(t)^R \times \frac{1}{2\pi i}\int_{\pounds} \Psi(l,t)\,(z)^l \times \frac{\mathfrak{q}^{\xi_1 l+\xi_2 R+1-\mu\lambda-1}}{\mu(\mathfrak{p}+(a+b)\mathfrak{q})^{\xi_1 l+\xi_2 R+1}\Gamma(\lambda)}$$
$$\times H_{1,\,2}^{2,\,1}\left[\mathfrak{q}[\mathfrak{p}+(a+b)\mathfrak{q}]\,\middle|\, \begin{matrix}(1,\frac{1}{\mu}),\\ (\xi_1 l+\xi_2 R+1,1),(\lambda,\frac{1}{\mu})\end{matrix}\right] dl$$
$$= \frac{\mathfrak{p}^{-\mu\lambda}}{\mu(\mathfrak{p}+(a+b)\mathfrak{q})}\sum_{R=0}^{[V/U]} \frac{(-V)_{UR}}{R!} A_{V,R}\left(t\left(\frac{\mathfrak{q}}{\mathfrak{p}+(a+b)\mathfrak{q}}\right)^{\xi_2}\right)^R$$
$$\times \frac{1}{2\pi i}\int_{\pounds} \Psi(l,t)\left(z\left(\frac{\mathfrak{q}}{\mathfrak{p}+(a+b)\mathfrak{q}}\right)^{\xi_1}\right)^l$$
$$\times \frac{1}{2\pi i}\int_{\Delta} \frac{\Gamma(\xi_1 l+\xi_2 R+1-s)\Gamma\left(\lambda-\frac{s}{\mu}\right)\Gamma\left(\frac{s}{\mu}\right)}{\Gamma(\lambda)}[\mathfrak{q}(\mathfrak{p}+(a+b)\mathfrak{q})]^s ds\, dl. \tag{2.29}$$

Change the order of integration and after some adjustment of terms, we achieve the intended outcomes. □

Theorem 2.12. *If $\lambda,\mathfrak{p}\in\mathbb{C},\Re(\lambda),\xi_1,\xi_2\geq 0,\mu\in\mathbb{N}$, $\mathfrak{q},U\in\mathbb{R}^+$, and $A_{V,R}$ are real or complex numbers arbitrary constant and with the condition presented in (1.2), then the result is as follows.*

$$M_{\lambda,\mu}\left[{}^{\gamma}I_{p,\,q}^{m,\,n}\left[z\,x^{\xi_1}\,e^{-\frac{ax}{l}}\,\middle|\,\begin{matrix}(\mathrm{u}_1,\mathcal{U}_1;\varpi_1:t),(\mathrm{u}_j,\mathcal{U}_j;\varpi_j)_{2,\,p}\\(\mathrm{e}_j,\mathcal{E}_j;\kappa_j)_{1,\,q}\end{matrix}\right]\times S_V^U[t\,x^{\xi_2}\,e^{\frac{-bx}{R}}]\right](\mathfrak{p},\mathfrak{q})$$
$$= \frac{\mathfrak{p}^{-\mu\lambda}}{\mu(\mathfrak{p}+(a+b)\mathfrak{q})}\sum_{R=0}^{[V/U]} \frac{(-V)_{UR}}{R!} A_{V,R}\left(t\left(\frac{\mathfrak{q}}{\mathfrak{p}+(a+b)\mathfrak{q}}\right)^{\xi_2}\right)^R$$
$$\times \frac{1}{2\pi i}\int_{\Delta} B\left(\lambda-\frac{s}{\mu},\frac{s}{\mu}\right)[\mathfrak{q}(\mathfrak{p}+(a+b)\mathfrak{q})]^s$$
$$\times\left({}^{\gamma}I_{p+1,\,q}^{m,\,n+1}\left[z\left(\frac{\mathfrak{q}}{\mathfrak{p}+(a+b)\mathfrak{q}}\right)^{\xi_1}\,\middle|\,\begin{matrix}(\mathrm{u}_1,\mathcal{U}_1;\varpi_1:t),(s-\xi_2 R,\xi_1;1),(\mathrm{u}_j,\mathcal{U}_j;\varpi_j)_{2,\,p}\\(\mathrm{e}_j,\mathcal{E}_j;\kappa_j)_{1,\,q}\end{matrix}\right]\right) ds. \tag{2.30}$$

Theorem 2.12 is proved in the same way as Theorem 2.11 with the same conditions.

