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Effect of Fractal Tree Structures on Fruit’s Vibration Amplitude in Mechanical Harvesting Process

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Abstract

Tree structures are complex and random in nature but they follow some specific rules according to their various kinds. The objective of this work is to model three fractal trees’ dynamic behavior in the mechanical shaking process and study the effect of the bifurcation ratio and the length-order ratio (pruning ratio) on the frequency response and the percentage of the harvested fruit. Experimentally the static loadings are carried out on the trunk and branches of the tree to obtain their important elastic properties. In order to obtain viscoelastic properties of the tree, its dynamic free vibration is examined. The fractal trees are modeled with an equivalent multi degree of freedom mass-spring models and the governing equations of motion are derived by means of Newton’s second law; then, they are solved numerically for some sample fractal trees. Consequently, for different values of the bifurcation ratio and the length-order ratio (fractal dimension), the displacement of the fruit is calculated and the displacement amplitude of the fruit is obtained. Also, the effects of the tree structures and pruning ratios on the displacement amplitude of the fruit are discussed.

Keyword: Fractal structures, Fractal dimension, Bifurcation ratio, Pruning ratio, Frequency response

اثر ساختارهای فراکتال درختان روی دامنه ارتعاش میوه در فرآیند برداشت مکانیکی

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چکیده

ساختارهای درختی در طبیعت پیچیده و تصادفی هستند اما با توجه به انواع مختلف شان از قوانین خاصی پیروی می کنند. هدف از این کار مدل سازی رفتار دینامیکی سه درخت فراکتال در فرآیند تکان دادن مکانیکی و مطالعه تأثیر نسبت دو شاخه شدن و نسبت هرس بر پاسخ فرکانسی و درصد میوه‌های برداشت شده، هست. به طور تجربی برای بدست آوردن خواص مهم الاستیکی، بارهای استاتیکی بر روی تنه و شاخه های درخت وارد می شوند. همچنین برای به دست آوردن خواص ویسکوالاستیک درخت، ارتعاش آزاد دینامیکی آن بررسی می شود. درختان فراکتال با مدل‌های معادل جرم-فنر چند درجه آزادی مدل سازی می‌شوند و معادلات حاکم بر حرکت با استفاده از قانون دوم نیوتن به دست می‌آیند؛ سپس، آنها به صورت عددی برای برخی از درختان فراکتال نمونه حل می شوند. در نتیجه برای مقادیر مختلف نسبت انشعاب و نسبت هرس (بعد فراکتال)، جابجایی میوه محاسبه شده و دامنه جابجایی میوه به دست می آید. همچنین اثرات ساختارهای درختی و نسبت های هرس بر دامنه جابجایی میوه مورد بحث قرار می‌گیرد.

واژه‌های کلیدی: ساختارهای فراکتالی، بعد فراکتال، نسبت دو شاخه شدن، نسبت هرس و پاسخ فرکانسی

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Introduction

Fruit shakers shake the trunk or branches or have contact heads with rods that extend into the canopy (Giametta and Bernardi, 2010; Lavee, 2010; Ravetti and Robb, 2010; Tous *et al.*, 2010; Vieri and Sarri, 2010; Sola-Guirado *et al.*, 2014; Moreno *et al.*, 2015; Sola-Guirado *et al.*, 2016; Zhang *et al.*, 2016; Sola-Guirado *et al.*, 2018; Peça *et al.*, 2019). Frequency and amplitude are among the principal operating parameters of the shakers concerning humans, trees, and fruit (El Attar *et al.*, 2004; Blanco-Roldán *et al.*, 2009; Zhou *et al.*, 2013; He *et al.*, 2017). Due to the damage of high frequency on humans, the manufacturers must declare the acceleration value in the machine instruction manual (Saraçoğlu *et al.*, 2011; Deboli *et al.*, 2014). However, manufacturers of shakers do not often know which variety of their machines will be used on; therefore, they design shakers with fixed acceleration and frequency values leaving the operators the responsibility of choosing the suitable shaking mode (Costa *et al.*, 2013).

Fractal trees have been employed in a wide variety of applications including drainage networks, actual plants and trees, root systems, bronchial systems, cardiovascular systems, and evolution (Newman *et al.,* 1997). Many scientists strongly have opinion that fractal geometry is a revolutionary area of mathematics which has deep impact on every branch of science fields. For two thousand years, it's been tried to describe nature with the help of Euclidean geometry. But it does not follow Euclidean geometry because biological systems are predominantly irregular, complex and non-linear. Therefore, irregularities of biological system cannot be quantified by means of classical Euclidean geometry. In order to overcome these limitations of Euclidean geometry, Mandelbrot first time formalized the idea of fractal dimension (Mandelbrot, 1967). He introduced the term "fractal geometry" and attempted to describe the behavior of chaos in nature. The fractal geometry is one of the important tools to explain true geometry of nature. In fact, this new area of mathematics enhances the power of Euclidean geometry. In addition, Euclidean geometry deals with objects in integer dimensions but fractal geometry deals with non-integer dimension.

