Dynamic output feedback fault-tolerant controller design for a class of generalized Takagi-Sugeno fuzzy nonlinear systems

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Abstract
A novel design approach to construct a fault-tolerant control (FTC) system for a class of nonlinear systems based on a generalized Takagi-Sugeno (GT-S) fuzzy model is proposed. The local rules of the GT-S fuzzy model consist of some multiplicative nonlinear terms. The nonlinear system is affected by actuator faults and unknown disturbances. A state/fault observer is designed and then, a dynamic output feedback scheme is proposed based on the estimated fault and state information. The sufficient conditions for observer and controller design are separately given in terms of linear matrix inequalities (LMIs). It can be shown that the number of LMIs and the computational burden is less than that of similar methods and the effectiveness of the proposed dynamic output feedback FTC approach is verified by proposing simulation results applied to an inverted pendulum system.

Keywords
nonlinear systems, Generalized Takagi-Sugeno fuzzy model, fault-tolerant control, dynamic output feedback.

1. Introduction

Fast detection of possible faults and attenuation of their effects on the closed-loop control systems is a challenging research issue. In the past decades, many researchers focused on the safe operation of the controlled systems when various faults occur [1]. Some good methods are proposed for linear systems [2], but most of the practical systems have nonlinear and complex dynamics and therefore, designing fault-tolerant controllers (FTC) for nonlinear systems is a very important challenge. This problem is investigated by many researchers, but most of the outcomes are only applicable for a limited class of systems with restrictive conditions (see for example the books [3-6], survey papers [7-8], and other works [9-18] and references therein).

Development of Takagi-Sugeno (T-S) fuzzy models inspired researchers to successfully extend the previous linear methods to nonlinear complex systems that could be modelled using T-S fuzzy models. The T-S fuzzy model can approximate nonlinear dynamic with some local linear IF-THEN rules as long as the number of rules is enough [19]. Many excellent works used T-S fuzzy modelling for solving the problems of nonlinear systems over the past decades and remarkable papers about FTC design for nonlinear systems with the aid of T-S fuzzy modelling are published in recent years [20-31].

Although the T-S fuzzy modelling changes the nonlinear dynamic of the complex systems into some local linear rules, the number of this rules augments exponentially when someone wants to describe the nonlinear system more precise. This results in more complication in analysis and design procedures. Recently, some methods have been proposed to allow some nonlinear terms to explicitly appear in the local models. This ends in decreasing necessary rules for describing the system, but the linearity shape of the overall system is lost. In [32], a very simple form of nonlinear local rules with an additive sinusoidal term to the linear part of the rules is proposed. More advanced work is performed in [33-35] where sector-bounded functions added to the rules and some recent papers used these ideas [36-42]. In [43-45], the authors suggest another nonlinear T-S fuzzy model in which, a nonlinear matrix was kept in the form of a multiplying term to the control input in local rules and the rest of the rules are linear. Besides decreasing the necessary number of needed local rules and hence less computational burden, the significant advantage of this model is that one can always have controllable models. The proposed Generalized T-S (GT-S) fuzzy model could overcome the difficulties concerning the uncontrollability issues in conventional T-S fuzzy models of some nonlinear systems.

The suggested GT-S fuzzy model in [45] is utilized in this paper for modelling nonlinear systems and a dynamic output feedback FTC system is designed based on this model. First, an observer is designed to simultaneously estimate actuator faults and system states and then, the estimated information is used to design FTC law which stabilizes the faulty system in the presence of unknown disturbances. Sufficient asymptotical stability conditions are derived in terms of linear matrix inequalities (LMIs). The proposed FTC law is a generalization of the work done in [46] and [47] where a dynamic output feedback FTC system is suggested based on conventional T-S fuzzy model. In the [46], a dynamic FTC law using a fault observer was proposed and the same problem has been solved in [47] using a set-theoretic description of T-S fuzzy model. So, if one could model the nonlinear system with the GT-S fuzzy model with less number of local rules, the proposed method may have fewer number of LMIs than that of [46] and the feasibility of them are more likely. A similar work is done in [48] where the GT-S fuzzy model is used to design a state-feedback FTC system based on the estimated fault information. Also, the nonlinear term of the local rules in [48] is supposed to satisfy a local Lipschitz condition. Although this is not a very restrictive condition, in this paper it is just assumed that the nonlinear term is norm-bounded in the working region. Using an inverted pendulum system, it is shown that the proposed method could stabilize the closed-loop faulty system with better performance in comparison to the existing approaches. The main contribution of this paper is to provide a dynamic output feedback FTC law for nonlinear systems based on the estimated information of faults and states in the presence of various types of additive actuator faults and unknown disturbances and for this goal, a GT-S fuzzy model is utilized. The proposed method could be applicable for a very wide class of nonlinear systems with less restrictive conditions.