Remark

- If we set $U=1, A_{V,0}=1$, and $A_{V,R}=0\ \forall\ R\neq 0$ in Theorems 2.3, 2.4, 2.7, 2.8, 2.11, and 2.12 then the result is same as that of Theorems 2.1, 2.2, 2.5, 2.6, 2.9, and 2.10 respectively.
- If we set $\xi_1,\xi_2=0$ in Theorems 2.11 and 2.12 then the result is same as that of Theorems 2.7 and 2.8 respectively.
- If we set $a,b=0$ in Theorems 2.11 and 2.12 then the result is same as that of Theorems 2.3 and 2.4 respectively.
- If we set $\varpi_j=1\forall j=1,2,...,p$, $\kappa_j=1\forall j=1,2,...,q$, and $\xi_1=1$ in Theorems 2.1 and 2.2 then the result is same as Theorem 2.2 and Theorem 2.1 respectively mentioned in Bansal et al. [2].
- If we take the substitutions $z=-z$,$\varpi_j=1\ \forall\ 1\leq j\leq p$, $\kappa_j=1\ \forall\ 1\leq j\leq q$, $\mathrm{u}_j\to(1-\mathrm{u}_j)\forall\ 1\leq j\leq p$, $\mathrm{e}_j\to(1-\mathrm{e}_j)\ \forall\ 1\leq j\leq q$ and $\xi_1=1$ in Theorems 2.1 and 2.2 then the result is same as corollary 1 mentioned in Bansal et al. [2].
- By using the Laplace-$M_{\lambda,\mu}[g(x)](\mathfrak{p},\mathfrak{q})$ transfrom duality relation defined in equation (1.11), in Theorems 2.1 and 2.2 then the result is same as Theorems 2.5 and 2.6 mentioned in Jangid et al. [11].

3. Special Cases

In this section, as a particular instance of Theorem 2.11 and Theorem 2.12, we establish Srivastava-Luo-Raina M-transform for the multiplication of Srivastava polynomial with the incomplete $\bar{I}$-function and the incomplete $\bar{H}$-function. Further, some special value will be given to Srivastava polynomial in order to get the outcomes in the form of Hermite and Laguerre polynomials. If we provide the parameter of particular features, we get the following special cases to delineate the use of fundamental outcomes.

(i) Incomplete $\bar{I}$-function: If we set $\kappa_k = 1$ for $1 \leq k \leq m$ in (1.2) and making use of the connection, that is (see [8, 12]).

$$ {}^{\Gamma}\overline{I}^{m,\,n}_{p,\,q}(z) = {}^{\Gamma}\overline{I}^{m,\,n}_{p,\,q}\left[z \,\middle|\, \begin{matrix} (\mathrm{u}_1, \mathcal{U}_1; \varpi_1 : t), (\mathrm{u}_j, \mathcal{U}_j; \varpi_j)_{2,\,p} \\ (\mathrm{e}_j, \mathcal{E}_j; 1)_{1,\,m}, (\mathrm{e}_j, \mathcal{E}_j; \kappa_j)_{m+1,\,q} \end{matrix} \right] = \frac{1}{2\pi i} \int_{\pounds} \overline{\Psi}(l,t)\, z^l \, dl, \tag{3.1} $$

where

$$ \overline{\Psi}(l,t) = \frac{\{\Gamma(1-\mathrm{u}_1+\mathcal{U}_1 l, t)\}^{\varpi_1} \prod\limits_{k=1}^{m} \{\Gamma(\mathrm{e}_k - \mathcal{E}_k l)\} \prod\limits_{k=2}^{n} \{\Gamma(1-\mathrm{u}_k+\mathcal{U}_k l)\}^{\varpi_k}}{\prod\limits_{k=n+1}^{p} \{\Gamma(\mathrm{u}_k - \mathcal{U}_k l)\}^{\varpi_k} \prod\limits_{k=m+1}^{q} \{\Gamma(1-\mathrm{e}_k+\mathcal{E}_k l)\}^{\kappa_k}}, \tag{3.2} $$

in (2.26) and (2.30), then we obtain the corollaries as follows:

Corollary 3.1. *If $\lambda, \mathfrak{p} \in \mathbb{C}, \Re(\lambda), \xi_1, \xi_2 \geq 0, \mu \in \mathbb{N}$, $\mathfrak{q}, U \in \mathbb{R}^+$, $A_{V,\,R}$ are real or complex numbers arbitrary constant and with the condition presented in (1.2), then the result is as follows.*

$$ \begin{aligned} & M_{\lambda,\mu}\left[{}^{\Gamma}\overline{I}^{m,\,n}_{p,\,q}\left[z\, x^{\xi_1}\, e^{-\frac{ax}{l}} \,\middle|\, \begin{matrix} (\mathrm{u}_1, \mathcal{U}_1; \varpi_1 : t), (\mathrm{u}_j, \mathcal{U}_j; \varpi_j)_{2,\,p} \\ (\mathrm{e}_j, \mathcal{E}_j; 1)_{1,\,m}, (\mathrm{e}_j, \mathcal{E}_j; \kappa_j)_{m+1,\,q} \end{matrix} \right] \times S_V^U[t\, x^{\xi_2}\, e^{\frac{-bx}{R}}] \right](\mathfrak{p}, \mathfrak{q}) \\ & = \frac{\mathfrak{q}^{-\mu\lambda}}{\mu(\mathfrak{p}+(a+b)\mathfrak{q})} \sum_{R=0}^{[V/U]} \frac{(-V)_{UR}}{R!} A_{V,R} \left(t \left(\frac{\mathfrak{q}}{\mathfrak{p}+(a+b)\mathfrak{q}} \right)^{\xi_2} \right)^R \\ & \times \frac{1}{2\pi i} \int_{\Delta} B\left(\lambda - \frac{s}{\mu}, \frac{s}{\mu} \right) [\mathfrak{q}(\mathfrak{p}+(a+b)\mathfrak{q})]^s \\ & \times \left({}^{\Gamma}\overline{I}^{m,\,n+1}_{p+1,\,q}\left[z \left(\frac{\mathfrak{q}}{\mathfrak{p}+(a+b)\mathfrak{q}} \right)^{\xi_1} \,\middle|\, \begin{matrix} (\mathrm{u}_1, \mathcal{U}_1; \varpi_1 : t), (s-\xi_2 R, \xi_1; 1), (\mathrm{u}_j, \mathcal{U}_j; \varpi_j)_{2,\,p} \\ (\mathrm{e}_j, \mathcal{E}_j; 1)_{1,\,m}, (\mathrm{e}_j, \mathcal{E}_j; \kappa_j)_{m+1,\,q} \end{matrix} \right] \right) ds. \end{aligned} \tag{3.3} $$

Corollary 3.2. *If $\lambda, \mathfrak{p} \in \mathbb{C}, \Re(\lambda), \xi_1, \xi_2 \geq 0, \mu \in \mathbb{N}$, $\mathfrak{q}, U \in \mathbb{R}^+$, $A_{V,\,R}$ are real or complex numbers arbitrary constant and with the condition presented in (1.2), then the result is as follows.*

