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The complex hyperbolic Schrödinger dynamical equation with a truncated M-fractional by using simplest equation method

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Abstract

This article studies a complex hyperbolic Schrodinger dynamical equation that is associated with nonlinear media via ultra short pulse propagation. The modified simplest equation method is executed to construct complex solitary wave and other solutions of the aforesaid equation by considering it in conformable M-fractional derivative sense. The acquired solutions are in the form of solitary and periodic waves and rational functions. These solutions are also described with their graphical representations by assuming appropriate values of required parameters. Moreover, the results show that the aforesaid approach can be effective for solving such nonlinear Schrodinger equations arising in nonlinear optics and physical sciences.

Keywords. The modified simplest equation method, Exact solutions, Truncated M-fractional derivative.
2010 Mathematics Subject Classification. 35C08, 35R11, 35M86.

1. Introduction

Most fundamental formations in the real world are shown by nonlinear partial differential equations (PDEs). Modeling of nonlinear PDEs with their solutions contributes an essential part in the study of mathematical modeling, mathematical physics, mathematical biology, optical fibers, fluid mechanics, heat transfer, plasma physics, and many more [10, 21, 25, 35, 36]. A variety of smart ansatze techniques have been developed to set solutions in traveling waveforms for nonlinear PDEs. In [15], researchers used the Exp-function and $(\frac{G'}{G})$-expansion scheme to gain traveling wave solutions for Kadomtsev-Petviashvili equation. In [16], authors applied the first integral scheme to get exact solutions for two dimensional GL equation. The two-dimensional fourth-order nonlinear Boussinesq equation has been explored in [21] for its solitary solutions and stability analysis. In [17], the researchers executed the sub-equation scheme to explore the Hirota-Satsuma coupled KdV and the Cahn-Allen equations. The authors used $(\frac{G'}{G})$-expansion scheme to obtain different kinds of solutions of nonlinear lattice equations in [14]. In [22], the researchers investigated the modified Kudryashov scheme to gain the solutions of Tzitzeica, DBM, and Tzitzéica–DB equations. The $exp(-\psi(\xi))$-expansion and the extended simple equation methods have been discussed for the SRL wave equation in [26]. In [37], researchers exercised the F-expansion scheme and the generalized extended tanh method to secure the solitary wave solutions of the KP equation and its modified form equations. The extended modified rational expansion scheme has been implemented to gain the solitary wave solutions for $(2+1)$-dim nonlinear Nizhnik-Novikov-Vesselov equation [23]. In [42], the extended Jacobi elliptic expansion function method has been exercised to gain the optical soliton solutions of the new Hamiltonian amplitude equation. The extended simplest equation method has been given in [29] to find the exact solutions of the perturbed Gerdjikov–Ivanov equation. With the help of symbolic computation, the first integral method has been practiced to construct solitary and periodic wave solutions for the

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complex nonlinear evolution equations [40]. One can also study some coupled systems in [18, 24, 38] and many more [1–3, 5–7, 11–13, 20, 28, 30–34, 39, 43, 44].

In this article, we will present a comprehensive approach named as the modified simplest equation method (mSEM) [19, 41] for investigating new solutions for the Schrodinger dynamical equation with conformable M-fractional derivative.

The complex hyperbolic nonlinear Schrödinger dynamical equation with a truncated M-fractional as follows [41]:

$$iD^{\mu,\beta}_{M,y}U + \frac{1}{2}(D^{2\mu,\beta}_{M,x}U - D^{2\mu,\beta}_{M,t}U) + |U|^2U = 0, \quad 0 < \mu < 1, \quad \beta > 0, \tag{1.1}$$

where

$$D^{\mu,\beta}_{M,t}u = \lim_{\tau\to 0} \frac{u\left(x, tE_\beta(\tau t^{1-\mu})\right) - u(x,t)}{\tau}, \quad 0 < \mu < 1, \ \ \beta > 0, \tag{1.2}$$

that $E_\beta(.)$ is a truncated Mittag-Leffler function with one parameter [27].

In Eq. (1.1), $U(x, y, t)$ is the complex valued function with the dimensionless independent variable x and the propagation coordinate y and the independent time variable t. The complex hyperbolic Schrödinger equation has been solved to extract solutions by the extended sinh-Gorden equation expansion method [37]. The generalized elliptic equation rational expansion scheme [9] and the classical symmetry approach [4] have been utilized to explore the said equation. Furthermore, the multipliers method and conservation laws [8], the modified simple equation and Exp-function methods [45] were also practiced.

1.1. **Description of the mSEM.** We exercise mSEM stepwise to explore the exact solutions of Eq. (1.3). Let us have a partial differential equation with conformable M-fractional derivative read as

$$H(u, D^{\mu,\beta}_{M,t}u, D^{\mu,\beta}_{M,x}u, D^{\mu,\beta}_{M,y}u, D^{2\mu,\beta}_{M,t}u, D^{2\mu,\beta}_{M,x}, \ldots) = 0, \tag{1.3}$$

here $u(x, y, t)$ is a dependent function involving the highest order derivatives and nonlinear terms.