The rest of the paper is organized as follows: the main problem to be solved in this paper is formulated in Section 2. In Section 3, a solution is provided and is summarized in the form of a constructive algorithm. Simulation results on an inverted pendulum system are illustrated in Section 4, and Section 5 concludes the paper.

The notations used in this paper are standard and are shown in Table I.

Table I. The notations used in the paper and their descriptions.

Parameter	Description
$\mathbf{A}^T$	Transpose of a given matrix $\mathbf{A}$
$\mathbf{A}^{-T}$	Inverse transpose of a given matrix $\mathbf{A}$
$\mathbf{I}_q$	$q \times q$ unity matrix
$\mathbf{0}_{s\times t}$	$s \times t$ matrix with zero elements
$\mathcal{L}_2[0,\infty)$ space	The set of signals satisfy $\int_0^\infty \boldsymbol{v}^T(t)\boldsymbol{v}(t)dt < \infty$
$\|v\|_2$	$\mathcal{L}_2$-norm of a given signal $v(t) \in \mathcal{L}_2[0,\infty)$
$*$	Matrix symmetric elements

2. Problem formulation

In this paper, a dynamic output feedback FTC scheme based on a generalized T-S fuzzy model that has a multiplicative state-dependent nonlinear term in the local rules is proposed. The form of ith local rule of the GT-S fuzzy model with additive actuator faults and unknown disturbances is given by

IF z_1 is M_{i1} and … and z_g is M_{ig}, THEN

$$\begin{aligned}\dot{\boldsymbol{x}}(t) &= \mathbf{A}_i\boldsymbol{x}(t) + \mathbf{B}_i\mathbf{F}\big(\boldsymbol{x}(t)\big)[\boldsymbol{u}(t) + \boldsymbol{f}(t)] \\ &\quad + \mathbf{D}_{1i}\boldsymbol{\omega}(t) \\ \boldsymbol{y}(t) &= \boldsymbol{C}_i\boldsymbol{x}(t) + \boldsymbol{D}_{2i}\boldsymbol{\omega}(t)\end{aligned} \tag{1}$$

where $\boldsymbol{x}(t) \in \mathbb{R}^n$, $\boldsymbol{u}(t) \in \mathbb{R}^m$, $\boldsymbol{f}(t) \in \mathbb{R}^m$, $\boldsymbol{y}(t) \in \mathbb{R}^p$, and $\boldsymbol{\omega}(t) \in \mathbb{R}^w$ are system states, the control input, the actuator faults, the output of the system, and unknown $\mathcal{L}_2$ disturbances, respectively, and $\mathbf{A}_i$, $\mathbf{B}_i$, $\mathbf{C}_i$, $\mathbf{D}_{1i}$ and $\mathbf{D}_{2i}$ are real constant matrices with appropriate dimensions. $\mathbf{F}\big(\boldsymbol{x}(t)\big)$ is an $m \times m$ nonlinear matrix and its presence in the local rules besides decreasing the number of rules and computational burden, provides controllable rules, which is a severe problem concerning with some conventional T-S fuzzy models [45] and then a very wide class of nonlinear systems could be dealt with. It is shown in [45], some of $\mathbf{B}_i$ matrices in the conventional T-S models for some nonlinear systems might be zero, so the corresponding local rules became uncontrollable. Use of the GT-S model for modelling this type of nonlinear systems often solves this problem.