Tree structures are complex and random in nature but they follow some specific rules according to their various kinds. A mathematical description of these structures seems to require a large number of independent parameters. However, it has been shown that geometrical features of most botanical structures can be described by only a few parameters using fractal theory (Mandelbrot, 1983). On the other hand, tree is truly a fractal object so that it is difficult to describe its shape in terms of Euclidean geometry.

The term, 'fractal,' comes from the Latin word 'Fractus' which means 'broken' or 'irregular' or 'unsmooth' (Rian *et al.*, 2007). Before one can apply fractal analysis to biological objects, it is necessary to understand the definition of a fractal (Schroeder, 1991; Peitgen *et al.*, 1992). Fractals or fractal objects are self-similar structures or scale-invariant structures (Mandelbrot, 1982; Mandelbrot, 1983) and they formed by a repetitive process in which each repetition builds on the prior result. In addition fractals have unique dimensions (roughness) that can be mathematically described. The properties of self-similar repetitions, abundance of textural details, and cascades of shape in architecture have been characterized by fractal geometry (Bovill, 1996). Fractals are objects that show self-similarity at different magnifications. The fractal dimension is a measure of the roughness of a fractal structure.

It can be understood as a form of symmetry. Fractals are symmetric under changes of scale, which means that fractals are invariant under a change of length scale. In other words, fractals look the same under various degrees of magnification or scale. This definition is true for regular or deterministic fractals, such as those that may be generated on a computer by joining together similar shapes according to an algorithm (Masters, 2004). Obviously, the dynamic response of a tree depends heavily on its actual geometric structure as well as its physical properties. Most trees have a branched structure, expressing repeating architectures on different scales. Fractal analysis is used especially for the evaluation of structural properties of soil. Oleschko *et al*. (1998) show the estimation of the fractal dimension of the soil solid and pore systems along the lines and across areas as useful parameter for monitoring the impact of tillage on physical properties of soil and also for evaluation of soil compaction degree.

Feder (1988) considerably developed the fractal analysis. The first Mandelbrot's works were from the field of geophysics. They were treated by the characteristics of seacoast relief. Mandelbrot showed that the curves observed at the different scales are able to refer one to another in form of power low. The exponent was denominating the fractal dimension. Afterward Mandelbrot applied the concept of fractal geometry on the areas as price diversification, frequency of words in the books, embranchment of respiration tubes, rivers, and tree branches (Mandelbrot, 1982). In this paper, the dynamic response of fractal trees with self-similar structures in harvesting process by trunk shakers is analyzed. The objective of this document is to improve force transferred to fruits in harvesting process and also, to evaluate fruit response frequency in the different tree structures (with different fractal dimension) in mechanical harvesting system by trunk shaker. The response amplitudes of the olive tree are determined in terms of shaker excitation frequency.

Materials and methods

Case study

This study is carried out on the six years old olive tree in Dez lake region (Iran). In order to evaluate realistic dynamic response of the olive trees (with different structures) in harvesting process; at first, the elastic and viscoelastic properties of the tree must be determined. Therefore, static and dynamic tests were carried out on the olive tree. A summary of static and dynamic (pull and release test) tests results are presented in Tables 1-3:

Table 1. Results of static pull test

Years Old Tree	Force (N)	Deflection of the Trunk (m)	Stiffness Coefficient of the Trunk (N/m)
For Five Years Old Tree	100	0.016	6250
	120	0.019	6315.789
	140	0.022	6363.636
For Six Years Old Tree	100	0.015	6666.667
	120	0.018	6666.667
	140	0.021	6666.667
For Seven Years Old Tree	100	0.015	6666.667
	120	0.017	7058.824
	140	0.02	7000

Table 2. Results of free vibration test for olive trunk

Years Old Tree	Force (N)	$\mathbf{x_1}$(mm)	$\mathbf{x_2}$(mm)	$\boldsymbol{\delta}_{\mathbf{Trunk}} = \ln\frac{\mathbf{x_1}}{\mathbf{x_2}}$
For Five Years Old Tree	100	3	1	1.098612289
	120	4	2	0.693147181
	140	5	2.2	0.820980552
For Six Years Old Tree	100	3	1	1.098612289
	120	4	2	0.693147181
	140	5	2.2	0.820980552
For Seven Years Old Tree	100	3	1	1.098612289
	120	4	2	0.693147181
	140	5	2.2	0.820980552

Table 3. Results of tree vibration test for olive fruit

	Force (N)	$\mathbf{x_1}$(mm)	$\mathbf{x_2}$(mm)	$\boldsymbol{\delta}_{\mathbf{Trunk}} = \ln\frac{\mathbf{x_1}}{\mathbf{x_2}}$
For Fruit of the Olive Tree	100	5.5	5	0.09531018
	120	5	4.5	0.10536052
	140	6	5	0.18232156

In the second place, to describe physical modeling of trees, and in particular the dynamic behavior of vibrant tree with different structures, theoretical fractal trees are considered (Exact self-similarity). These fractals contain exact copies of them self through all scales. These are idealized structures with self-repeating structures. As shown in Fig 1, a fractal olive tree with seven levels of branching is used for this paper:

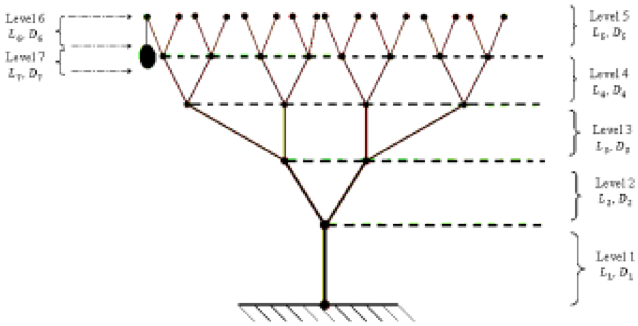


Fig 1. A schematic side view of the seven levels of branching for a fractal tree structure.

The bottom level belongs to the trunk and other levels are considered for the canopy tree. A level is a group of branches that can be considered a node in tree and at a node where an originating branch divides into two or n subsequent branches. The branching angle for a subsequent branch is defined by the angle between the originating and subsequent branches. For the describe branching between branch N and branches N+1 in a fractal model, it's required to obtain geometric parameters. Geometric parameters of the olive tree were determined that consist of three parameters: diameters and lengths of each branch in different levels, the angle of branching, and the number of levels of branching. In addition, for using the aforementioned parameters the bifurcation ratio is introduced as follow (Horton, 1945):

$$R_b = \frac{N_i}{N_{i+1}} \qquad (1)$$

Also, the length-order ratio or pruning ratio is as follows:

$$R_r = \frac{r_{i+1}}{r_i} \qquad (2)$$

Where, N_i is the number of branches of level i, and r_i is the mean length of branches of level i. In order to illustrate relationship the bifurcation ratio and pruning ratio of the tree, fractal dimension is introduced as follow (Newman *et al.*, 1997):

$$D = \frac{\ln R_b}{\ln R_r} \tag{3}$$

In order to construct fractal algorithms for this paper, it's considered the olive tree (with a variety of fractal structures) grows from bottom up by branching over and over again and then branches can be split into two (binary fractal), or three (ternary fractal) and or four (quadruple fractal) lateral segments that each branch away from the original branch's axis by an angle. Then this process repeats in canopy's five successive levels. In addition, each branch in a given level shares an identical (equal) length and diameters with other branches in that level and for this model tree branches are assumed not to taper. A summary of main data of the sample fractal tree is presented in Table 4:

Table 4. Main data of the sample fractal tree

The Number of the Levels	The Diameter of the Levels (**m**)	The Length of the Levels (**m**)		
		$R_r = 0.25$	$R_r = 0.5$	$R_r = 0.75$
Level (1)	0.22	1.00	1.00	1.00
Level (2)	0.11	0.25	0.5	0.75
Level (3)	0.063	0.0625	0.25	0.5625
Level (4)	0.0367	0.0156	0.125	0.4218
Level (5)	0.0211	0.003906	0.625	0.3164
Level (6)	0.002	0.014	0.014	0.014
Level (7)	0.04	0.02	0.02	0.02

Absolute Binary Fractal Model

In order to construct this fractal tree, it's used from a generator that its structure has constant bifurcation ratio ($R_b = 1/2$) and a variety of the pruning ratio ($R_r = 0.25, 0.5, 0.75$) so that fractal dimensions of this case are 0.5, 1.00 and 2.414 respectively. The first-level branch extends from tree base to its trunk end and there is a tip node at its end. The tip node is then divided into two equal branches. Also the second-level branches extend from its tip to the other tip node at the third-level. It can be noted that in this case, highest nodes are then divided into new four equal branches. The end of each new branch at the third-level, are separated into eight equal branches. So that the generated eight equal branches of the fourth-level are divided into sixteen new identical branches. In the final-levels, there are sixteen stem and fruit. As it can be seen from above, binary tree grows up very regularly with constant bifurcation ratio.

Absolute Ternary Fractal Model

The construction of presented fractal model has constant bifurcation ratio ($R_b = 1/3$) and a variety of the pruning ratio ($R_r = 0.25, 0.5, 0.75$) so that the fractal dimensions of this case are 0.778, 1.556 and 3.758 respectively. In this case, the first-level branch extends from its base to stalk end and there is a tip node at its end and tip node is then divided into three equal branches. Also, the second-level branches extend from its base to the other tip node at the third-level. In order to illustrate third-level, highest nodes are then divided into new nine equal branches. The end of each new branch at the third-level, are separated into 27 equal branches. So that the generated 27 equal branches of the fourth-level are divided into 81 new identical branches. In the final-levels, there are 81 stem and fruit. In both aforementioned cases, the variety of tree shapes could be captured by change in the pruning ratio meanwhile its bifurcation ratio was constant. Also, the generator can be considered with a variety of bifurcation ratio and constant pruning ratio that the captured models in both states are analyzed.