Step 1 Using the traveling wave transform in Eq. (1.3), one have

$$u(x, y, t) = U(\eta), \ \eta = \frac{\Gamma(\beta+1)}{\mu}\left(\alpha x^\mu + \rho y^\mu + \lambda\, t^\mu\right), \tag{1.4}$$

here α, λ, and ρ are nonzero constants. Eq. (1.4) can simplify Eq. (1.3) to produce a nonlinear ODE:

$$N(U, U_\eta, U_{\eta\eta}, ...) = 0, \tag{1.5}$$

where $U = \frac{dU}{d\eta}, U_{\eta\eta} = \frac{dU^2}{d\eta^2}, ...$

Step 2 Taking the solution of Eq.(1.5) in the form of finite series as

$$U(\eta) = \sum_{i=0}^{m} d_i\varphi^i(\eta), \tag{1.6}$$

where $d_i (i = 0, 1, 2, ..., m)$ are constants to be found later and $d_m \neq 0$. The Riccati equation is used as the simplest equation for the function $\varphi(\eta)$ to be satisfied.

$$\varphi'(\eta) = \varphi^2(\eta) + \epsilon, \tag{1.7}$$

here ϵ is a constant and the prime represent derivative w.r.t η. A family of solutions to Eq. (1.7) are procured depending upon the variations of ϵ:

When $\epsilon < 0$, **(Solitary wave solutions)**

$$\varphi(\eta) = -\sqrt{-\epsilon}\tanh\sqrt{-\epsilon}\ \eta, \tag{1.8}$$

$$\varphi(\eta) = -\sqrt{-\epsilon}\coth\sqrt{-\epsilon}\ \eta, \tag{1.9}$$

$$\varphi(\eta) = \sqrt{-\epsilon}(-\tanh(2\sqrt{-\epsilon}\ \eta) \pm i\,\mathrm{sech}(2\sqrt{-\epsilon}\ \eta)), \tag{1.10}$$

$$\varphi(\eta) = \sqrt{-\epsilon}\left(-\coth\left(2\sqrt{-\epsilon}\,\eta\right) \pm \operatorname{csch}\left(2\sqrt{-\epsilon}\,\eta\right)\right), \tag{1.11}$$

$$\varphi(\eta) = -\frac{\sqrt{-\epsilon}}{2}\left(\tanh\left(\frac{\sqrt{-\epsilon}}{2}\,\eta\right) + \coth\left(\frac{\sqrt{-\epsilon}}{2}\,\eta\right)\right). \tag{1.12}$$

When $\epsilon > 0$, **(Periodic function solutions)**

$$\varphi(\eta) = \sqrt{\epsilon}\tan\sqrt{\epsilon}\,\eta, \tag{1.13}$$

$$\varphi(\eta) = -\sqrt{\epsilon}\cot\sqrt{\epsilon}\,\eta, \tag{1.14}$$

$$\varphi(\eta) = \sqrt{\epsilon}\left(\tan\left(2\sqrt{\epsilon}\,\eta\right) \pm \sec\left(2\sqrt{\epsilon}\,\eta\right)\right), \tag{1.15}$$

$$\varphi(\eta) = \sqrt{\epsilon}\left(-\cot\left(2\sqrt{\epsilon}\,\eta\right) \pm \csc\left(2\sqrt{\epsilon}\,\eta\right)\right), \tag{1.16}$$

$$\varphi(\eta) = \frac{\sqrt{\epsilon}}{2}\left(\tan\left(\frac{\sqrt{\epsilon}}{2}\,\eta\right) - \cot\left(\frac{\sqrt{\epsilon}}{2}\,\eta\right)\right). \tag{1.17}$$

When $\epsilon = 0$, **(Rational function soltuion)**

$$\varphi(\eta) = -\frac{1}{\eta}. \tag{1.18}$$

Step 3
By inserting Eq. (1.6) in Eq. (1.5) with Eq. (1.7) and collect all the coefficients of function φ^i equal to zero for determining the values of $d_i, \alpha, \rho, \lambda$.
Step 4
Replacing the determined values of d_i, α, ρ, and λ in Eq. (1.6) and using the appropriate solutions of Eq. (1.7), we acquire the required wave solutions of Eq. (1.3).

2. Complex exact solution of Eq. (1.1)

We now apply the modified SE method to explore Eq. (1.1) by using the following transformations

$$\begin{aligned} &u(x,y,t) = U(\eta)e^{i\vartheta}, \quad \eta = \tfrac{\Gamma(\beta+1)}{\mu}\left(x^{\mu} + ay^{\mu} - \epsilon\, t^{\mu}\right), \\ &\vartheta = \tfrac{\Gamma(\beta+1)}{\mu}\left(\vartheta_1 x^{\mu} + \vartheta_2 y^{\mu} + \vartheta_3\, t^{\mu}\right) + \vartheta_4, \end{aligned} \tag{2.1}$$

where $\vartheta_1, \vartheta_2, \vartheta_3, \vartheta_4, a, \epsilon$ are non-variables and a plays the role of connector between two stable states of the solution. Moreover, ϵ and ϑ_1 are the speed and the frequency whereas ϑ_2 and ϑ_3 represent the wave number and the phase.