$$ \begin{aligned} & M_{\lambda,\mu}\left[{}^{\gamma}\overline{I}^{m,\,n}_{p,\,q}\left[z\, x^{\xi_1}\, e^{-\frac{ax}{l}} \,\middle|\, \begin{matrix} (\mathrm{u}_1, \mathcal{U}_1; \varpi_1 : t), (\mathrm{u}_j, \mathcal{U}_j; \varpi_j)_{2,\,p} \\ (\mathrm{e}_j, \mathcal{E}_j; 1)_{1,\,m}, (\mathrm{e}_j, \mathcal{E}_j; \kappa_j)_{m+1,\,q} \end{matrix} \right] \times S_V^U[t\, x^{\xi_2}\, e^{\frac{-bx}{R}}] \right](\mathfrak{p}, \mathfrak{q}) \\ & = \frac{\mathfrak{q}^{-\mu\lambda}}{\mu(\mathfrak{p}+(a+b)\mathfrak{q})} \sum_{R=0}^{[V/U]} \frac{(-V)_{UR}}{R!} A_{V,R} \left(t. \left(\frac{\mathfrak{q}}{\mathfrak{p}+(a+b)\mathfrak{q}} \right)^{\xi_2} \right)^R \\ & \times \frac{1}{2\pi i} \int_{\Delta} B\left(\lambda - \frac{s}{\mu}, \frac{s}{\mu} \right) [\mathfrak{q}(\mathfrak{p}+(a+b)\mathfrak{q})]^s \\ & \times \left({}^{\gamma}\overline{I}^{m,\,n+1}_{p+1,\,q}\left[z \left(\frac{\mathfrak{q}}{\mathfrak{p}+(a+b)\mathfrak{q}} \right)^{\xi_1} \,\middle|\, \begin{matrix} (\mathrm{u}_1, \mathcal{U}_1; \varpi_1 : t), (s-\xi_2 R, \xi_1; 1), (\mathrm{u}_j, \mathcal{U}_j; \varpi_j)_{2,\,p} \\ (\mathrm{e}_j, \mathcal{E}_j; 1)_{1,\,m}, (\mathrm{e}_j, \mathcal{E}_j; \kappa_j)_{m+1,\,q} \end{matrix} \right] \right) ds. \end{aligned} \tag{3.4} $$

(ii) Incomplete $\bar{H}$-function: If we set $\varpi_j = 1$ for $n+1 \le j \le p$ and $\kappa_j = 1$ for $1 \le j \le m$ in (1.2) and making use of the connection, that is (see [14]).

$$\begin{aligned}\overline{\Gamma}_{p,\,q}^{m,\,n}(z) &= \overline{\Gamma}_{p,\,q}^{m,\,n}\left[z \,\middle|\, \begin{matrix}(\mathrm{u}_1,\mathcal{U}_1;\varpi_1:t),(\mathrm{u}_j,\mathcal{U}_j;\varpi_j)_{2,\,n},(\mathrm{u}_j,\mathcal{U}_j;1)_{n+1,\,p}\\ (\mathrm{e}_j,\mathcal{E}_j;1)_{1,\,m},(\mathrm{e}_j,\mathcal{E}_j;\kappa_j)_{m+1,\,q}\end{matrix}\right]\\ &= \overline{\Gamma}_{p,\,q}^{m,\,n}\left[z \,\middle|\, \begin{matrix}(\mathrm{u}_1,\mathcal{U}_1;\varpi_1:t),(\mathrm{u}_j,\mathcal{U}_j;\varpi_j)_{2,\,n},(\mathrm{u}_j,\mathcal{U}_j)_{n+1,\,p}\\ (\mathrm{e}_j,\mathcal{E}_j)_{1,\,m},(\mathrm{e}_j,\mathcal{E}_j;\kappa_j)_{m+1,\,q}\end{matrix}\right]\\ &= \frac{1}{2\pi i}\int_{\pounds}\overline{\Psi}(l,t)\,z^l\,dl,\end{aligned} \tag{3.5}$$

where

$$\overline{\Psi}(l,t) = \frac{\{\Gamma(1-\mathrm{u}_1+\mathcal{U}_1 l,t)\}^{\varpi_1}\prod_{k=1}^{m}\Gamma(e_j-\mathcal{E}_j l)\prod_{k=2}^{n}\{\Gamma(1-\mathrm{u}_j+\mathcal{U}_j l)\}^{\varpi_j}}{\prod_{k=n+1}^{p}\Gamma(\mathrm{u}_j-\mathcal{U}_j l)\prod_{k=m+1}^{q}\{\Gamma(1-e_j+\mathcal{E}_j l)\}^{\kappa_j}}, \tag{3.6}$$

in (2.26) and (2.30), then we obtain the corollaries as follows.