Absolute Quadruple Fractal Model

For the design of this fractal tree model, its structure has constant bifurcation ratio ($R_b = 1/4$) and a variety of pruning ratio ($R_r = 0.25, 0.5, 0.75$) so that fractal dimensions of this case are 1.00, 2.00 and 4.829 respectively. In this case, the first-level branch extends from its base to stalk end and there is a tip node at its end and tip node is then divided into four equal branches. Also, the second-level branches extend from its base to the other tip node at the third-level. In order to illustrate third-level, highest nodes are then divided into new 16 equal branches. The end of each new branch at the third-level, are separated into 64 equal branches. So that the generated 64 equal branches of the fourth-level are divided into 256 new identical branches. In the final-levels, there are 256 stem and fruit.

Ternary-Binary compound Fractal Model

In order to construct this fractal tree, it's used from a generator that its structure has two constant bifurcation ratio ($R_b = 1/3, R_b = 1/2$) and a variety of the pruning ratio ($R_r = 0.25, 0.5, 0.75$) so that fractal dimensions of the ternary and binary levels have two different quantities. Obviously, it can be noted that the fractal dimension of the ternary levels are more than the binary levels. This model has three samples that each sample is distinct by a variety of pruning ratio as shown in Table 5:

Table 5. The branch's number of each level and its fractal dimensions in three sample olive trees
(Ternary-Binary Compound Fractal Tree Model)

The Number of the Levels	1	2	3	4	5	6	7	Fractal Dimension					
								$R_r = 0.2$		$R_r = 0.5$		$R_r = 0.75$	
Sample								D_1^*	D_2^*	D_1	D_2	D_1	D_2
1	1	3	9	27	54	54	54	0.778	0.5	1.556	1.0	3.758	2.414
2	1	3	9	18	36	36	36	0.778	0.5	1.556	1.0	3.758	2.414
3	1	3	6	12	24	24	24	0.778	0.5	1.556	1.0	3.758	2.414

* Fractal dimension of the ternary levels
** Fractal dimension of the binary levels

Quadruple-Ternary- Binary compound Fractal Model

The generator for this construction is made from combining bifurcation ratios of the quadruple, ternary and binary models ($R_b = 1/4,\ R_b = 1/3$, $R_b = 1/2$) and a variety of the pruning ratio ($R_r = 0.25, 0.5, 0.75$). Fractal dimension of the tree in this model has three different dimensions in various levels that include dimensions of the quadruple, ternary and binary levels. Also, this fractal tree model has six samples that each sample is distinct by a specific pruning ratio as shown in Table 6:

Table 6. The branch's number of each level and its fractal dimensions in six sample olive trees
(Quadruple-Ternary- Binary Compound Fractal Model)

The Number of the Levels	Level (1)	Level (2)	Level (3)	Level (4)	Level (5)	Level (6)	Level (7)	Fractal Dimension								
								$R_r = 0.25$			$R_r = 0.5$			$R_r = 0.75$		
Sample								D_1^*	D_2^{**}	D_3^{***}	D_1	D_2	D_3	D_1	D_2	D_3
1	1	4	16	64	192	192	192	1.0	0.778	0	2.0	1.556	0	4.829	3.758	0
2	1	4	16	48	144	144	144	1.0	0.778	0	2.0	1.556	0	4.829	3.758	0
3	1	4	12	36	108	108	108	1.0	0.778	0	2.0	1.556	0	4.829	3.758	0
4	1	4	12	36	72	72	72	1.0	0.778	0.5	2.0	1.556	1.0	4.829	3.758	2.414
5	1	4	12	24	48	48	48	1.0	0.778	0.5	2.0	1.556	1.0	4.829	3.758	2.414
6	1	4	8	16	32	32	32	1.0	0	0.5	2.0	0	1.0	4.829	0	2.414

* Fractal dimension of the quadruple levels
** Fractal dimension of the ternary levels
*** Fractal dimension of the binary levels

Mathematical Modeling:

As shown in Fig 2, a fractal tree with masses of the trunk, branch and fruits is considered. The lower part of the fractal tree is coupled to a set of shaker, which vibrates fractal tree by a harmonic force. The equivalent mass of the trunk, branches and fruit are $M_1, M_2, \ldots$ and M_n, respectively:

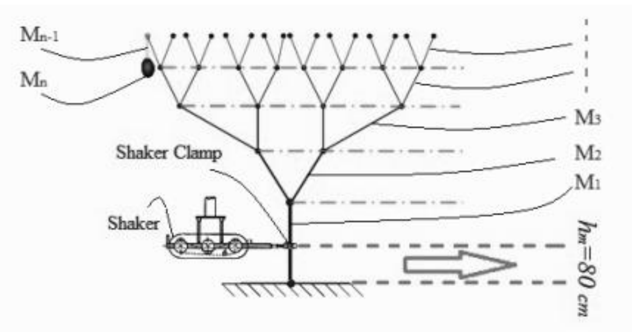


Fig 2. A schematic side view of the fruit tree in mechanical harvesting process

For comparison, mathematical modeling and analysis dynamic behavior of the vibrant-fractal tree by shaker (rotating eccentric weights type) is considered in a mass-spring system with Multiple-degree-of-freedom (MDOF) and as a fractal structure, that it was used to determine a dynamic structural analysis of fractal trees as shown in Fig 3 (Alstrup *et al.*, 2005):