Then, Eq. (1.1) is converted to a nonlinear ODE by using (1.4), and secure the following real and imaginary parts:

$$(\epsilon^2 - 1)U'' - 2U^3 + (\vartheta_1^2 + 2\vartheta_2 - \vartheta_3^2)U = 0, \tag{2.2}$$

$$a = -(\vartheta_2 + \vartheta_3\epsilon).$$

Putting Eq. (1.6) into Eq. (2.2) and one can acquires the following system of algebraic equations.

$$\begin{aligned} d_0\vartheta_1^2 - d_0\vartheta_3^2 + 2d_0\vartheta_2 - 2d_0^3 &= 0,\\ 2d_1\epsilon^2\epsilon - 2d_1\epsilon + d_1\vartheta_1^2 - d_1\vartheta_3^2 + 2d_1\vartheta_2 - 6d_0^2 d_1 &= 0,\\ -6d_0 d_1^2 &= 0,\\ 2d_1\epsilon^2 - 2d_1^3 - 2d_1 &= 0. \end{aligned}$$

On solving the above system for d_0, d_1, ϵ by utilizing Mathematica, the following results are produced:
Case 1:

$$d_0 = 0,\ d_1 = -\sqrt{\frac{-\vartheta_1^2 + \vartheta_3^2 - 2\vartheta_2}{2\epsilon}},\ \epsilon = \mp\sqrt{\frac{2\epsilon - \vartheta_1^2 + \vartheta_3^2 - 2\vartheta_2}{2\epsilon}}.$$

When $\epsilon < 0$, we arrive at the solitary wave solutions

$$\begin{aligned} &u(x,y,t) = \sqrt{\tfrac{\vartheta_1^2-\vartheta_3^2+2\vartheta_2}{2}} \times \exp\left[i(\tfrac{\Gamma(\beta+1)}{\mu}\left(\vartheta_1 x^\mu + \vartheta_2 y^\mu + \vartheta_3\, t^\mu\right) + \vartheta_4)\right]\\ &\times \tanh\left(\sqrt{-\epsilon}\left(\tfrac{\Gamma(\beta+1)}{\mu}\left(x^\mu + a y^\mu - \epsilon\, t^\mu\right)\right)\right), \end{aligned} \tag{2.3}$$

or

$$\begin{aligned} &u(x,y,t) = \sqrt{\tfrac{\vartheta_1^2-\vartheta_3^2+2\vartheta_2}{2}} \times \exp\left[i(\tfrac{\Gamma(\beta+1)}{\mu}\left(\vartheta_1 x^\mu + \vartheta_2 y^\mu + \vartheta_3\, t^\mu\right) + \vartheta_4)\right]\\ &\times \coth\left(\sqrt{-\epsilon}\left(\tfrac{\Gamma(\beta+1)}{\mu}\left(x^\mu + a y^\mu - \epsilon\, t^\mu\right)\right)\right), \end{aligned} \tag{2.4}$$

or

$$\begin{aligned} &u(x,y,t) = -\sqrt{\tfrac{\vartheta_1^2-\vartheta_3^2+2\vartheta_2}{2}} \times \exp\left[i(\tfrac{\Gamma(\beta+1)}{\mu}\left(\vartheta_1 x^\mu + \vartheta_2 y^\mu + \vartheta_3\, t^\mu\right) + \vartheta_4)\right]\\ &\times \left(-\tanh\left(2\sqrt{-\epsilon}(\tfrac{\Gamma(\beta+1)}{\mu}\left(x^\mu + a y^\mu - \epsilon\, t^\mu\right)\right) \pm i\,\mathrm{sech}\left(2\sqrt{-\epsilon}(\tfrac{\Gamma(\beta+1)}{\mu}\left(x^\mu + a y^\mu - \epsilon\, t^\mu\right))\right)\right), \end{aligned} \tag{2.5}$$

or

$$\begin{aligned} &u(x,y,t) = -\sqrt{\tfrac{\vartheta_1^2-\vartheta_3^2+2\vartheta_2}{2}} \times \exp\left[i(\tfrac{\Gamma(\beta+1)}{\mu}\left(\vartheta_1 x^\mu + \vartheta_2 y^\mu + \vartheta_3\, t^\mu\right) + \vartheta_4)\right]\\ &\times \left(-\coth\left(2\sqrt{-\epsilon}(\tfrac{\Gamma(\beta+1)}{\mu}\left(x^\mu + a y^\mu - \epsilon\, t^\mu\right)\right) \pm \mathrm{csch}\left(2\sqrt{-\epsilon}(\tfrac{\Gamma(\beta+1)}{\mu}\left(x^\mu + a y^\mu - \epsilon\, t^\mu\right))\right)\right), \end{aligned} \tag{2.6}$$

