Time Resource Management in Cognitive Radar Using Adaptive Waveform Design

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Abstract
Cognitive Radar is a recently presented research topic, in which most efforts has been done for its conceptual description and the adaptive waveform design feature of these radars, while other aspects of additivity for optimum performance of cognitive radars has been ignored. In this paper, a framework for adaptive time resource management in Cognitive Radars is proposed. The main purpose of this paper is proposing an algorithm for time resource management, with incorporation of adaptive waveform design capability of cognitive radars, to enhance the radar performance for an efficient time resource usage. After developing the equations of radar time resource management using adaptive waveform design, an implementable algorithm is proposed for this purpose and its performance is simulated and analysed. The results show that the proposed algorithm resulted in more efficient time resource management compared to the existing ones.

Keywords
Cognitive radar, radar resource management, adaptive waveform design, radar target tracking.

1. Introduction

Electronically beam steering feature of phased array radars provides controlling the radar beam instantly, and thus there would be no waiting period to direct the beam or mechanical inertia to overcome. These phased array radars, namely Multi-function radar (MFR), can handle a variety of tasks that previously multiple single function radars was used to perform, such as surveillance, multi-target tracking and missile guidance. Each of these tasks consume radar resources, so to use the MFR capabilities efficiently, an effective radar resource management (RRM) algorithm is required [1].

In 2006, Cognitive Radar concept was illustrated for the first time in [2]. The key point in any cognition-based system is the perception-action cycle [3]. In this concept, a cognitive radar "continuously learns about the environment through experience gained from interactions with the environment; the transmitter adjusts its illumination of the environment in an intelligent manner; and the whole radar system constitutes a dynamic closed feedback loop encompassing the transmitter, environment, and receiver." The feature of a cognitive radar that differs from a classical radar is the active feedback between receiver to transmitter [4]. A classical adaptive radar is only able to extract information from the target and the disturbance signals through appropriate signal processing algorithms and to apply that information at the receive level to improve its performance [5]. Conversely, a cognitive radar is able to use all of the extracted information not only at the receive level but also at the transmit level by changing the transmit frequency channel, waveform shape, time on target, pulse repetition frequency, transmitted power, number of pulses, polarization, and so forth [6].

Previous researches in cognitive radars mainly concentrated on adaptive waveform design for monostatic cognitive radars [7, 8], monostatic cognitive radars for wideband environment (like sonar) [9], multi-static cognitive radars [10, 11, 12], multi-static cognitive radars for extended targets [13] and for multi-static cognitive radars with for wideband environment [14].

In all of the previously presented investigations on cognitive radars, time resource management was not included in the proposed algorithms and their performance analysis. Thus, the effect of the environmental perceptions on radar time resources is not included in the previously proposed cost functions. In other words, there can be more improvement in radar performance, using the prior environmental information for optimum time resource usage in cognitive radars [15]. As stated in the definition of the cognitive radar, the loop-forward from receiver to transmitter can include the target revisit time, using the prior information about target, to manage radar resources more efficiently. Also, the previous attempts for task scheduling in resource managements incorporated only the target tracking quality and the waveform dependency of tracking error was not considered [16]. This paper concentrates on the dependency of tracking error, and thus tracking radar task scheduling, on adaptive waveform design feature of cognitive radars. This paper proposes a novel algorithm for resource management and adaptive waveform design in cognitive radars.

In this paper, a general framework for fully use of former information to mitigate a reduced tracking error and time resource saving algorithm for cognitive radars is proposed. In addition, its feasibility study using the Bayesian tracker equations is developed. Then an implementable algorithm for this purpose is presented and its performance is compared to the tracking algorithms in adaptive update rate case and adaptive waveform design situation.

This paper is organized as follows: In Section 2 cognitive tracking radar principles are being reviewed. In Section 3, the proposed framework for time resource management in cognitive radar and its feasibility study is presented. Section 4 includes the proposed implementable algorithm for target tracking resource management in cognitive radar. Section 5 presents simulation results and, finally, Section 6 concludes the paper.

2. Cognitive Tracking Radar

The state vector, in state-space model, contains all relevant information required to describe the system under investigation. For example, in tracking problems, this information include the kinematic characteristics of the target. The state equations are [17]:

$$\mathbf{x}_{k+1} = f(\mathbf{x}_k) + \mathbf{w}_k \tag{1}$$

$$\mathbf{z}_k = h(\mathbf{x}_k) + \mathbf{n}_k \tag{2}$$

In these equations, $\mathbf{x_k}$ is the target state at kth time instant, $\mathbf{z_k}$ is radar observations at kth time instant, $\mathbf{w_k}$ and $\mathbf{n_k}$ are model noise and observation noise, with covariance matrix of $\mathbf{Q}$ and $\mathbf{R}$ respectively.

The answer to the state space equations are obtained using the Bayesian equations [18]. The state estimation has two steps: update (estimation) step, in which the estimation of target state is obtained using the prior information and the current observations, and prediction step, in which the prediction of target state is obtained using the posterior density of the current state and target motion model. In addition, each step provides a measure of incertitude of the estimations and predictions.

There are two main groups of tracking algorithms based on state-space. One group are the Gaussian approximation approaches, and the other is the Bayesian algorithms with the Monte-Carlo simulation. In [19], an optimum filtering algorithm is presented for linear systems with Gaussian measurement and model noise. In the Gaussian approximation approaches, one tries to approximate the system model, so that the conditions for optimum filtering in [19] are satisfied. The Extended Kalman filter (EKF) [20], Unscented Kalman filter (UKF) [21] and Gauss-Hermit filter (GHF) [22] are the most common filters from this group. In Monte-Carlo simulation filters, one tries to solve the basic Bayesian equation using as much sample points as possible. Particle filter [23], cognitive structure adaptive particle filter [24], unscented particle filter (UPF) [25] and EKF-PF [26] are the most famous filters from the Monte-Carlo simulation category. In [27] a complete comparison of these algorithms is presented.