The form of overall fuzzy model, which is achieved by fuzzy summation of the local rules (1), is as follows

$$\begin{aligned}\dot{\boldsymbol{x}}(t) &= \sum_{i=1}^{r} h_i(\boldsymbol{z})\big\{\mathbf{A}_i\boldsymbol{x}(t) \\ &\quad + \mathbf{B}_i\mathbf{F}\big(\boldsymbol{x}(t)\big)[\boldsymbol{u}(t) + \boldsymbol{f}(t)] \\ &\quad + \mathbf{D}_{1i}\boldsymbol{\omega}(t)\big\} \\ \boldsymbol{y}(t) &= \sum_{i=1}^{r} h_i(\boldsymbol{z})\{\boldsymbol{C}_i\boldsymbol{x}(t) + \boldsymbol{D}_{2i}\boldsymbol{\omega}(t)\}\end{aligned} \tag{2}$$

where

$$h_i(\boldsymbol{z}) = \frac{\beta_i(\boldsymbol{z})}{\sum_{i=1}^{r}\beta_i(\boldsymbol{z})}, \qquad \beta_i(\boldsymbol{z}) = \prod_{j=1}^{g} M_{ij}(\boldsymbol{z}) \tag{3}$$

in which, $\boldsymbol{z} = \left[z_1, \dots, z_g\right]^T$ is premise variables vector, and M_{ij} $(i = 1, \dots, r; j = 1, \dots, g)$ are the fuzzy sets of membership functions and $M_{ij}(.)$ is the grade of corresponding membership function where $0 \leq M_{ij}(.) < 1$ $(i = 1, \dots, r; j = 1, \dots, g)$. The T-S fuzzy model can approximate any nonlinear system with arbitrary precision and the same discussion can easily be made for the GT-S fuzzy model [19]. In order to simply deal with nonlinear term, a filtered version of fault is defined as follows

$$\bar{\boldsymbol{f}}(t) = \boldsymbol{F}\big(\boldsymbol{x}(t)\big)\boldsymbol{f}(t) \tag{4}$$

therefore, the nonlinear system (2) including additive actuator faults and unknown disturbances could be represented by:

$$\begin{aligned}\dot{\boldsymbol{x}}(t) &= \sum_{i=1}^{r} h_i(\boldsymbol{z})\big\{\mathbf{A}_i\boldsymbol{x}(t) + \mathbf{B}_i\mathbf{F}\big(\boldsymbol{x}(t)\big)\boldsymbol{u}(t) \\ &\quad + \mathbf{B}_i\bar{\boldsymbol{f}}(t) + \mathbf{D}_{1i}\boldsymbol{\omega}(t)\big\}\end{aligned} \tag{5}$$

$$\boldsymbol{y}(t) = \sum_{i=1}^{r} h_i(\boldsymbol{z})\{\boldsymbol{C}_i\boldsymbol{x}(t) + \boldsymbol{D}_{2i}\boldsymbol{\omega}(t)\}$$

It is assumed in this paper that the nonlinear term $\mathbf{F}(\boldsymbol{x}(t))$ satisfies

$$\delta_1 \le \left\|\boldsymbol{F}(\boldsymbol{x}(t))\right\|_2 \le \delta_2, \qquad \forall \boldsymbol{x}(t) \in \Omega \subseteq \mathbb{R}^n \tag{6}$$

where $\delta_1, \delta_2 > 0$ are two real numbers and Ω is a local region. In [48], the term $\mathbf{F}(\boldsymbol{x}(t))$ is supposed to satisfy a local Lipschitz condition. Although this assumption is not very restrictive, assuming norm-bounded term makes the proposed method applicable to wider class of nonlinear systems.

The problem of designing a dynamic output feedback FTC law for nonlinear systems that modelled by GT-S fuzzy model that is solved in this paper could be presented as follows.

Problem 1. Consider the GT-S model (5) with depicted in Fig. 1. Design a state/fault observer to estimate states and faults simultaneously and then, design a dynamic FTC law that stabilizes the system in the presence of faults and disturbances based on estimated information of faults and states.

Before presenting the main results, the following useful lemmas for proving the Theorems of the paper are presented.

Lemma 1 [49]: The inequality

$$\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\boldsymbol{z})h_j(\boldsymbol{z})\,\boldsymbol{\Theta}_{ij} < 0 \tag{7}$$

holds, if the following inequalities are satisfied

$$\boldsymbol{\Theta}_{ii} < 0, i = 1, \dots, r \tag{8}$$
$$\frac{1}{r-1}\boldsymbol{\Theta}_{ii} + \frac{1}{2}\left(\boldsymbol{\Theta}_{ij} + \boldsymbol{\Theta}_{ji}\right) < 0, 1 \le i < j \le r$$

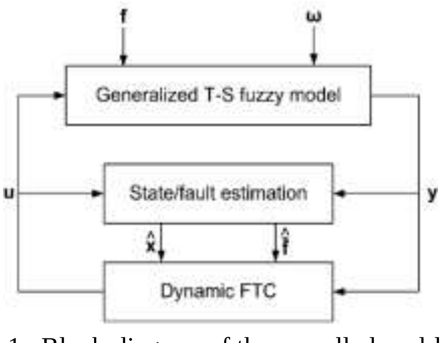

Fig. 1. Block-diagram of the overall closed-loop control system.