Corollary 3.3. *If $\lambda, \mathfrak{p} \in \mathbb{C}, \Re(\lambda), \xi_1, \xi_2 \ge 0, \mu \in \mathbb{N}$, $\mathfrak{q}, U \in \mathbb{R}^+$, $A_{V,R}$ are real or complex numbers arbitrary constant and with the condition presented in (1.2), then the result is as follows.*

$$\begin{aligned}&M_{\lambda,\mu}\left[\overline{\Gamma}_{p,\,q}^{m,\,n}\left[z\,x^{\xi_1}\,e^{-\frac{ax}{l}} \,\middle|\, \begin{matrix}(\mathrm{u}_1,\mathcal{U}_1;\varpi_1:t),(\mathrm{u}_j,\mathcal{U}_j;\varpi_j)_{2,\,n},(\mathrm{u}_j,\mathcal{U}_j;1)_{n+1,\,p}\\ (\mathrm{e}_j,\mathcal{E}_j;1)_{1,\,m},(\mathrm{e}_j,\mathcal{E}_j;\kappa_j)_{m+1,\,q}\end{matrix}\right]\right.\\ &\left.\times S_V^U[t\,x^{\xi_2}\,e^{\frac{-bx}{R}}]\right](\mathfrak{p},\mathfrak{q}) = \frac{\mathfrak{q}^{-\mu\lambda}}{\mu(\mathfrak{p}+(a+b)\mathfrak{q})}\sum_{R=0}^{[V/U]}\frac{(-V)_{UR}}{R!}A_{V,R}\left(t\left(\frac{\mathfrak{q}}{\mathfrak{p}+(a+b)\mathfrak{q}}\right)^{\xi_2}\right)^R\\ &\times\frac{1}{2\pi i}\int_{\Delta}B\left(\lambda-\frac{s}{\mu},\frac{s}{\mu}\right)[\mathfrak{q}(\mathfrak{p}+(a+b)\mathfrak{q})]^s\\ &\times\left(\overline{\Gamma}_{p+1,\,q}^{m,\,n+1}\left[z\left(\frac{\mathfrak{q}}{\mathfrak{p}+(a+b)\mathfrak{q}}\right)^{\xi_1} \,\middle|\, \begin{matrix}(\mathrm{u}_1,\mathcal{U}_1;\varpi_1:t),(s-\xi_2R,\xi_1;1),\\ (\mathrm{e}_j,\mathcal{E}_j;1)_{1,\,m},\end{matrix}\right.\right.\\ &\qquad\left.\left.\begin{matrix}(\mathrm{u}_j,\mathcal{U}_j;\varpi_j)_{2,\,n},(\mathrm{u}_j,\mathcal{U}_j;1)_{n+1,\,p}\\ (\mathrm{e}_j,\mathcal{E}_j;\kappa_j)_{m+1,\,q}\end{matrix}\right]\right)ds.\end{aligned} \tag{3.7}$$

Corollary 3.4. *If $\lambda, \mathfrak{p} \in \mathbb{C}, \Re(\lambda), \xi_1, \xi_2 \ge 0, \mu \in \mathbb{N}$, $\mathfrak{q}, U \in \mathbb{R}^+$, $A_{V,R}$ are real or complex numbers arbitrary constant and with the condition presented in (1.2), then the result is as follows.*