As proposed in [3 &28], Bayesian tracker is used in cognitive radars. Also, in [8], mathematical representation of cognitive radars is proposed. In this representation, observations vector, $\mathbf{z_k}$, is a function of target real state and environmental parameters (clutter, noise, etc.) and transmitted signal, $\boldsymbol{\theta}_\mathbf{k}$ [29]. Target state is a Markov model with probability density function of $P(\mathbf{x}_{k+1}|\mathbf{x}_k)$. In addition, observations have probability density function of $P(\mathbf{z}_k|\mathbf{x}_k)$. Probability density function of target state estimation is [30]:

$$P(\mathbf{x}_k|\mathbf{z}_{1:k}) = \frac{P(\mathbf{z}_k|\mathbf{x}_k,\mathbf{z}_{1:k-1})P(\mathbf{x}_k|\mathbf{z}_{1:k-1})}{\int P(\mathbf{z}_k|\mathbf{x}_k,\mathbf{z}_{1:k-1})P(\mathbf{x}_k|\mathbf{z}_{1:k-1})d\mathbf{x}_k} \tag{3}$$

Thus target state estimation via MMSE estimator would be:

$$\mathbf{x}_{\mathbf{k|k}} = \int \mathbf{x}_k\, P(\mathbf{x}_k|\mathbf{z}_{1:k})d\mathbf{x}_k \tag{4}$$

In addition, probability density function of target prediction at the next time instant is:

$$P(\mathbf{x}_{k+1}|\mathbf{z}_{1:k}) = \int P(\mathbf{x}_{k+1}|\mathbf{x}_k)\, P(\mathbf{x}_k|\mathbf{z}_{1:k})d\mathbf{x}_k \tag{5}$$

Therefore, MMSE estimation of target prediction is:

$$\mathbf{x}_{\mathbf{k+1|k}} = \int \mathbf{x}_{\mathbf{k+1}}\, P(\mathbf{x}_{\mathbf{k+1}}|\mathbf{z}_{\mathbf{1:k}})d\mathbf{x}_{\mathbf{k+1}} \tag{6}$$

As mentioned in the introduction section, there are many proposed cost functions for adaptive waveform design in cognitive radars. In [10], the following cost function is proposed:

$$\boldsymbol{\theta}_{\mathbf{k+1}} = \arg\min \int (\mathbf{x}_{\mathbf{k+1|k+1}} - \mathbf{x}_{\mathbf{k+1}})(\mathbf{x}_{\mathbf{k+1|k+1}} - \mathbf{x}_{\mathbf{k+1}})^T P(\mathbf{x}_{\mathbf{k+1}}|\mathbf{z}_{1:k})\, P(\mathbf{z}_{\mathbf{k+1}}|\mathbf{x}_{\mathbf{k+1}})d\mathbf{x}_{\mathbf{k+1}} \tag{7}$$

Also in [7, 8, 9, 11, 13, 14] this cost function has been adapted to the multi-static cognitive radars, extended targets and so on.

3. Resource Management Framework for Cognitive Radar

This section presents the proposed framework for resource management in cognitive radar and its feasibility study. Suppose target state at kth time instant is $\mathbf{x_k}$. As presented in previous radar resource management algorithms [16], if prediction error of k+1th time instant, provided by the tracking algorithm, was less than the tolerable error in tracking system, the state prediction might be used and time resources for receiving observations can be saved. Table 1 describes this concept.

Table 1. Adaptive update rate target tracking

	…	$(\mathbf{k}-\mathbf{1})^{\text{th}}$ time instant	$\mathbf{k}^{\text{th}}$ time instant	$(\mathbf{k}+\mathbf{1})^{\text{th}}$ time instant	…	$(\mathbf{k}+\mathbf{n})^{\text{th}}$ time instant
Update	-	…	$\mathrm{P}(\mathbf{x}_{\mathrm{k}}\mid\mathbf{z}_{1:\mathrm{k}})$	-	-	$\mathrm{P}(\mathbf{x}_{\mathbf{k+n}}\mid\mathbf{z}_{1:\mathrm{k}},\mathbf{z}_{\mathbf{k+n}})$
Prediction	-	$\mathrm{P}(\mathbf{x}_{\mathrm{k}}\mid\mathbf{z}_{1:\mathrm{k-1}})$	$\mathrm{P}(\mathbf{x}_{\mathbf{k+n}}\mid\mathbf{z}_{1:\mathrm{k}})$	-	-	…

This paper proposed a framework that causes further time resource preservations, due to the reduction of target tracking error using the adaptive waveform design. Fig. 1 presents the flow diagram of the proposed framework. The diagram shows that based on the estimation of tracking error and system tolerable tracking error, the next time for observing the target is determined. Then based on the prediction of target state in the determined time, the transmitted signal for this time instant should be designed. The challenges in this procedure are:

1- Prediction of target state in k+n$^{\text{th}}$ time instant, without using target observations from k$^{\text{th}}$ to k+n$^{\text{th}}$ time instant.
2- Estimation of target state at k+n$^{\text{th}}$ time instant after receiving observation in k+n$^{\text{th}}$ time instant, without any information about the previous target state and observations.