or

$$\begin{aligned} &u(x,y,t) = \sqrt{-\tfrac{\vartheta_1^2-\vartheta_3^2+2\vartheta_2}{2}} \times \exp\left[i(\tfrac{\Gamma(\beta+1)}{\mu}\left(\vartheta_1 x^\mu + \vartheta_2 y^\mu + \vartheta_3\, t^\mu\right) + \vartheta_4)\right]\\ &\tfrac{1}{2}\left(\tanh\left(\tfrac{\sqrt{-\epsilon}}{2}(\tfrac{\Gamma(\beta+1)}{\mu}\left(x^\mu + a y^\mu - \epsilon\, t^\mu\right)\right) + \coth\left(\tfrac{\sqrt{-\epsilon}}{2}(\tfrac{\Gamma(\beta+1)}{\mu}\left(x^\mu + a y^\mu - \epsilon\, t^\mu\right))\right)\right). \end{aligned} \tag{2.7}$$

When $\epsilon > 0$, we acquire the periodic function solutions as

$$\begin{aligned} &u(x,y,t) = -\sqrt{\tfrac{-\vartheta_1^2+\vartheta_3^2-2\vartheta_2}{2}} \times \exp\left[i(\tfrac{\Gamma(\beta+1)}{\mu}\left(\vartheta_1 x^\mu + \vartheta_2 y^\mu + \vartheta_3\, t^\mu\right) + \vartheta_4)\right]\\ &\times \tan\left(\sqrt{\epsilon}\left(\tfrac{\Gamma(\beta+1)}{\mu}\left(x^\mu + a y^\mu - \epsilon\, t^\mu\right)\right)\right), \end{aligned} \tag{2.8}$$

or

$$\begin{aligned} &u(x,y,t) = \sqrt{\tfrac{-\vartheta_1^2+\vartheta_3^2-2\vartheta_2}{2}} \times \exp\left[i(\tfrac{\Gamma(\beta+1)}{\mu}\left(\vartheta_1 x^\mu + \vartheta_2 y^\mu + \vartheta_3\, t^\mu\right) + \vartheta_4)\right]\\ &\times \cot\left(\sqrt{\epsilon}\left(\tfrac{\Gamma(\beta+1)}{\mu}\left(x^\mu + a y^\mu - \epsilon\, t^\mu\right)\right)\right), \end{aligned} \tag{2.9}$$

or

$$\begin{aligned} &u(x,y,t) = -\sqrt{\tfrac{-\vartheta_1^2+\vartheta_3^2-2\vartheta_2}{2}} \times \exp\left[i(\tfrac{\Gamma(\beta+1)}{\mu}\left(\vartheta_1 x^\mu + \vartheta_2 y^\mu + \vartheta_3\, t^\mu\right) + \vartheta_4)\right]\\ &\times \left(\tan\left(2\sqrt{-\epsilon}(\tfrac{\Gamma(\beta+1)}{\mu}\left(x^\mu + a y^\mu - \epsilon\, t^\mu\right)\right) \pm \sec\left(2\sqrt{\epsilon}(\tfrac{\Gamma(\beta+1)}{\mu}\left(x^\mu + a y^\mu - \epsilon\, t^\mu\right))\right)\right), \end{aligned} \tag{2.10}$$

or

$$u(x,y,t) = -\sqrt{\frac{-\vartheta_1^2+\vartheta_3^2-2\vartheta_2}{2}} \times \exp\left[i(\frac{\Gamma(\beta+1)}{\mu}\left(\vartheta_1 x^\mu + \vartheta_2 y^\mu + \vartheta_3\, t^\mu\right) + \vartheta_4)\right] \times \left(-\cot\left(2\sqrt{\epsilon}(\frac{\Gamma(\beta+1)}{\mu}\left(x^\mu + a y^\mu - \epsilon\, t^\mu\right)\right) \pm \csc\left(2\sqrt{\epsilon}(\frac{\Gamma(\beta+1)}{\mu}\left(x^\mu + a y^\mu - \epsilon\, t^\mu\right)\right)\right), \tag{2.11}$$

or

$$u(x,y,t) = -\sqrt{\frac{-\vartheta_1^2+\vartheta_3^2-2\vartheta_2}{2}} \times \exp\left[i(\frac{\Gamma(\beta+1)}{\mu}\left(\vartheta_1 x^\mu + \vartheta_2 y^\mu + \vartheta_3\, t^\mu\right) + \vartheta_4)\right] \times \frac{1}{2}\left(\tan\left(\frac{\sqrt{\epsilon}}{2}(\frac{\Gamma(\beta+1)}{\mu}\left(x^\mu + a y^\mu - \epsilon\, t^\mu\right)\right) - \cot\left(\frac{\sqrt{\epsilon}}{2}(\frac{\Gamma(\beta+1)}{\mu}\left(x^\mu + a y^\mu - \epsilon\, t^\mu\right)\right)\right), \tag{2.12}$$

where $a = -(\vartheta_2 + \vartheta_3\epsilon)$.