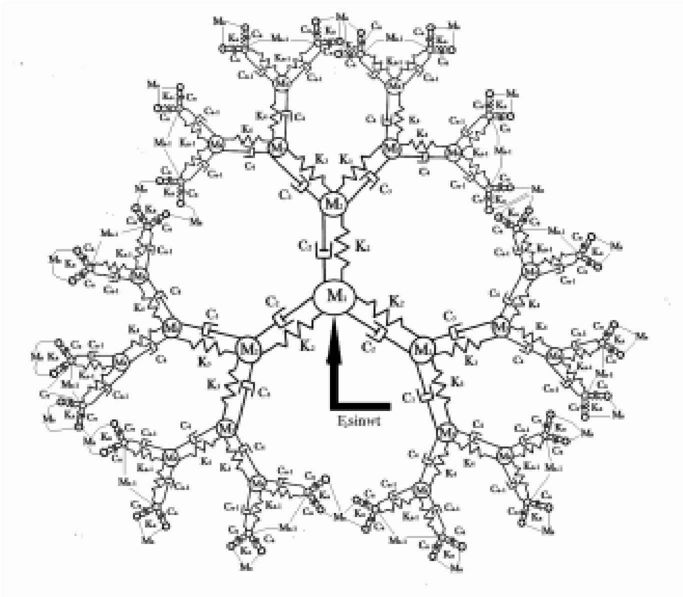


Fig 3. A schematic top view of the mass-spring system for a fractal tree structure

For measuring displacement amplitude of the each level, symbolic points are shown (one for each level) in Fig 1.The differential equations motion of the equivalent mass-spring linear model of the fractal tree is derived by means of Newton's second law in each level as follow:

$$\begin{aligned}&M_1\ddot{x}_1 - k_1x_1 - Sk_2(x_1-x_2) - c_1\dot{x}_1 - \\ &Sc_2(\dot{x}_1-\dot{x}_2) + F_0 \sin \omega t = 0\\ &SM_2\ddot{x}_2 + Sk_2(x_1-x_2) - Zk_3(x_2-x_3) + \\ &Sc_2(\dot{x}_1-\dot{x}_2) - Zc_3(\dot{x}_2-\dot{x}_3) = 0\\ &\vdots\\ &\vdots\\ &GM_{n-1}\ddot{x}_{n-1} + Gk_{n-1}(x_{n-2}-x_{n-1}) - Gk_n(x_{n-1}-x_n)\\ &Gc_{n-1}(\dot{x}_{n-2}-\dot{x}_{n-1}) - Gc_n(\dot{x}_{n-1}-\dot{x}_n) = 0\\ &GM_n\ddot{x}_n + Gk_n(x_{n-1}-x_n) + Gc_n(\dot{x}_{n-1}-\dot{x}_n) = 0\end{aligned} \tag{4}$$

Also, these equations can be expressed in matrix form as:

$$[\boldsymbol{M}]\ddot{\vec{x}} + [\boldsymbol{c}]\dot{\vec{x}} + [\boldsymbol{k}]\vec{x} = \vec{\boldsymbol{F}} \tag{5}$$

where $[\mathbf{M}]$ are the components of the mass matrix depending on the values of the stalk, branch or fruit tree in each level (kg), $[\mathbf{c}]$ are the elements of the equivalent viscous damping matrix in each level $(\mathrm{N.s/m})$, $[\mathbf{k}]$ are the components of stiffness matrix depending on the values of the apparent spring constant of the stalk, branch or fruit tree in each level ($\mathrm{N/m}$), ω is the shaking frequency or excitation frequency ($\mathrm{rad/s}$), t is the time (s) and the vectors $\vec{\mathrm{x}}$, $\dot{\vec{\mathrm{x}}}$ and $\ddot{\vec{\mathrm{x}}}$ indicate, respectively, the vectors of displacements (m), velocities $(\mathrm{m/s})$, and accelerations $(\mathrm{m/s^2})$ of the stalk, branch or fruit tree in each level, and $\vec{\mathbf{F}}$ represents the vector of force acting on the trunk mass (N). The components of the mass, viscose damping, stiffness, force matrices as follow:

$$\mathbf{M} = \begin{bmatrix} M_1 & 0 & 0 & \dots & 0 & 0\\ 0 & SM_2 & 0 & \dots & 0 & 0\\ 0 & 0 & ZM_3 & \dots & 0 & 0\\ \vdots & & & \ddots & & \\ 0 & 0 & 0 & \dots & GM_{n-1} & 0\\ 0 & 0 & 0 & \dots & 0 & GM_n \end{bmatrix} \tag{6}$$

And

$$\mathbf{C} = \begin{bmatrix} c_1+Sc_2 & -Sc_2 & 0 & \dots & 0 & 0\\ -Sc_2 & Sc_2+Zc_3 & -Zc_3 & \dots & 0 & 0\\ 0 & -Zc_3 & Zc_3+Yc_4 & \dots & 0 & 0\\ \vdots & & & & & \\ 0 & 0 & 0 & \dots & Gc_{n-1}+Gc_n & -Gc_n\\ 0 & 0 & 0 & \dots & -Gc_n & Gc_n \end{bmatrix} \tag{7}$$