Lemma 2 [50]: Let $\mathbf{D}$, $\mathbf{E}$, and $\mathbf{S}$ be real matrices with appropriate dimensions and $\mathbf{S}$ satisfying $\mathbf{S}^T\mathbf{S} \le \mathbf{I}$. For any scalar $\varepsilon > 0$ and vectors $\boldsymbol{x}, \boldsymbol{y} \in \mathbb{R}^n$

$$2\boldsymbol{x}^T\boldsymbol{DSEy} \le \varepsilon^{-1}\boldsymbol{x}^T\boldsymbol{DD}^T\boldsymbol{x} + \varepsilon\boldsymbol{y}^T\boldsymbol{E}^T\boldsymbol{Ey} \tag{9}$$

3. Main idea

To solve Problem 1, a constructive algorithm is proposed in this section. The proposed solution is composed of two parts: (i) State/fault observer design, and (ii) Constructing a dynamic output feedback FTC law.

3.1. State/Fault Observer Design

First of all, a state/fault observer for the system (5) is constructed as follows

$$\begin{aligned}
\dot{\hat{\boldsymbol{x}}}(t) &= \sum_{i=1}^{r} h_i(\boldsymbol{z})\Big\{\mathbf{A}_i\hat{\boldsymbol{x}}(t) + \mathbf{B}_i\mathbf{F}(\hat{\boldsymbol{x}}(t))\boldsymbol{u}(t) \\
&\qquad + \mathbf{B}_i\hat{\bar{\boldsymbol{f}}}(t) \\
&\qquad - \mathbf{L}_i(\hat{\boldsymbol{y}}(t) - \boldsymbol{y}(t))\Big\} \\
\hat{\boldsymbol{y}}(t) &= \sum_{i=1}^{r} h_i(\boldsymbol{z})\{\mathbf{C}_i\hat{\boldsymbol{x}}(t)\} \\
\dot{\hat{\bar{\boldsymbol{f}}}}(t) &= \sum_{i=1}^{r} h_i(\boldsymbol{z})\{-\mathbf{G}_i(\hat{\boldsymbol{y}}(t) - \boldsymbol{y}(t))\}
\end{aligned} \tag{10}$$

where $\hat{\boldsymbol{x}}(t) \in \mathbb{R}^n$ and $\hat{\boldsymbol{y}}(t) \in \mathbb{R}^p$ are the observer state and output, respectively, $\hat{\bar{\boldsymbol{f}}}(t) \in \mathbb{R}^m$ is an estimation of filtered fault $\bar{\boldsymbol{f}}(t)$, and $\mathbf{L}_i$ and $\mathbf{G}_i$ are observer gain matrices, to be designed. Denoting $\boldsymbol{e}_x = \hat{\boldsymbol{x}}(t) - \boldsymbol{x}(t)$ and $\boldsymbol{e}_f = \hat{\bar{\boldsymbol{f}}}(t) - \bar{\boldsymbol{f}}(t)$, the dynamics of the error is given by

$$\begin{aligned}
\dot{\boldsymbol{e}}_x &= \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\boldsymbol{z})h_j(\boldsymbol{z})\{(\mathbf{A}_i - \mathbf{L}_i\mathbf{C}_j)\boldsymbol{e}_x \\
&\qquad + \mathbf{B}_i\check{\mathbf{F}}\boldsymbol{u}(t) + \mathbf{B}_i\boldsymbol{e}_f \\
&\qquad + (\mathbf{L}_i\mathbf{D}_{2j} - \mathbf{D}_{1i})\boldsymbol{\omega}(t)\} \\
\dot{\boldsymbol{e}}_f &= \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\boldsymbol{z})h_j(\boldsymbol{z})\Big\{-\mathbf{G}_i\mathbf{C}_j\boldsymbol{e}_x \\
&\qquad + \mathbf{G}_i\mathbf{D}_{2j}\boldsymbol{\omega}(t) - \dot{\bar{\boldsymbol{f}}}(t)\Big\}
\end{aligned} \tag{11}$$