Case 2:

$$d_0 = 0,\ d_1 = \sqrt{\frac{-\vartheta_1^2+\vartheta_3^2-2\vartheta_2}{2\epsilon}},\ \epsilon = \mp\sqrt{\frac{2\epsilon-\vartheta_1^2+\vartheta_3^2-2\vartheta_2}{2\epsilon}}.$$

When $\epsilon < 0$, we have the solitary wave solutions as

$$u(x,y,t) = -\sqrt{\frac{\vartheta_1^2-\vartheta_3^2+2\vartheta_2}{2}} \times \exp\left[i(\frac{\Gamma(\beta+1)}{\mu}\left(\vartheta_1 x^\mu + \vartheta_2 y^\mu + \vartheta_3\, t^\mu\right) + \vartheta_4)\right] \times \tanh\left(\sqrt{-\epsilon}\left(\frac{\Gamma(\beta+1)}{\mu}\left(x^\mu + a y^\mu - \epsilon\, t^\mu\right)\right)\right), \tag{2.13}$$

or

$$u(x,y,t) = -\sqrt{\frac{\vartheta_1^2-\vartheta_3^2+2\vartheta_2}{2}} \times \exp\left[i(\frac{\Gamma(\beta+1)}{\mu}\left(\vartheta_1 x^\mu + \vartheta_2 y^\mu + \vartheta_3\, t^\mu\right) + \vartheta_4)\right] \times \coth\left(\sqrt{-\epsilon}\left(\frac{\Gamma(\beta+1)}{\mu}\left(x^\mu + a y^\mu - \epsilon\, t^\mu\right)\right)\right), \tag{2.14}$$

or

$$u(x,y,t) = \sqrt{\frac{\vartheta_1^2-\vartheta_3^2+2\vartheta_2}{2}} \times \exp\left[i(\frac{\Gamma(\beta+1)}{\mu}\left(\vartheta_1 x^\mu + \vartheta_2 y^\mu + \vartheta_3\, t^\mu\right) + \vartheta_4)\right] \times \left(-\tanh\left(2\sqrt{-\epsilon}(\frac{\Gamma(\beta+1)}{\mu}\left(x^\mu + a y^\mu - \epsilon\, t^\mu\right)\right) \pm i\,\mathrm{sech}\left(2\sqrt{-\epsilon}(\frac{\Gamma(\beta+1)}{\mu}\left(x^\mu + a y^\mu - \epsilon\, t^\mu\right)\right)\right), \tag{2.15}$$

or

$$u(x,y,t) = \sqrt{\frac{\vartheta_1^2-\vartheta_3^2+2\vartheta_2}{2}} \times \exp\left[i(\frac{\Gamma(\beta+1)}{\mu}\left(\vartheta_1 x^\mu + \vartheta_2 y^\mu + \vartheta_3\, t^\mu\right) + \vartheta_4)\right] \times \left(-\cot\left(2\sqrt{\epsilon}(\frac{\Gamma(\beta+1)}{\mu}\left(x^\mu + a y^\mu - \epsilon\, t^\mu\right)\right) \pm \csc\left(2\sqrt{\epsilon}(\frac{\Gamma(\beta+1)}{\mu}\left(x^\mu + a y^\mu - \epsilon\, t^\mu\right)\right)\right), \tag{2.16}$$

or

$$u(x,y,t) = -\sqrt{\frac{\vartheta_1^2-\vartheta_3^2+2\vartheta_2}{2}} \times \exp\left[i(\frac{\Gamma(\beta+1)}{\mu}\left(\vartheta_1 x^\mu + \vartheta_2 y^\mu + \vartheta_3\, t^\mu\right) + \vartheta_4)\right] \times \frac{1}{2}\left(\tan\left(\frac{\sqrt{\epsilon}}{2}(\frac{\Gamma(\beta+1)}{\mu}\left(x^\mu + a y^\mu - \epsilon\, t^\mu\right)\right) - \cot\left(\frac{\sqrt{\epsilon}}{2}(\frac{\Gamma(\beta+1)}{\mu}\left(x^\mu + a y^\mu - \epsilon\, t^\mu\right)\right)\right). \tag{2.17}$$

When $\epsilon > 0$, we procure the periodic function solutions as

$$u(x,y,t) = \sqrt{\frac{-\vartheta_1^2+\vartheta_3^2-2\vartheta_2}{2}} \times \exp\left[i(\frac{\Gamma(\beta+1)}{\mu}\left(\vartheta_1 x^\mu + \vartheta_2 y^\mu + \vartheta_3\, t^\mu\right) + \vartheta_4)\right] \times \tan\left(\sqrt{\epsilon}\left(\frac{\Gamma(\beta+1)}{\mu}\left(x^\mu + a y^\mu - \epsilon\, t^\mu\right)\right)\right), \tag{2.18}$$