And

$$\mathbf{K} = \begin{bmatrix} k_1+Sk_2 & -Sk_2 & 0 & \dots & 0 & 0\\ -Sk_2 & Sk_2+Zk_3 & -Zk_3 & \dots & 0 & 0\\ 0 & -Zk_3 & Zk_3+Yk_4 & \dots & 0 & 0\\ \vdots & & & & & \\ 0 & 0 & 0 & \dots & Gk_{n-1}+Gk_n & -Gk_n\\ 0 & 0 & 0 & \dots & -Gk_n & Gk_n \end{bmatrix} \tag{8}$$

Where S, Z, Y, … and G are numbers of the branches in each level, respectively. The applied force in this study is frequency proportional actuation:

$$\vec{\boldsymbol{F}} = [mr\omega^2 \quad 0 \quad 0 \quad 0 \quad \dots \quad 0 \quad 0 \quad 0 \quad 0]^T \tag{9}$$

Which m is the unbalanced mass (kg), r is the distance from the axis to the center of mass (m) in the inertia shaker with rotating eccentric weights mechanism.

Eigen Frequency Analysis

In order to determine the natural frequencies and the normal modes, the eigenvalue problem corresponding to the vibration of the undamped system is solved. The free vibration of the undamped system can be governed by the following (10):

$$[\boldsymbol{M}]\ddot{\vec{x}} + [\boldsymbol{k}]\vec{x} = \vec{0} \tag{10}$$

The solution of (10) is assumed to be harmonic as:

$$\vec{x}(t) = \vec{A}\sin(\omega t + \phi) \tag{11}$$

So that:

$$\ddot{\vec{x}}(t) = -\omega^2\vec{A}\sin(\omega t + \phi) \tag{12}$$

Where $\vec{A}$ is the vector of amplitudes of the $\vec{x}(t)$, ϕ is the phase angle. Substituting (11) and (12) into (10) gives:

$$\big[[\,\boldsymbol{k}\,] - \omega^2[\mathbf{m}]\big]\vec{A} = \vec{0} \tag{13}$$

Equation (13) represents a system of n algebraic homogeneous equations with unknown coefficients $\mathrm{A_1, A_2, \dots}$ and $\mathrm{A_n}$ (amplitudes of $\mathrm{x_1, x_2, \dots}$ and $\mathrm{x_n}$.) For a nontrivial solution of the vector of coefficients $\vec{\mathrm{A}}$, the determinant of the coefficient matrix must be equal to zero:

$$|[\boldsymbol{K}] - [\boldsymbol{M}]\omega^2| = 0 \tag{14}$$

Equation (14) is the characteristic equation or frequency equation. The roots of this equation give the n Eigenvalues $(\omega_1^2, \omega_2^2, \dots, \omega_n^2)$. The positive square roots of the Eigenvalues yield the natural frequencies of the system $(\omega_1, \omega_2, \dots, \omega_n)$.

Frequency Response of the System

In order to determine frequency response of the each level's component with Multiple-degree-of-freedom (MDOF) in considered fractal tree, matrix methods are indispensable. For solution of the (5) is assumed to be harmonic as $(\vec{x}(t) = \vec{A}\sin(\omega t + \phi))$ and substituting this equation into it and rearranging takes the following form:

$$[\vec{A}] = \left[-\omega^2[\boldsymbol{M}] + i\omega[\boldsymbol{c}] + [\boldsymbol{k}]\right]^{-1}[\vec{\boldsymbol{F}}] \tag{15}$$

Equation (15) denotes displacement amplitude of each level's components in fractal tree system when it is harvested by inertia shaker with rotating eccentric weights mechanism.

Results and discussion

The olive tree (with different fractal structures) vibrated by shaker (rotating eccentric weights type) is modeled with an equivalent Multiple-degree-of-freedom (MDOF) mass-spring system. The displacement amplitude of the fruit (D.A.F) is calculated for different values of the shaker excitation frequency. Then the D.A.F is calculated for olive trees with three different bifurcation ratio ($R_b = 1/4, R_b = 1/3$, $R_b = 1/2$) and pruning ratios ($R_r = 0.25, 0.5, 0.75$). In the sixth and seventh levels, the fruit is modeled with a two degree of freedom mass-spring system (in which the stem and fruit are modeled with a simple pendulum). So the considered DOF is one number more than the age the tree. The amplitude and area under the frequency response curve represent the transferred energy in the oscillations. The greatest energy occurs at the fundamental or first mode (James, 2010). The frequency response of the fruit for three fractal tree models with different pruning ratios is depicted in Figs 4 (a-c). Fig 4(a) shows the frequency response of the fruit for the binary, ternary and quadruple fractal models with short pruning ratio ($R_r = 0.25$). As shown from this figure, these models have exactly the same quantity of the maximum D.A.F. In addition, it can be seen from Fig 4b that the maximum D.A.F of the quadruple fractal tree is more than other models. Also in Fig 4c, the maximum D.A.F belongs to binary model. So it can be said that transferred force into the fruit will have significant increase by tall pruning operation in binary model.