Therefore, the algorithm would have to perform the followings:

3.1. Target State Estimation

Equations (3-4) estimate the target state as usual.

3.2. Determination of Tracking Time

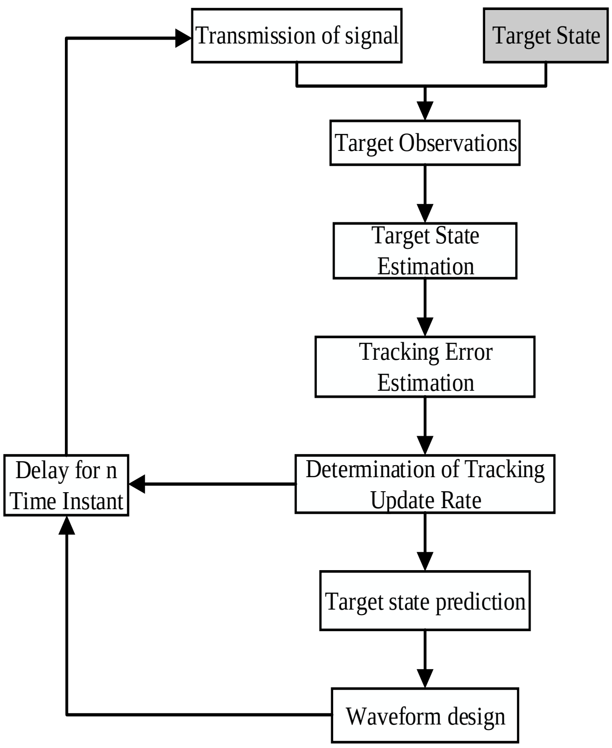

Fig. 1. Target Tracking Framwork For Cognitive Radar

There are many adaptive algorithms for determination of update rate for tracking systems [31 & 32]. We propose to incorporate tracking residual error, to make use of the dependency of observations on the target tracking. Therefore, we would have:

$$\mathrm{e_k} = \mathbf{x}_{\mathbf{k|k}} - \mathbf{x}_{\mathbf{k|k-1}} \tag{8}$$

$$\mathrm{T_{next}} = \mathrm{f}(\mathrm{T_{previous}}, \mathrm{e_k}) \tag{9}$$

$$\mathrm{n} = \frac{\mathrm{T_{next}}}{\mathrm{T_{min}}} \tag{10}$$

H. Shin in [33] proposes the function in (9) as:

$$f(a,b) = \frac{a}{\sqrt[3]{|b|}} \tag{11}$$

3.3. Target State Prediction

Now, we have to predict target state at k+n$^{\text{th}}$ time instant using observations of k$^{\text{th}}$ time instant. So, first $\mathrm{P}(\mathbf{x}_{\mathbf{k+n}}|\mathbf{z}_{1:\mathbf{k}})$ might be determined. The Chapman-Kolmogorov stated that:

$$\mathrm{P}(\boldsymbol{A}|\mathbf{C}) = \int \mathrm{P}(\mathbf{A}|\mathbf{B})\,\mathrm{P}(\mathbf{B}|\mathbf{C})\mathrm{d}\mathbf{B} \tag{12}$$

Thus, we have:

$$\mathrm{P}(\mathbf{x}_{\mathbf{k+n}}|\mathbf{z}_{1:\mathbf{k}}) = \int \mathrm{P}(\mathbf{x}_{\mathbf{k+n}}|\mathbf{x}_{\mathbf{k}})\,\mathrm{P}(\mathbf{x}_{\mathbf{k}}|\mathbf{z}_{1:\mathbf{k}})\mathrm{d}\mathbf{x}_{\mathbf{k}} \tag{13}$$

In (13), $\mathrm{P}(\mathrm{x_{k+n}}|\mathrm{x_k})$ is unknown. In order to determine this pdf, using the recursive calculations, we would have:

$$\mathrm{P}(\mathbf{x}_{\mathbf{k+2}}|\mathbf{x}_{\mathbf{k}}) = \int \mathrm{P}(\mathbf{x}_{\mathbf{k+2}}|\mathbf{x}_{\mathbf{k+1}})\,\mathrm{P}(\mathbf{x}_{\mathbf{k+1}}|\mathbf{x}_{\mathbf{k}})\mathrm{d}\mathbf{x}_{\mathbf{k+1}} \tag{14}$$

$$\mathrm{P}(\mathbf{x}_{\mathbf{k+3}}|\mathbf{x}_{\mathbf{k}}) = \int \mathrm{P}(\mathbf{x}_{\mathbf{k+3}}|\mathbf{x}_{\mathbf{k+2}})\,\mathrm{P}(\mathbf{x}_{\mathbf{k+2}}|\mathbf{x}_{\mathbf{k}})\mathrm{d}\mathbf{x}_{\mathbf{k+2}} \tag{15}$$

...

$$\mathrm{P}(\mathbf{x}_{\mathbf{k+n-1}}|\mathbf{x}_{\mathbf{k}}) = \int \mathrm{P}(\mathbf{x}_{\mathbf{k+n-1}}|\mathbf{x}_{\mathbf{k+n-2}})\,\mathrm{P}(\mathbf{x}_{\mathbf{k+n-2}}|\mathbf{x}_{\mathbf{k}})\mathrm{d}\mathbf{x}_{\mathbf{k+n-2}} \tag{16}$$

$$\mathrm{P}(\mathbf{x}_{\mathbf{k+n}}|\mathbf{x}_{\mathbf{k}}) = \int \mathrm{P}(\mathbf{x}_{\mathbf{k+n}}|\mathbf{x}_{\mathbf{k+n-1}})\,\mathrm{P}(\mathbf{x}_{\mathbf{k+n-1}}|\mathbf{x}_{\mathbf{k}})\mathrm{d}\mathbf{x}_{\mathbf{k+n-1}} \tag{17}$$