where $\check{\mathbf{F}} = \mathbf{F}(\hat{\boldsymbol{x}}(t)) - \mathbf{F}(\boldsymbol{x}(t))$. Motivated by the developments in [46], the dynamic equations (11) are rewritten as follows

$$\begin{aligned}
\dot{\bar{\boldsymbol{e}}} &= \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\boldsymbol{z})h_j(\boldsymbol{z})\{(\bar{\boldsymbol{A}}_i - \bar{\boldsymbol{L}}_i\bar{\boldsymbol{C}}_j)\bar{\boldsymbol{e}} \\
&\qquad + (\bar{\boldsymbol{L}}_i\bar{\boldsymbol{D}}_{2j} - \bar{\boldsymbol{D}}_{1i})\boldsymbol{v}\}
\end{aligned} \tag{12}$$

where

$$\begin{aligned}
&\bar{\boldsymbol{e}} = \begin{bmatrix}\boldsymbol{e}_x \\ \boldsymbol{e}_f\end{bmatrix}, \qquad \boldsymbol{v} = \begin{bmatrix}\check{\boldsymbol{F}}\boldsymbol{u}(t) \\ \boldsymbol{\omega}(t) \\ \dot{\bar{\boldsymbol{f}}}(t)\end{bmatrix} \\
&\bar{\boldsymbol{A}}_i = \begin{bmatrix}\boldsymbol{A}_i & \boldsymbol{B}_i \\ \mathbf{0}_{m\times n} & \mathbf{0}_{m\times m}\end{bmatrix}, \bar{\boldsymbol{C}}_i = [\boldsymbol{C}_i \quad \mathbf{0}_{p\times m}] \\
&\bar{\boldsymbol{D}}_{1i} = \begin{bmatrix}-\boldsymbol{B}_i & \boldsymbol{D}_{1i} & \mathbf{0}_{n\times m} \\ \mathbf{0}_{m\times m} & \mathbf{0}_{m\times w} & \boldsymbol{I}_m\end{bmatrix} \\
&\bar{\boldsymbol{D}}_{2i} = [\mathbf{0}_{p\times m} \quad \boldsymbol{D}_{2i} \quad \mathbf{0}_{p\times m}], \qquad \bar{\boldsymbol{L}}_i = \begin{bmatrix}\boldsymbol{L}_i \\ \boldsymbol{G}_i\end{bmatrix}
\end{aligned} \tag{13}$$

The following Theorem summerizes the conditions for robust stability of the augmented error dyamics (12) with an H_∞ performance index.

Theorem 1. The augmented error dynamics (12) is asymptotically stable with H_∞ performance index $\|\bar{\boldsymbol{e}}\|_2 \le \gamma_1 \|\boldsymbol{\nu}\|_2$, where γ_1 is a scalar, if there exists a symmetric positive definite matrix $\bar{\mathbf{P}} \in \mathbb{R}^{(n+m)\times(n+m)}$ and matrices $\bar{\mathbf{Y}}_i \in \mathbb{R}^{(n+m)\times p}$ $(i = 1, \cdots, r)$, which solve the following optimization problem minimize γ_1, subject to:

$$\boldsymbol{\Psi}_{ii} < 0, \qquad i = 1, \dots, r$$

$$\frac{1}{r-1}\boldsymbol{\Psi}_{ii} + \frac{1}{2}\left(\boldsymbol{\Psi}_{ij} + \boldsymbol{\Psi}_{ji}\right) < 0, 1 \le i < j \le r \tag{14}$$

where

$$\boldsymbol{\Psi}_{ij} = \begin{bmatrix} \boldsymbol{\psi}_{11} & \bar{\boldsymbol{Y}}_i \bar{\boldsymbol{D}}_{2j} - \bar{\boldsymbol{P}}\bar{\boldsymbol{D}}_{1i} & \boldsymbol{I}_{n+m} \\ * & -\gamma_1 \boldsymbol{I}_{2m+w} & \mathbf{0}_{(2m+w)\times(n+m)} \\ * & * & -\gamma_1 \boldsymbol{I}_{n+m} \end{bmatrix} \tag{15}$$

$$\boldsymbol{\psi}_{11} = \bar{\boldsymbol{A}}_i^T \bar{\boldsymbol{P}} + \bar{\boldsymbol{P}}\bar{\boldsymbol{A}}_i - \bar{\boldsymbol{Y}}_i \bar{\boldsymbol{C}}_j - \bar{\boldsymbol{C}}_j^T \bar{\boldsymbol{Y}}_i^T$$