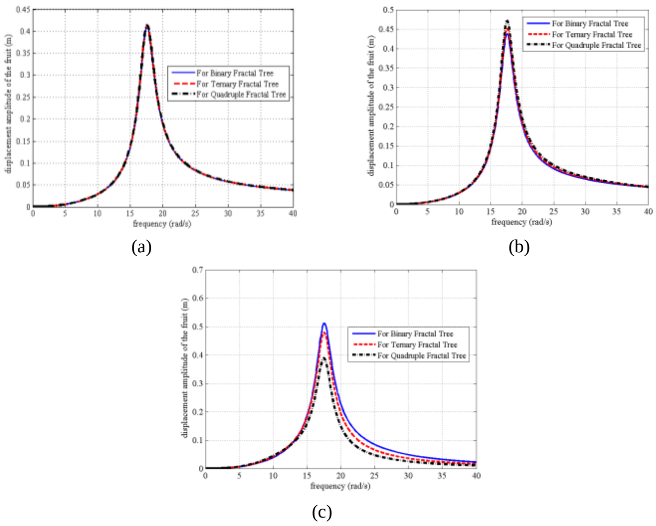


Fig 4. D.A.F of the fractal tree with the same pruning ratio and different bifurcation ratios, (a): R_r=0.25, (b): R_r=0.5 and (c): R_r=0.75

Fig 5 shows the D.A.F in different fractal tree models with a variety of the length-order ratios (pruning ratios) and constant bifurcation ratio. As shown in Figs 5(a-b) by increasing quantity of pruning ratio, D.A.F of the binary and ternary models ($R_b = 1/2, 1/3$) is increased respectively. But at the quadruple model ($R_b = 1/4$) as shown in Fig 5c, the maximum and minimum of the D.A.F are occurred in pruning ratios ($R_r = 0.5, 0.75$) respectively. Also, the results of the Figs 4, 5 show that in two states (fractal tree with the same pruning ratio-different bifurcation ratios and fractal tree with different pruning ratios-the same bifurcation ratios), the D.A.F of both models have an equal in quantity of the transferred force.

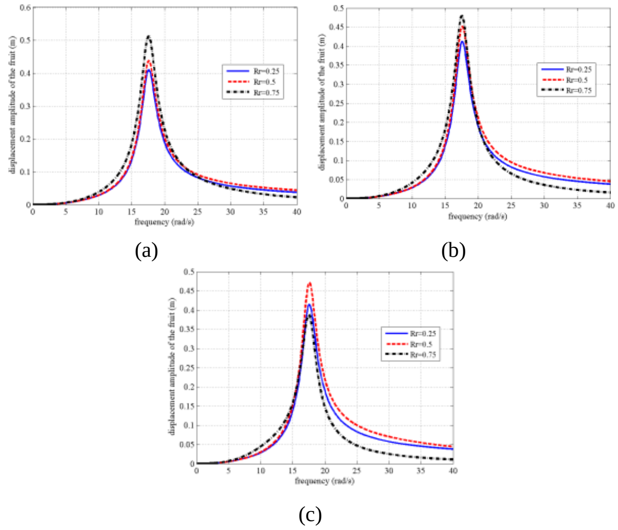


Fig 5. D.A.F of the fractal tree with different the pruning ratios and the same bifurcation ratios, (a): binary fractal (R_b = 1/2), (b): ternary fractal (R_b = 1/3) and (c): quadruple fractal (R_b = 1/4)

Fig. 6 shows frequency response of the ternary-binary fractal tree model with different pruning ratios and the bifurcation ratios. It can be figured out from Fig. 6 that the maximum of the D.A.F is gotten by applying tall pruning ratio ($R_r = 0.75$) in ternary-binary model. On the other hand, increasing the number of the binary branches is affected sharply on the maximum D.A.F of this model.