or

$$u(x,y,t) = -\sqrt{\frac{-\vartheta_1^2+\vartheta_3^2-2\vartheta_2}{2}} \times \exp\left[i(\frac{\Gamma(\beta+1)}{\mu}\left(\vartheta_1 x^\mu + \vartheta_2 y^\mu + \vartheta_3\, t^\mu\right) + \vartheta_4)\right] \times \cot\left(\sqrt{\epsilon}\left(\frac{\Gamma(\beta+1)}{\mu}\left(x^\mu + a y^\mu - \epsilon\, t^\mu\right)\right)\right), \tag{2.19}$$

or

$$u(x,y,t) = \sqrt{\frac{-\vartheta_1^2+\vartheta_3^2-2\vartheta_2}{2}} \times \exp\left[i(\frac{\Gamma(\beta+1)}{\mu}\left(\vartheta_1 x^\mu + \vartheta_2 y^\mu + \vartheta_3\, t^\mu\right) + \vartheta_4)\right] \times \left(\tan\left(2\sqrt{-\epsilon}(\frac{\Gamma(\beta+1)}{\mu}\left(x^\mu + a y^\mu - \epsilon\, t^\mu\right)\right) \pm \sec\left(2\sqrt{-\epsilon}(\frac{\Gamma(\beta+1)}{\mu}\left(x^\mu + a y^\mu - \epsilon\, t^\mu\right)\right)\right), \tag{2.20}$$

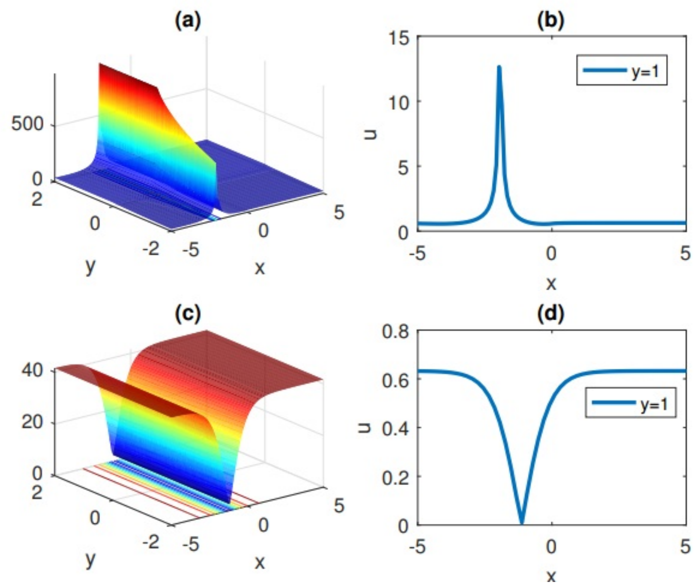

FIGURE 1. $3D$ and $2D$ wave profiles of solution (2.3), are displayed corresponding to $\mu = 0.75$ and $t = 1 = y$.

or

$$\begin{aligned} u(x,y,t) &= \sqrt{\tfrac{-\vartheta_1^2+\vartheta_3^2-2\vartheta_2}{2}} \times \exp\left[i(\tfrac{\Gamma(\beta+1)}{\mu}(\vartheta_1 x^\mu + \vartheta_2 y^\mu + \vartheta_3 t^\mu) + \vartheta_4)\right] \\ &\times \left(-\cot\left(2\sqrt{\epsilon}(\tfrac{\Gamma(\beta+1)}{\mu}(x^\mu + a y^\mu - \epsilon t^\mu)\right) \pm \csc\left(2\sqrt{\epsilon}(\tfrac{\Gamma(\beta+1)}{\mu}(x^\mu + a y^\mu - \epsilon t^\mu))\right)\right), \end{aligned} \tag{2.21}$$

or

$$\begin{aligned} u(x,y,t) &= \sqrt{\tfrac{-\vartheta_1^2+\vartheta_3^2-2\vartheta_2}{2}} \times \exp\left[i(\tfrac{\Gamma(\beta+1)}{\mu}(\vartheta_1 x^\mu + \vartheta_2 y^\mu + \vartheta_3 t^\mu) + \vartheta_4)\right] \\ &\times \tfrac{1}{2}\left(\tan\left(\tfrac{\sqrt{\epsilon}}{2}(\tfrac{\Gamma(\beta+1)}{\mu}(x^\mu + a y^\mu - \epsilon t^\mu)\right) - \cot\left(\tfrac{\sqrt{\epsilon}}{2}(\tfrac{\Gamma(\beta+1)}{\mu}(x^\mu + a y^\mu - \epsilon t^\mu))\right)\right), \end{aligned} \tag{2.22}$$

where $a = -(\vartheta_2 + \vartheta_3 \epsilon)$.

3. Physical explanation

The dynamics of obtained solutions are presented here via $2D$ and $3D$ graphics in 1-4. For the sake of simplicity we take $\vartheta_1 = 0.3 = \vartheta_3$, $\vartheta_2 = 0.4$, $\vartheta_4 = 0.2$ and $\beta = 1$. Furthermore, the obtained solutions given by Eqs. (2.18), (2.19), (2.20), and (2.21) are presented here via $2D$ and $3D$ graphics 5-6. For the sake of simplicity we take $y = 1, \vartheta_1 = 1 = \vartheta_3$, $\vartheta_2 = 1$ and $\vartheta_4 = 0.2$. We now take $t = 1 = \beta$, $\vartheta_1 = 0.2 = \vartheta_3$, $\vartheta_2 = 0.3$ and $\vartheta_4 = 0.2$.