If the optimization problem (14) has feasible solutions, then the gains $\bar{\mathbf{L}}_i$ are given by

$$\bar{\boldsymbol{L}}_i = \bar{\boldsymbol{P}}^{-1}\bar{\boldsymbol{Y}}_i, \qquad i = 1, \dots, r \tag{16}$$

Proof. Consider the following Lyapunov function candidate

$$V_1 = \bar{\boldsymbol{e}}^T \bar{\boldsymbol{P}} \bar{\boldsymbol{e}} \tag{17}$$

the time derivative of V_1 is given by

$$\begin{aligned}\dot{V}_1 = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\boldsymbol{z}) h_j(\boldsymbol{z}) \Big\{ \bar{\boldsymbol{e}}^T \Big[\left(\bar{\boldsymbol{A}}_i - \bar{\boldsymbol{L}}_i \bar{\boldsymbol{C}}_j\right)^T \bar{\boldsymbol{P}} \\ + \bar{\boldsymbol{P}}\left(\bar{\boldsymbol{A}}_i - \bar{\boldsymbol{L}}_i \bar{\boldsymbol{C}}_j\right) \Big] \bar{\boldsymbol{e}} \\ + 2\bar{\boldsymbol{e}}^T \bar{\boldsymbol{P}}\left(\bar{\boldsymbol{L}}_i \bar{\boldsymbol{D}}_{2j} - \bar{\boldsymbol{D}}_{1i}\right)\boldsymbol{\nu} \Big\}\end{aligned} \tag{18}$$

Define the cost function

$$J_1 = \dot{V}_1 + \gamma_1^{-1} \bar{\boldsymbol{e}}^T \bar{\boldsymbol{e}} - \gamma_1 \boldsymbol{\nu}^T \boldsymbol{\nu} \tag{19}$$

then

$$\begin{aligned}J_1 \le \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\boldsymbol{z}) h_j(\boldsymbol{z}) \Big\{ \bar{\boldsymbol{e}}^T \Big[\left(\bar{\boldsymbol{A}}_i - \bar{\boldsymbol{L}}_i \bar{\boldsymbol{C}}_j\right)^T \bar{\boldsymbol{P}} \\ + \bar{\boldsymbol{P}}\left(\bar{\boldsymbol{A}}_i - \bar{\boldsymbol{L}}_i \bar{\boldsymbol{C}}_j\right) \Big] \bar{\boldsymbol{e}} \\ + 2\bar{\boldsymbol{e}}^T \bar{\boldsymbol{P}}\left(\bar{\boldsymbol{L}}_i \bar{\boldsymbol{D}}_{2j} - \bar{\boldsymbol{D}}_{1i}\right)\boldsymbol{\nu} \\ + \gamma_1^{-1} \bar{\boldsymbol{e}}^T \bar{\boldsymbol{e}} - \gamma_1 \boldsymbol{\nu}^T \boldsymbol{\nu} \Big\}\end{aligned} \tag{20}$$

or

$$J_1 \le \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\boldsymbol{z}) h_j(\boldsymbol{z}) \left\{ \tilde{\boldsymbol{e}}^T \boldsymbol{\Omega}_{ij} \tilde{\boldsymbol{e}} \right\} \tag{21}$$

where

$$\tilde{\boldsymbol{e}} = \begin{bmatrix} \bar{\boldsymbol{e}} \\ \boldsymbol{\nu} \end{bmatrix}, \qquad \boldsymbol{\Omega}_{ij} = \begin{bmatrix} \boldsymbol{\omega}_{11} & \bar{\boldsymbol{P}}\left(\bar{\boldsymbol{L}}_i \bar{\boldsymbol{D}}_{2j} - \bar{\boldsymbol{D}}_{1i}\right) \\ * & -\gamma_1 \boldsymbol{I}_{2m+w} \end{bmatrix} \tag{22}$$

$$\begin{aligned}\boldsymbol{\omega}_{11} = \left(\bar{\boldsymbol{A}}_i - \bar{\boldsymbol{L}}_i \bar{\boldsymbol{C}}_j\right)^T \bar{\boldsymbol{P}} + \bar{\boldsymbol{P}}\left(\bar{\boldsymbol{A}}_i - \bar{\boldsymbol{L}}_i \bar{\boldsymbol{C}}_j\right) \\ + \gamma_1^{-1} \boldsymbol{I}_{n+m}\end{aligned}$$