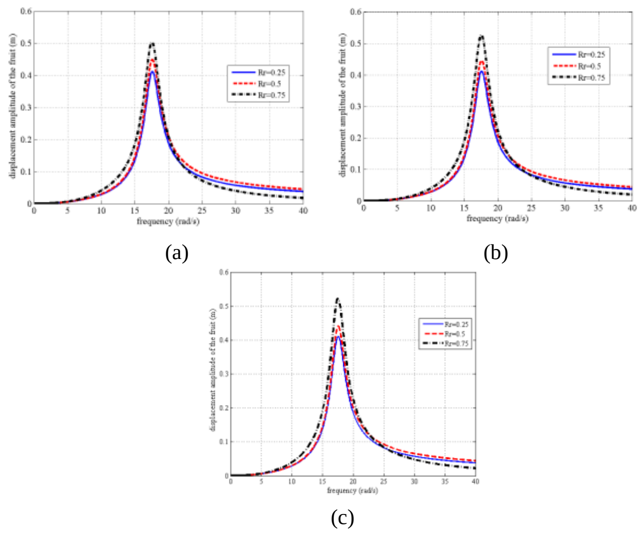
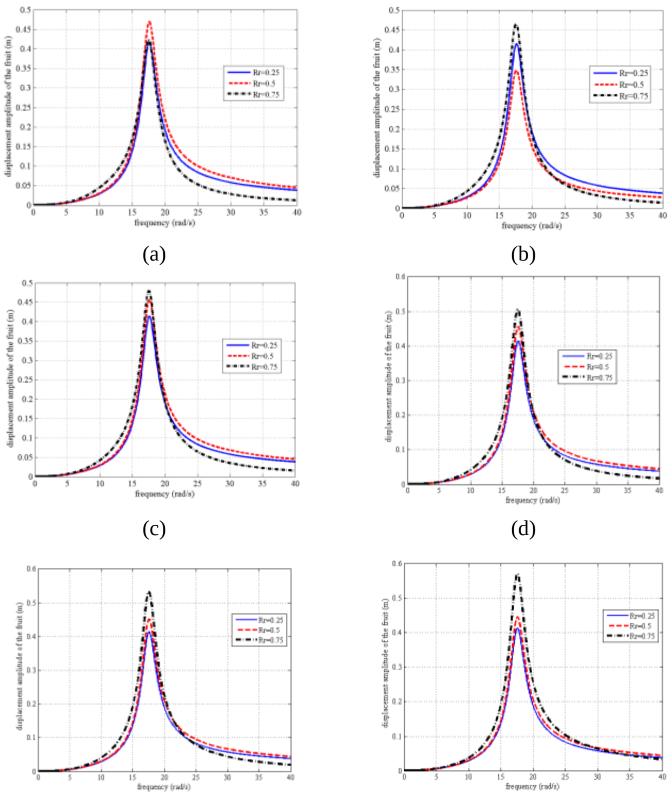


Fig 6. D.A.F of the compound fractal model (ternary-binary fractal tree) with different the pruning ratios and the bifurcation ratios (Rb=1/2, 1/3), (a): sample 1, (b): sample 2 and (c): sample 3

The frequency response of the fruit for compound fractal models (quadruple-ternary-binary fractal trees) with different pruning and the bifurcation ratios in six cases is depicted in Figs 7 (a-f). Fig. 7a shows that the fruit of this fractal model (sample 1) is received significant transferred force by applying specific pruning ratio 0.5. Therefore, it is true to say that quadruple branches have main role into transfer the maximum shaker force to the fruit in this model. As it is shown in Fig 7 (b-c), by increasing the number of the ternary branches, dominant fractal is ternary type in this model so that transferred force to the fruit is received significant by applying pruning ratio 0.75 to other pruning ratios. The results of Fig 7 (d-f) demonstrate that the maximum of the D.A.F is function of the binary branches in these models. As shown in these Figs, specific pruning ratio ($R_r = 0.75$) has global maximum D.A.F to other applied pruning ratios.



(a) (b) (c) (d) (e) (f)

Fig 7. D.A.F of the compound fractal model (quadruple–ternary- binary fractal tree) with different the pruning ratios and the bifurcation ratios (R_b=1/4, 1/3, 1/2), (a): sample 1, (b): sample 2, (c): sample 3, (d): sample 4, (e): sample 5 and (f): sample 6

4. Conclusions

A wide variety of fractal tree models have been developed for modeling the dynamic behavior of the vibrant olive tree with different structures. The obtained results of the mathematical modeling (with the same pruning ratio and different bifurcation ratios) have shown that by applying short pruning ratio ($R_r = 0.25$) to the three fractal models, fruit's vibration amplitude and value of the force transferred to it could be equaled in each three models. On the other hand, bifurcation ratio's role had not significant effect on force transferred to fruits whereas pruning ratio has a major effect on force transferred to fruits in harvest process.

It has been observed from results that the maximum amplitude of the fruit, selected tall ($R_r = 0.75$) and average ($R_r = 0.5$) pruning ratio could increase the transferred force rate to fruit in binary, ternary and quadruple tree models, respectively. In addition, the findings of the figures (the fractal tree with the different pruning ratios and the same bifurcation ratios) have demonstrated that fruit's vibration amplitude or D.A.F and value of the force transferred to it could be increased by applying the tall pruning ratio ($R_r = 0.75$) in binary and ternary tree models and the appropriate D.A.F of the quadruple model is affected by average pruning ratio ($R_r = 0.5$). In general, in order to have an effective mechanical harvest by trunk shaker, the proper pruning ratio should be selected considering type of the fractal tree models to obtain maximum fruit's vibration amplitude and the value of the force transferred to it. The results of the mathematical modeling for compound fractal trees have confirmed that the maximum of the D.A.F is function of the type bifurcation ratios of branches in the final levels. The binary and ternary branch types in final levels have transferred the maximum value of the force transferred rate to fruit, therefore, transferred force's dissipation will decrease to fruit which can increase the stress in the pedicle and the stem bottom and fruit removal of the trees will raise in shaking process.

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