Now, the prediction of target state would be:

$$\mathbf{x}_{\mathbf{k+n|k}} = \int \mathbf{x}_{\mathbf{k+n}}\mathrm{P}(\mathbf{x}_{\mathbf{k}+\boldsymbol{n}}|\mathbf{z}_{1:\mathbf{k}})\,\mathrm{d}\mathbf{x}_{\mathrm{k+n}} \tag{18}$$

3.4. Waveform Design

Waveform is designed based on minimization of the following cost function

$$\boldsymbol{\theta}_{\mathbf{k+n}} = \arg\min \int (\mathbf{x}_{\mathbf{k+n|k}} - \mathbf{x}_{\mathbf{k+n}})(\mathbf{x}_{\mathbf{k+n|k}} - \mathbf{x}_{\mathbf{k+n}})^{\mathrm{T}}\mathrm{P}(\mathbf{x}_{\mathbf{k+n}}|\mathbf{z}_{1:\mathbf{k}})\,\mathrm{P}(\mathbf{z}_{\mathbf{k+n}}|\mathbf{x}_{\mathbf{k+n}})\mathrm{d}\mathbf{x}_{\mathbf{k+n}} \tag{19}$$

$P(\mathbf{x_{k+n}}|\mathbf{z_{1:k}})$ and $P(\mathbf{z_{k+n}}|\mathbf{x_{k+1}})$ in (19), due to their dependency on target observations, are waveform dependent [29]. $(\mathbf{x_{k+n|k}}-\mathbf{x_{k+n}})(\mathbf{x_{k+n|k}}-\mathbf{x_{k+n}})^T$ represents the prediction mean square error at $k+n^{th}$ time instant, using the observations up to k^{th} time instant. $P(\mathbf{x_{k+n}}|\mathbf{z_{1:k}})$ in this equation weighens the error with respect to different predictions of target state and $P(\mathbf{z_{k+n}}|\mathbf{x_{k+n}})$ is the likelihood function and models the radar receiver and matched filter.

3.5. Target State Estimation

After elapsing n time instant and transmission of the signal designed in 2.4, target state must be estimated.

$$P(\mathbf{x_{k+n}}|\mathbf{z_{1:k+n}}) = P(\mathbf{x_{k+n}}|\mathbf{z_{k+n}},\mathbf{z_{k+n-1}},\dots,\mathbf{z_1}) \quad (20)$$

$$= \frac{P(\mathbf{z_{k+n}},\mathbf{z_{k+n-1}},\dots,\mathbf{z_{k+1}}|\mathbf{x_{k+n}},\mathbf{z_{1:k}}).P(\mathbf{x_{k+n}}|\mathbf{z_{1:k}})}{\int P(\mathbf{z_{k+n}},\mathbf{z_{k+n-1}},\dots,\mathbf{z_{k+1}}|\mathbf{x_{k+n}},\mathbf{z_{1:k}}).P(\mathbf{x_{k+n}}|\mathbf{z_{1:k}})d\mathbf{x_{k+n}}} \quad (21)$$

Observations in (21) are independent from each other and independent from target state in future, so we have:

$$P(\mathbf{x_{k+n}}|\mathbf{z_{1:k+n}})$$

$$= \frac{P(\mathbf{z_{k+n}}|\mathbf{x_{k+n}},\mathbf{z_{1:k}}).P(\mathbf{z_{k+n-1}}|\mathbf{x_{k+n}},\mathbf{z_{1:k}})\dots P(\mathbf{x_{k+n}}|\mathbf{z_{1:k}})}{\int P(\mathbf{z_{k+n}}|\mathbf{x_{k+n}},\mathbf{z_{1:k}}).P(\mathbf{z_{k+n-1}}|\mathbf{x_{k+n}},\mathbf{z_{1:k}})\dots P(\mathbf{x_{k+n}}|\mathbf{z_{1:k}})d\mathbf{x_{k+n}}}$$

$$= \frac{P(\mathbf{z_{k+n}}|\mathbf{x_{k+n}}).P(\mathbf{z_{k+n-1}}|\mathbf{z_{1:k}})P(\mathbf{z_{k+n-2}}|\mathbf{z_{1:k}})\dots P(\mathbf{x_{k+n}}|\mathbf{z_{1:k}})}{\int P(\mathbf{z_{k+n}}|\mathbf{x_{k+n}}).P(\mathbf{z_{k+n-1}}|\mathbf{z_{1:k}})P(\mathbf{z_{k+n-2}}|\mathbf{z_{1:k}})\dots P(\mathbf{x_{k+n}}|\mathbf{z_{1:k}})d\mathbf{x_{k+n}}}$$

$$= \frac{P(\mathbf{z_{k+n}}|\mathbf{x_{k+n}}).P(\mathbf{z_{k+n-1}}|\mathbf{z_{1:k}})P(\mathbf{z_{k+n-2}}|\mathbf{z_{1:k}})\dots P(\mathbf{x_{k+n}}|\mathbf{z_{1:k}})}{P(\mathbf{z_{k+n-1}}|\mathbf{z_{1:k}})P(\mathbf{z_{k+n-2}}|\mathbf{z_{1:k}})\dots \int P(\mathbf{z_{k+n}}|\mathbf{x_{k+n}}).P(\mathbf{x_{k+n}}|\mathbf{z_{1:k}})d\mathbf{x_{k+n}}}$$

$$= \frac{P(\mathbf{z_{k+n}}|\mathbf{x_{k+n}}).P(\mathbf{x_{k+n}}|\mathbf{z_{1:k}})}{\int P(\mathbf{z_{k+n}}|\mathbf{x_{k+n}}).P(\mathbf{x_{k+n}}|\mathbf{z_{1:k}})d\mathbf{x_{k+n}}} \quad (22)$$

Therefore, (22) shows that target state at $k+n^{th}$ time instant is independent of observations between k^{th} and $k+n^{th}$ time instant.