Defining $\bar{\mathbf{Y}}_i = \bar{\mathbf{P}}\bar{\mathbf{L}}_i$, by Schur complement, the inequality $\boldsymbol{\Omega}_{ij} < 0$ becomes

$$\begin{bmatrix} \boldsymbol{\kappa}_{11} & \bar{\boldsymbol{Y}}_i \bar{\boldsymbol{D}}_{2j} - \bar{\boldsymbol{P}}\bar{\boldsymbol{D}}_{1i} & \boldsymbol{I}_{n+m} \\ * & -\gamma_1 \boldsymbol{I}_{2m+w} & \mathbf{0}_{(2m+w)\times(n+m)} \\ * & * & -\gamma_1 \boldsymbol{I}_{n+m} \end{bmatrix} < 0 \tag{23}$$

$$\boldsymbol{\kappa}_{11} = \bar{\boldsymbol{A}}_i^T \bar{\boldsymbol{P}} + \bar{\boldsymbol{P}}\bar{\boldsymbol{A}}_i - \bar{\boldsymbol{Y}}_i \bar{\boldsymbol{C}}_j - \bar{\boldsymbol{C}}_j^T \bar{\boldsymbol{Y}}_i^T$$

Taking Lemma 1 into account, it is obvious that if (14) hold, then (12) is asymptotically stable with H_∞ performance index $\|\bar{\boldsymbol{e}}\|_2 \le \gamma_1 \|\boldsymbol{\nu}\|_2$. The proof is completed. ■

3.2. *Dynamic Output Feedback FTC Design*

After state/fault observer design, now we suggest a dynamic output feedback FTC law based on the estimated states and faults. It will be shown that this control law guarantees asymptotical stability of the closed-loop system that depicted in Fig. 1 in the presence of actuator faults and unknown disturbances.

The suggested dynamic output feedback FTC law is as follows

$$\begin{aligned}\dot{\boldsymbol{\zeta}}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{\ell=1}^{r} h_i(\boldsymbol{z}) h_j(\boldsymbol{z}) h_\ell(\boldsymbol{z}) \left\{ \mathbf{N}_{1ij\ell}\boldsymbol{\zeta}(t) \right. \\ \left. + \mathbf{N}_{2i}\boldsymbol{y}(t) \right\}\end{aligned}$$

$$\begin{aligned}\boldsymbol{u}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\boldsymbol{z}) h_j(\boldsymbol{z}) \mathbf{F}^{-1}\left(\hat{\boldsymbol{x}}(t)\right) \left\{ \mathbf{K}_{1ij}\boldsymbol{\zeta}(t) \right. \\ \left. + \mathbf{K}_{2i}\boldsymbol{y}(t) - \hat{\bar{\boldsymbol{f}}}(t) \right\}\end{aligned} \tag{24}$$

where $\boldsymbol{\zeta}(t) \in \mathbb{R}^n$ is the state of the controller and $\mathbf{N}_{1ij\ell}$, $\mathbf{N}_{2i}$, $\mathbf{K}_{1ij}$, and $\mathbf{K}_{2i}$ $(i, j, \ell = 1, \cdots, r)$ are controller gain matrices with appropriate dimensions, to be designed. Substituting (24) in the system dynamics (5), we have (25) at the bottom of the page

$$\begin{aligned}\dot{\boldsymbol{x}}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{\ell=1}^{r} h_i(\boldsymbol{z}) h_j(\boldsymbol{z}) h_\ell(\boldsymbol{z}) \left\{ \left(\mathbf{A}_i + \mathbf{B}_i \tilde{\mathbf{F}} \mathbf{K}_{2j} \mathbf{C}_\ell\right)\boldsymbol{x}(t) + \mathbf{B}_i \tilde{\mathbf{F}} \mathbf{K}_{1j\ell}\boldsymbol{\zeta}(t) + \left(\mathbf{B}_i - \mathbf{B}_i \tilde{\mathbf{F}}\right)\bar{\boldsymbol{f}}(t) - \mathbf{B}_i \tilde{\mathbf{F}} \boldsymbol{e}_f \right. \\ \left. + \left(\mathbf{B}_i \tilde{\mathbf{F}} \mathbf{K}_{2j} \mathbf{D}_{2\ell} + \mathbf{D}_{1i}\right)\boldsymbol{\omega}(t) \right\}\end{aligned} \tag{25}$$