$$\begin{aligned}&M_{\lambda,\mu}\left[\overline{\gamma}_{p,\,q}^{m,\,n}\left[z\,x^{\xi_1}\,e^{-\frac{ax}{l}} \,\middle|\, \begin{matrix}(\mathrm{u}_1,\mathcal{U}_1;\varpi_1:t),(\mathrm{u}_j,\mathcal{U}_j;\varpi_j)_{2,\,n},(\mathrm{u}_j,\mathcal{U}_j;1)_{n+1,\,p}\\ (\mathrm{e}_j,\mathcal{E}_j;1)_{1,\,m},(\mathrm{e}_j,\mathcal{E}_j;\kappa_j)_{m+1,\,q}\end{matrix}\right]\right.\\ &\left.\times S_V^U[t\,x^{\xi_2}\,e^{\frac{-bx}{R}}]\right](\mathfrak{p},\mathfrak{q}) = \frac{\mathfrak{q}^{-\mu\lambda}}{\mu(\mathfrak{p}+(a+b)\mathfrak{q})}\sum_{R=0}^{[V/U]}\frac{(-V)_{UR}}{R!}A_{V,R}\left(t\left(\frac{\mathfrak{q}}{\mathfrak{p}+(a+b)\mathfrak{q}}\right)^{\xi_2}\right)^R\\ &\times\frac{1}{2\pi i}\int_{\Delta}B\left(\lambda-\frac{s}{\mu},\frac{s}{\mu}\right)[\mathfrak{q}(\mathfrak{p}+(a+b)\mathfrak{q})]^s\\ &\times\left(\overline{\gamma}_{p+1,\,q}^{m,\,n+1}\left[z\left(\frac{\mathfrak{q}}{\mathfrak{p}+(a+b)\mathfrak{q}}\right)^{\xi_1} \,\middle|\, \begin{matrix}(\mathrm{u}_1,\mathcal{U}_1;\varpi_1:t),(s-\xi_2R,\xi_1;1),\\ (\mathrm{e}_j,\mathcal{E}_j;1)_{1,\,m},\end{matrix}\right.\right.\\ &\qquad\left.\left.\begin{matrix}(\mathrm{u}_j,\mathcal{U}_j;\varpi_j)_{2,\,n},(\mathrm{u}_j,\mathcal{U}_j;1)_{n+1,\,p}\\ (\mathrm{e}_j,\mathcal{E}_j;\kappa_j)_{m+1,\,q}\end{matrix}\right]\right)ds.\end{aligned} \tag{3.8}$$

(iii) Laguerre Polynomial: If we set $A_{V,\,R} = \binom{V+\alpha}{V}\frac{1}{(\alpha+1)_R}$ and $U = 1$ in (1.8) then $S_V^1[t] \to L_V^{(\alpha)}(t)$ and making use of the connection, that is (see [19]).

$$L_V^{\alpha}(t) = \sum_{R=0}^{V} \binom{V+\alpha}{V-R} \frac{(-t)^R}{R!}, \tag{3.9}$$

in (2.26) and (2.30), then we obtain the corollaries as follows.

Corollary 3.5. *If $\lambda, \mathfrak{p} \in \mathbb{C}, \Re(\lambda), \xi_1, \xi_2 \geq 0, \mu \in \mathbb{N}$, $\mathfrak{q} \in \mathbb{R}^+$ and with the condition presented in (1.2), then the result is as follows.*

$$\begin{aligned}
&M_{\lambda,\mu}\left[{}^{\Gamma}I_{p,\,q}^{m,\,n}\left[z\,x^{\xi_1}\,e^{-\frac{ax}{l}} \;\middle|\; \begin{matrix}(\mathrm{u}_1,\mathcal{U}_1;\varpi_1:t),(\mathrm{u}_j,\mathcal{U}_j;\varpi_j)_{2,\,p}\\ (\mathrm{e}_j,\mathcal{E}_j;\kappa_j)_{1,\,q}\end{matrix}\right] L_V^{\alpha}[t\,x^{\xi_2}\,e^{\frac{-bx}{R}}]\right](\mathfrak{p},\mathfrak{q})\\
&= \frac{\mathfrak{q}^{-\mu\,\lambda}}{\mu(\mathfrak{p}+(a+b)\mathfrak{q})} \sum_{R=0}^{[V]} \binom{V+\alpha}{V-R} \left(-t\left(\frac{\mathfrak{q}}{\mathfrak{p}+(a+b)\mathfrak{q}}\right)^{\xi_2}\right)^R\\
&\times \frac{1}{2\pi i}\int_{\Delta} B\left(\lambda-\frac{s}{\mu},\frac{s}{\mu}\right)[\mathfrak{q}(\mathfrak{p}+(a+b)\mathfrak{q})]^s\\
&\times \left({}^{\Gamma}I_{p+1,\,q}^{m,\,n+1}\left[z\left(\frac{\mathfrak{q}}{\mathfrak{p}+(a+b)\mathfrak{q}}\right)^{\xi_1} \;\middle|\; \begin{matrix}(\mathrm{u}_1,\mathcal{U}_1;\varpi_1:t),(s-\xi_2R,\xi_1;1),(\mathrm{u}_j,\mathcal{U}_j;\varpi_j)_{2,\,p}\\ (\mathrm{e}_j,\mathcal{E}_j;\kappa_j)_{1,\,q}\end{matrix}\right]\right) ds.
\end{aligned} \tag{3.10}$$