The equations presented in this section shows that adaptive waveform design is possible in the absence of target state estimation and target observation in a period of time, that due to time resource management is skipped. Due to the computationally complex integrals involved in the equations presented in this section, implementation of the presented framework is actually impossible. Thus, considering these complexities, next we propose an implementable algorithm for this purpose.

4. Proposed Resource Management Algorithm

As stated in Section 2, the Particle Filter (PF) is a numerical algorithm for implementation of Bayesian filter using Monte-Carlo simulation. Main idea in PF is to represent the posterior pdf as a set of random samples, called particles. Thus, this filter reduces the difficult integrals to finite summations [23]. Obviously, as the number of samples become very large, the achieved posterior pdf becomes more accurate. In addition, if there is little overlap between the prior and likelihood, the failure to use the latest measurement leads to large tracking error [27]. To overcome this problem, the proposed algorithm incorporates UKF with the PF. We use this idea to develop an implantable algorithm for the presented framework in section 3, as shown in Fig. 2. In addition, Table 2 presents the complete proposed algorithm for resource management in cognitive radar. In this algorithm, $\underline{\hat{\mathbf{x}}}$ is the particles state vector of size N, n is the number of sigma points and J is the length of waveform library.

In this algorithm, the importance sampling and resampling is used to overcome the degeneracy phenomenon in particle filter based tracking algorithms [23]. Due to this phenomenon, after a few iterations, all but one particle would have a weight value near to zero. The basic idea in resampling is to eliminate particles that have small weights and concentrate on particles with large weights. Then, a new set of modified particles are obtained.

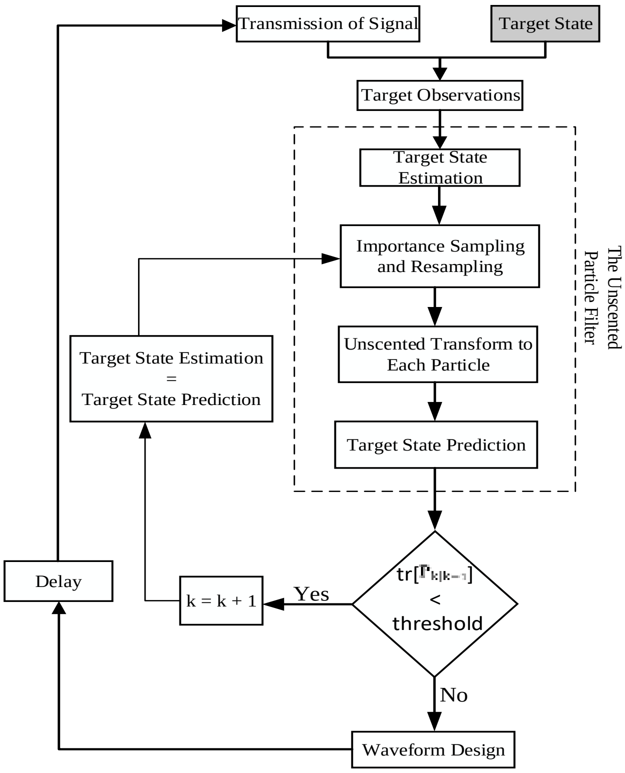

Fig. 2. Proposed Target Tracking Algorithm For Cognitive Radar

5. Simulation

In this section, the performance of the proposed algorithm for time resource management in cognitive radars is evaluated. Consider a target moving a famous 8-scenario as shown in Fig. 3 (green line). In this scenario target moves in different motion scenarios. The transmitted waveform is a single tone pulse with minimum and maximum value of 1μs and 100μs, respectively, for adaptive waveform design case and a constant value of 20μs for fixed waveform transmission and κ in Table 2 has been set as unity. In the entire simulations environment is supposed to be narrowband. The radar measurement vector is $[r \quad \theta]^T$ and target state is considered as $[x_x \quad \dot{x}_x \quad \ddot{x}_x \quad x_y \quad \dot{x}_y \quad \ddot{x}_y]^T$. The target model matrix and measurement matrix in (1) and (2) are:

$$\mathbf{h} = \left[\sqrt{\mathbf{x}_x^2 + \mathbf{x}_y^2} \quad \operatorname{atan}\left(\frac{\mathbf{x}_y}{\mathbf{x}_x}\right)\right] \tag{23}$$

$$\mathbf{F} = \begin{bmatrix} 1 & T & 0.5T^2 & 0 & 0 & 0 \\ 0 & 1 & T & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & T & 0.5T^2 \\ 0 & 0 & 0 & 0 & 1 & T \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{24}$$

Table 2. The Proposed Algorithm for Time Resource Management in Cognitive Radars

initialize: $\underline{\hat{\mathbf{x}}}_{k-1|k-1}$

$\underline{\hat{\mathbf{x}}}_{k|k} =$ function proposed algorithm ($\underline{\hat{\mathbf{x}}}_{k-1|k-1}$, $\mathbf{z}_k$)