4. Conclusion

The modified simplest equation method has been executed to construct a new complex solitary wave solutions for a complex hyperbolic Schrodinger dynamical equation. We have discussed the aforesaid equation in the sense of a conformable M-fractional derivative operator. Such nonlinear Schrodinger equations have an important role in physics and many other nonlinear sciences. The acquired solutions might have a significant role in the above-mentioned areas as they have the complex solitary wave, periodic, and rational functions forms. These solutions have also been presented with their graphical demonstrations by choosing suitable values of involved parameters.

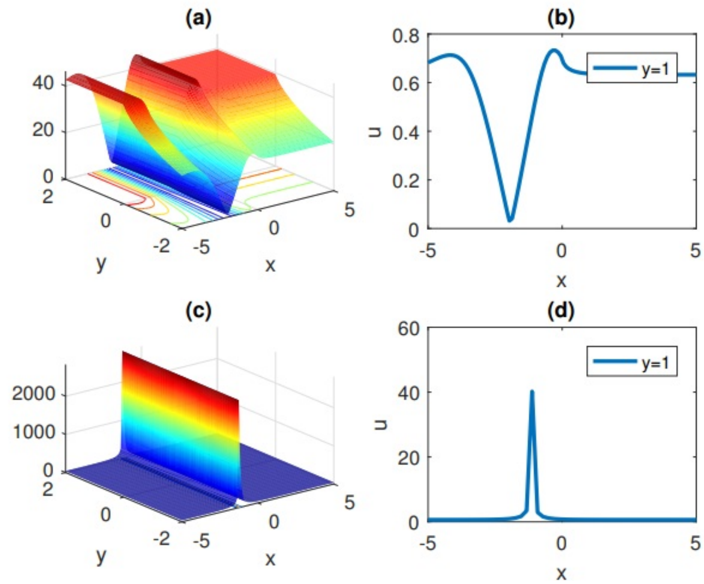

FIGURE 2. $3D$ and $2D$ wave profiles of solution (2.4), are displayed corresponding to $\mu = 0.75$ and $t = 1 = y$

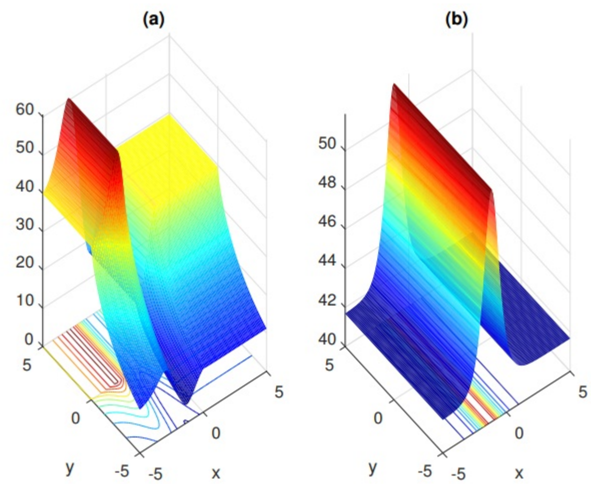

FIGURE 3. $3D$ wave profiles of solution (2.5), are displayed corresponding to $\mu = 0.75, 1$ and $t = 1$.

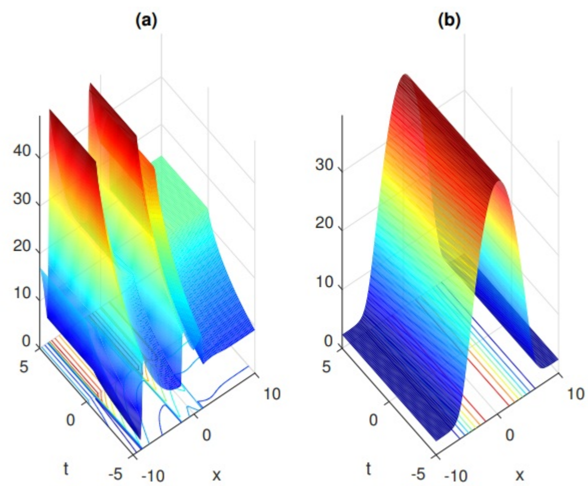

FIGURE 4. $3D$ wave profiles of solution (2.6), are displayed corresponding to $\mu = 0.75, 1$ and $t = 1$.