Corollary 3.6. *If $\lambda, \mathfrak{p} \in \mathbb{C}, \Re(\lambda), \xi_1, \xi_2 \geq 0, \mu \in \mathbb{N}$, $\mathfrak{q} \in \mathbb{R}^+$ and with the condition presented in (1.2), then the result is as follows.*

$$\begin{aligned}
&M_{\lambda,\mu}\left[{}^{\gamma}I_{p,\,q}^{m,\,n}\left[z\,x^{\xi_1}\,e^{-\frac{ax}{l}} \;\middle|\; \begin{matrix}(\mathrm{u}_1,\mathcal{U}_1;\varpi_1:t),(\mathrm{u}_j,\mathcal{U}_j;\varpi_j)_{2,\,p}\\ (\mathrm{e}_j,\mathcal{E}_j;\kappa_j)_{1,\,q}\end{matrix}\right] L_V^{\alpha}[t\,x^{\xi_2}\,e^{\frac{-bx}{R}}]\right](\mathfrak{p},\mathfrak{q})\\
&= \frac{\mathfrak{q}^{-\mu\,\lambda}}{\mu(\mathfrak{p}+(a+b)\mathfrak{q})} \sum_{R=0}^{[V]} \binom{V+\alpha}{V-R} \left(-t\left(\frac{\mathfrak{q}}{\mathfrak{p}+(a+b)\mathfrak{q}}\right)^{\xi_2}\right)^R\\
&\times \frac{1}{2\pi i}\int_{\Delta} B\left(\lambda-\frac{s}{\mu},\frac{s}{\mu}\right)[\mathfrak{q}(\mathfrak{p}+(a+b)\mathfrak{q})]^s\\
&\times \left({}^{\gamma}I_{p+1,\,q}^{m,\,n+1}\left[z\left(\frac{\mathfrak{q}}{\mathfrak{p}+(a+b)\mathfrak{q}}\right)^{\xi_1} \;\middle|\; \begin{matrix}(\mathrm{u}_1,\mathcal{U}_1;\varpi_1:t),(s-\xi_2R,\xi_1;1),(\mathrm{u}_j,\mathcal{U}_j;\varpi_j)_{2,\,p}\\ (\mathrm{e}_j,\mathcal{E}_j;\kappa_j)_{1,\,q}\end{matrix}\right]\right) ds.
\end{aligned} \tag{3.11}$$

Remark

- If we set $U = 1, A_{V,0} = 1$, and $A_{V,R} = 0\ \forall\ R \neq 0$ in Corollaries 3.1 and 3.2 then we get the special case(in terms of incomplete $\overline{I}$- function) of Theorems 2.9 and 2.10 respectively.
- If we set $U = 1, A_{V,0} = 1$, and $A_{V,R} = 0\ \forall\ R \neq 0$ in Corollaries 3.3 and 3.4 then we get the special case(in terms of incomplete $\overline{H}$- function) of Theorems 2.9 and 2.10 respectively.

4. Concluding Remarks

In this article, we obtain the Srivastava-Luo-Raina M-transform for the incomplete I-function which is the extension of the I-function investigated by Jangid et al. [11] and we also study Srivastava-Luo-Raina M-transform for the product of incomplete I-function and Srivastava Polynomial. As the incomplete I-function generalize varity of incomplete functions like (see [3–5, 8, 9, 13, 16, 19]): I-function, Meijer G-function, hypergeometric function, H-function, $\overline{I}$-function and many other functions. Also, Srivastava Polynomial generalize various other polynomial like: Hermite polynomial, Jacobi polynomial, Gegenbauer polynomial, Legendre polynomial, Tchebycheff polynomial, Gould-Hopper Polynomial and several other polynomials. Hence our primary outcomes are significant and can help to determine the number of different Srivastava-Luo-Raina M-transform linked with numerous kinds of special functions and polynomials.

It is interesting to note that if we put $\mathfrak{p} = 0$ in (1.9) and use the equation (1.11), (1.14), (1.17) one may easily obtain the well known transform like classical Stieltjes, Laplace, Natural and Sumudu transform involving the incomplete I-functionn and the family of polynomials as a special case of our main findings.

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