$\boldsymbol{x}^0_{k-1|k-1} = \underline{\hat{\mathbf{x}}}_{k-1|k-1}$, $\mathbf{W}_0 = \frac{\kappa}{n+\kappa}$

$\boldsymbol{x}^0_{k|k-1} = f(\boldsymbol{x}^0_{k-1|k-1})$

for i = 1: n

$\boldsymbol{x}^i_{k-1|k-1} = \underline{\hat{\mathbf{x}}}_{k-1|k-1} + \left(\sqrt{(n+\kappa)\mathbf{P}_{k-1|k-1}}\right)_i, \mathbf{W}_i = \frac{1}{2(n+\kappa)}$

$\boldsymbol{x}^{i+n}_{k-1|k-1} = \underline{\hat{\mathbf{x}}}_{k-1|k-1} - \left(\sqrt{(n+\kappa)\mathbf{P}_{k-1|k-1}}\right)_i, \mathbf{W}_i = \frac{1}{2(n+\kappa)}$

$\boldsymbol{x}^i_{k|k-1} = f(\boldsymbol{x}^i_{k-1|k-1})$

$\boldsymbol{z}^i_{k|k-1} = h(\boldsymbol{x}^i_{k+1|k})$

end

$\hat{\mathbf{x}}_{k|k-1} = \sum_{i=0}^{2n} \mathbf{W}_i \boldsymbol{x}^i_{k|k-1}$

$\mathbf{P}_{k|k-1} = \mathbf{Q}_{k-1} + \sum_{i=0}^{2n} \mathbf{W}_i(\boldsymbol{x}^i_{k|k-1} - \hat{\mathbf{x}}_{k|k-1})(\boldsymbol{x}^i_{k|k-1} - \hat{\mathbf{x}}_{k|k-1})^T$

if $\mathrm{trac}(\mathbf{P}_{k|k-1}) < \mathrm{thr}$

$\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1}$

$\mathbf{P}_{k|k} = \mathbf{P}_{k|k-1}$

$k = k + 1$

else

$\hat{\mathbf{z}}_{k+1|k} = \sum_{i=0}^{2n} \mathbf{W}_i \boldsymbol{z}^i_{k+1|k}$

for j = 1: J

$\mathbf{P}^j_{zz} = \sum_{i=0}^{2n} \mathbf{W}_i(\boldsymbol{z}^i_{k+1|k} - \hat{\mathbf{z}}_{k+1|k})(\boldsymbol{z}^i_{k+1|k} - \hat{\mathbf{z}}_{k+1|k})^T + \mathbf{R}(\boldsymbol{\theta}_j)$

$\mathbf{P}_{xz} = \sum_{i=0}^{2n} \mathbf{W}_i(\boldsymbol{x}^i_{k+1|k} - \hat{\mathbf{x}}_{k+1|k})(\boldsymbol{z}^i_{k+1|k} - \hat{\mathbf{z}}_{k+1|k})^T$

$\mathbf{K}^j_{k+1} = \mathbf{P}_{xz}{\mathbf{P}^j_{zz}}^{-1}$

${\mathbf{P}^j}_{k+1|k+1} = \mathbf{P}_{k+1|k} - \mathbf{K}^j_{k+1}\mathbf{P}_{zz}{\mathbf{K}^j_{k+1}}^T$

end

$JJ = \arg\min_{\boldsymbol{\theta}_j}[\mathrm{tr}({\mathbf{P}^j}_{k+1|k+1})]$

$k = k + 1$

$\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + \mathbf{K}_k(\mathbf{z}_k - \hat{\mathbf{z}}_{k|k-1})$

$\mathbf{P}_{k|k} = \mathbf{P}_{k|k-1} - \mathbf{K}^{JJ}_k \mathbf{P}^j_{zz} {\mathbf{K}^{JJ}_k}^T$

end

$\underline{\hat{\mathbf{x}}}_{k|k} =$ function resampling ($\underline{\hat{\mathbf{x}}}_{k-1|k-1}$)

The covariance matrix of model and measurement noise are:

$$\mathbf{Q} = \begin{bmatrix} \frac{T^5}{20} & \frac{T^4}{8} & \frac{T^3}{6} & 0 & 0 & 0 \\ \frac{T^4}{8} & \frac{T^3}{6} & \frac{T^2}{2} & 0 & 0 & 0 \\ \frac{T^3}{6} & \frac{T^2}{2} & T & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{T^5}{20} & \frac{T^4}{8} & \frac{T^3}{6} \\ 0 & 0 & 0 & \frac{T^4}{8} & \frac{T^3}{6} & \frac{T^2}{2} \\ 0 & 0 & 0 & \frac{T^3}{6} & \frac{T^2}{2} & T \end{bmatrix} \tag{25}$$

$$\mathbf{R} = \begin{bmatrix} \sigma_r^2 & 0 \\ 0 & \sigma_\theta^2 \end{bmatrix} \tag{26}$$

where $T = 1$, $\sigma_r = \frac{c\tau}{2\sqrt{2\eta}}$ and $\sigma_\theta = \frac{\varphi_{3dB}}{\kappa\sqrt{2\eta}}$ are waveform dependent and $\eta = \frac{P_t G_t G_r \lambda^2 \sigma \tau N_p}{(4\pi)^3 R^4 K T F_n} = \frac{K\tau}{R^4}$ is the received signal to noise ratio and τ is the pulse length value in waveform design. So $\sigma_r \sim \sqrt{\tau}$ and $\sigma_\theta \sim \frac{1}{\sqrt{\tau}}$ have reverse dependencies on pulse length.