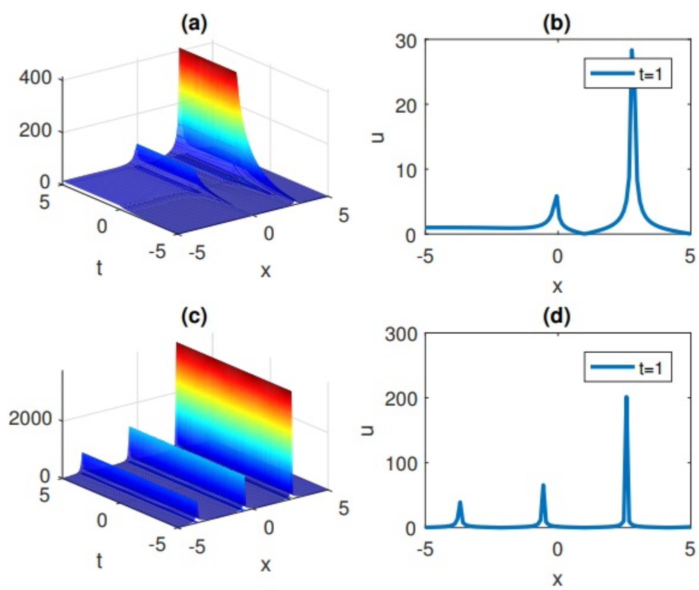

FIGURE 5. $2D$ and $3D$ wave profiles of Eq. (2.18), are displayed corresponding to $\mu = 0.75, 1$, $\beta = 1$ and $\epsilon = 1$.

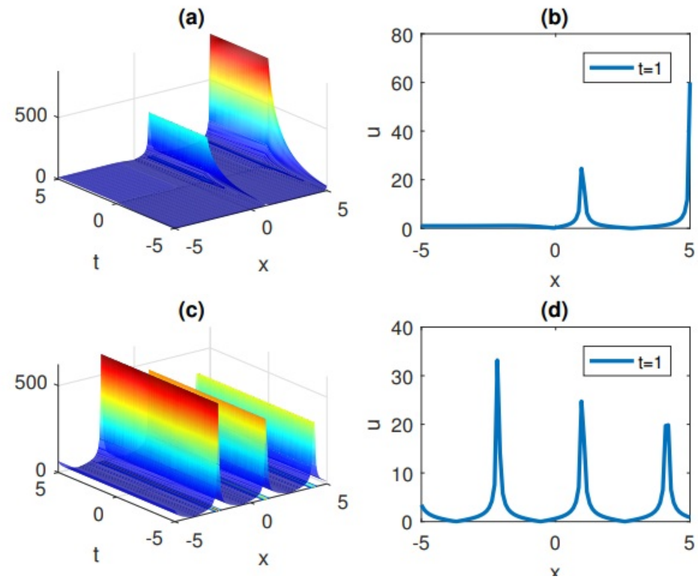

FIGURE 6. $2D$ and $3D$ wave profiles of Eq. (2.19), are displayed corresponding to $\mu = 0.75, 1$, $\beta = 1$ and $\epsilon = 1$.

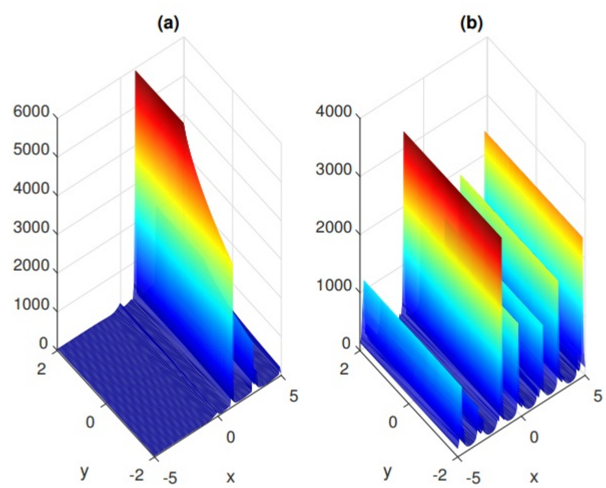

FIGURE 7. Two different $3D$ wave profiles of Eq. (2.20), are displayed corresponding to $\mu = 0.75, 1$ and $\epsilon = 1$.

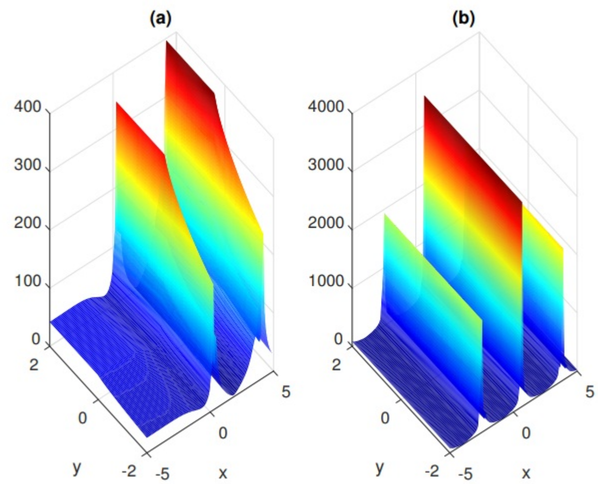

FIGURE 8. Two different $3D$ wave profiles of Eq. (2.21), are given corresponding to $\mu = 0.75, 1$ and $\epsilon = 1$.

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