For evaluation of the proposed framework, we consider four cases. Simulation of the UPF tracker, simulation of the UPF tracker with adaptive update rate tracking, simulation of the UPF tracker using adaptive waveform design, and finally, simulation of the proposed adaptive update rate UPF tracker using adaptive waveform design. Fig. 3 presented target tracking using the UPF tracker. In this figure the green line, presents the target real state, the blue line presents the radar measurement of target state, the red line is the target state prediction, and the black line is the estimation of target state.

Next, we present target tracking using the adaptive update rate. Fig. 4 shows target tracking results. In this figure, the black dots represent target tracking with target state estimation using target observation, and red dot represents the moments that state prediction error was tolerable and target was tracked using the predictions, instead of using radar observations of target, thus time resources are reserved. Fig. 5 shows tracking error standard deviation. It is seen from this figure that in time snapshots that target tracking is achieved using prediction of target state, there are spikes in tracking error, although the value of this spikes are tolerable.

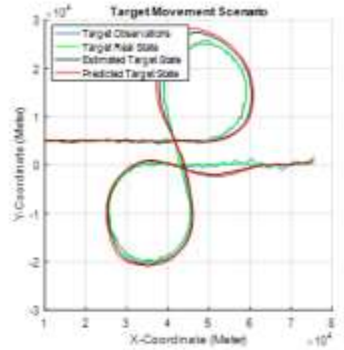

Fig. 3. Target Tracking Using UPF tracker for Movement Scenario # 1

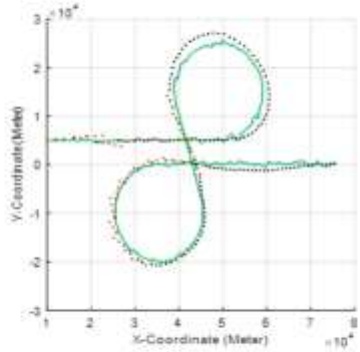

Fig. 4. Target Tracking Using Adaptive Update Rate UPF Tracker

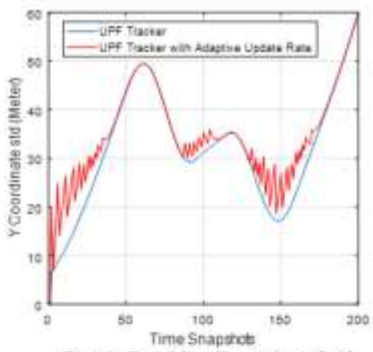

Fig. 5. Tracking Error Standard Deviation Using UPF Tracker and Adpative Update Rate UPF Tracker

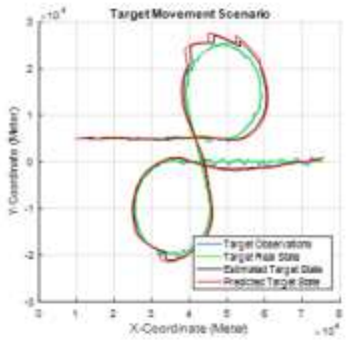

Fig. 6. Target Tracking Using Adaptive Waveform Design UPF Tracker

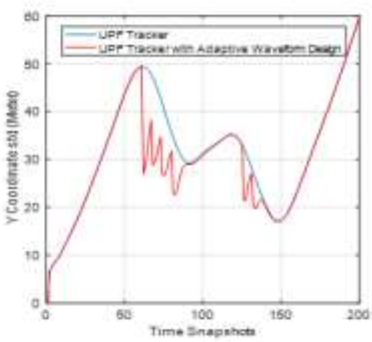

Fig. 7. Tracking Error Standard Deviation Using UPF Tracker and Adpative Waveform Design UPF Tracker

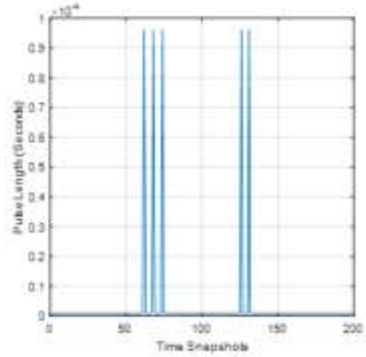

Fig. 8. Pulse Length Variation for UPF Tracker With Adaptive Waveform Design for Movement Scenario # 1

Fig. 6 presents target tracking with UPF tracker using adaptive waveform design. Also in Fig. 7, its tracking error variance is compared to the classic UPF tracker. As shown in Fig. 8, it is seen that in moments of target manoeuvre, [35~75] and [110~130] time snapshots, tracking error has increased. Thus, pulse length increase in Fig. 8 causes a decrease in angular error, and thus total tracking error, as shown in Fig. 7 (red line).

It is worthy to note that the increase in pulse length causes a decrease in angular measurement error and increase in range measurement error, as shown in Fig. 9. When angular error is increasing (like a manoeuvre in angular coordinate), pulse length must increase to enhance the angular measurement quality. Thus, the pulse length variations must be adopted to the target motion scenario, with respect to the measurement error variations.

Fig. 10 presents simulation results of the proposed algorithm for adaptive waveform design target tracking with adaptive update rate. In addition, Fig. 11 presents comparison of error standard deviation for adaptive update rate UPF tracker with fixed waveform and adaptive waveform. It is seen that adaptive transmission in the proposed algorithm causes a reduction in tracking error.

Fig. 12 shows the comparison of tracking error of the UPF Tracker and adaptive update rate UPF tracker with adaptive waveform design on transmission.

Table 3 presents comparison of tracking error and time resource consumption for each of the tracking algorithms.

Table 3. Comparison of Tracking Error and Elapsed Time

	Range Error	Angular Error	Target Revisits
UPF Tracker	35	0.082	200
Adaptive Update Rate UPF Tracker	55	0.095	147
UPF Tracker with Adaptive Transmission	33	0.064	200
Adaptive Update Rate UPF Tracker with Adaptive Transmission	36	0.083	146

We repeat the above simulations for another target movement scenario. A target starts moving along a line and speeds up to reach its nominal velocity and then starts manoeuvring in angular coordinate. This is shown in Fig. 13. Pulse length variations for adaptive waveform design algorithm is shown in Fig. 14. In addition, Fig. 15 shows tracking error standard deviation in. It is seen from these figures that pulse length variations in Fig. 14 caused a reduced tracking error in Fig. 15, compared to the fixed waveform.

In addition, these waveform variations causes an equal tracking error compared to the conventional UPF tracker, while saving about 25% of radar time resources.

From above simulations and Table 3, the following points are concluded:

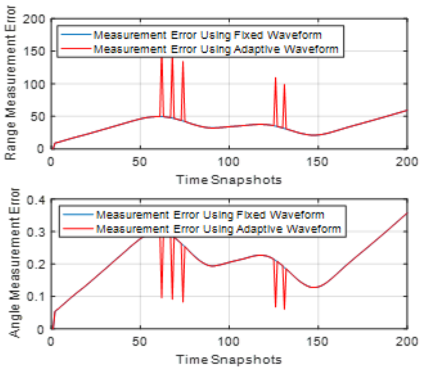

Fig. 9. Comparison of Measurement Errors Using Fixed and Adaptive Waveform

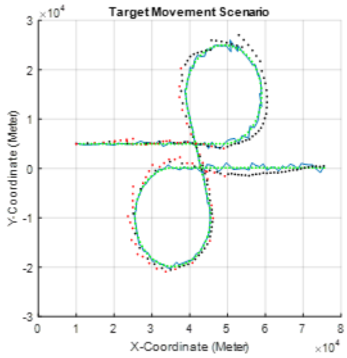

Fig. 10. Target Tracking Using the Proposed Adaptive Waveform Design UPF Tracker with Adpative Update Rate

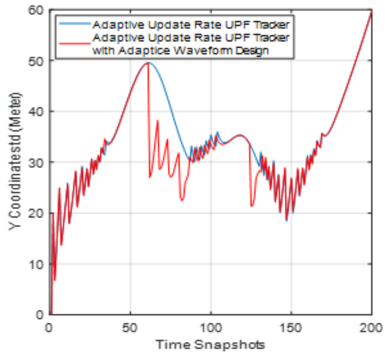

Fig. 11. Tracking Error Standard Deviation Using Adaptive Update Rate UPF Tracker with Fixed and Adaptive Transmitted Waveform

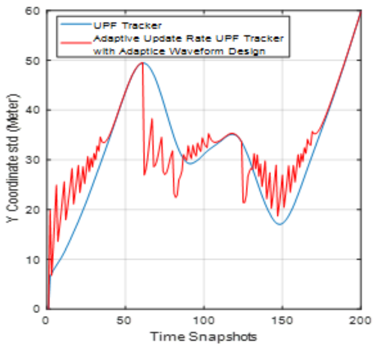

Fig. 12. Tracking Error Standard Deviation Using UPF Tracker and The Proposed Adaptive Update Rate UPF Tracker with Adaptive Transmitted Waveform

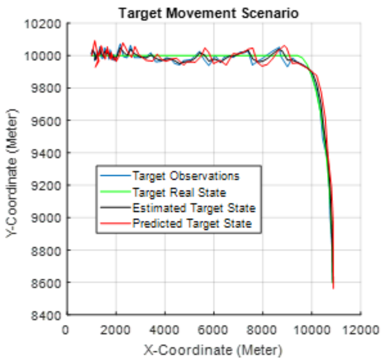

Fig. 13. Target Tracking Using UPF tracker for Movement Scenario # 2

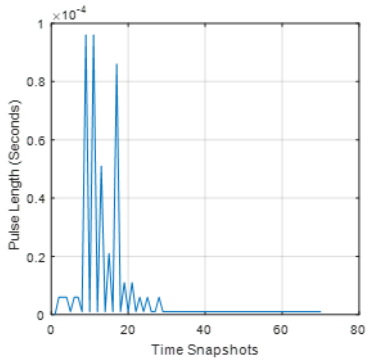

Fig. 14. Pulse Length Variation for UPF Tracker With Adaptive Waveform Design for Movement Scenario # 2

- Using adaptive waveform a reduction in tracking error is achieved, with respect to fixed waveform transmission.
- Adaptive update rate tracking causes saving radar time resources, although tracking errors are increased.

Using the proposed adaptive update rate tracking with adaptive waveform design can reach the fixed waveform transmission tracking error, while about 25% of radar time resources are saved.

6. Conclusion

This paper studied time resource management in cognitive radars. The main idea in the proposed framework is to use adaptive waveform on transmission to compensate the increase in tracking error for time saving in adaptive update rate tracking. For this purpose, a framework for time resource management with adaptive waveform design was proposed and the feasibility of the adaptive waveform design with adaptive update rate tracking were studied. The mathematical equations of time resource management for cognitive radar framework was fully developed. In addition, an implementable algorithm for this purpose was proposed. The simulation results showed that keeping a constant tracking error, the proposed algorithm can save a great amount of radar time resources. This time saving in the presented scenario was about 25%, but this saving could be much more in other scenarios.

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