where $\tilde{\mathbf{F}} = \mathbf{F}\left(\boldsymbol{x}(t)\right)\mathbf{F}^{-1}\left(\hat{\boldsymbol{x}}(t)\right)$. Now we can write

$$\begin{aligned}\dot{\tilde{\boldsymbol{x}}}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{\ell=1}^{r} h_i(\boldsymbol{z}) h_j(\boldsymbol{z}) h_\ell(\boldsymbol{z}) \left\{ \left(\tilde{\mathbf{A}}_{ij\ell} \right.\right. \\ \left. + \tilde{\mathbf{B}}_i \tilde{\mathbf{F}} \tilde{\mathbf{K}}_{1j\ell} \tilde{\mathbf{C}}_\ell\right)\tilde{\boldsymbol{x}}(t) \\ \left. + \left(\tilde{\mathbf{D}}_{i\ell} + \tilde{\mathbf{B}}_i \tilde{\mathbf{F}} \tilde{\mathbf{K}}_{2j} \tilde{\mathbf{E}}_\ell\right)\boldsymbol{\mu} \right\}\end{aligned} \tag{26}$$

where

$$\tilde{\boldsymbol{x}}(t) = \begin{bmatrix} \boldsymbol{x}(t) \\ \boldsymbol{\zeta}(t) \end{bmatrix}, \qquad \boldsymbol{\mu} = \begin{bmatrix} \boldsymbol{\omega}(t) \\ \bar{\boldsymbol{f}}(t) \\ \boldsymbol{e}_f \end{bmatrix} \tag{27}$$

$$\tilde{\boldsymbol{A}}_{ij\ell} = \begin{bmatrix} \boldsymbol{A}_i & \mathbf{0}_{n\times n} \\ \boldsymbol{N}_{2i}\boldsymbol{C}_\ell & \boldsymbol{N}_{1ij\ell} \end{bmatrix}, \tilde{\boldsymbol{B}}_i = \begin{bmatrix} \boldsymbol{B}_i \\ \mathbf{0}_{n\times m} \end{bmatrix}$$

$$\widetilde{\boldsymbol{D}}_{i\ell} = \begin{bmatrix} \boldsymbol{D}_{1i} & \boldsymbol{B}_i & \mathbf{0}_{n\times m} \\ \boldsymbol{N}_{2i}\boldsymbol{D}_{2\ell} & \mathbf{0}_{n\times m} & \mathbf{0}_{n\times m} \end{bmatrix}$$

$$\widetilde{\boldsymbol{C}}_{\ell} = \begin{bmatrix} \boldsymbol{C}_{\ell} & \mathbf{0}_{p\times n} \\ \mathbf{0}_{n\times n} & \boldsymbol{I}_n \end{bmatrix}$$

$$\widetilde{\boldsymbol{E}}_{\ell} = \begin{bmatrix} \boldsymbol{D}_{2\ell} & \mathbf{0}_{p\times m} & \mathbf{0}_{p\times m} \\ \mathbf{0}_{m\times w} & -\boldsymbol{I}_m & \mathbf{0}_{m\times m} \\ \mathbf{0}_{m\times w} & \mathbf{0}_{m\times m} & -\boldsymbol{I}_m \end{bmatrix}$$

$$\widetilde{\boldsymbol{K}}_{1j\ell} = [\boldsymbol{K}_{2j} \quad \boldsymbol{K}_{1j\ell}]$$

$$\widetilde{\boldsymbol{K}}_{2j} = [\boldsymbol{K}_{2j} \quad \boldsymbol{I}_m \quad \boldsymbol{I}_m]$$

and the output equation becomes

$$\boldsymbol{y}(t) = \sum_{i=1}^{r} h_i(\boldsymbol{z})\{\widetilde{\mathbf{M}}_{1i}\widetilde{\boldsymbol{x}}(t) + \widetilde{\mathbf{M}}_{2i}\boldsymbol{\mu}\} \tag{28}$$

where

$$\widetilde{\boldsymbol{M}}_{1i} = [\boldsymbol{C}_i \quad \mathbf{0}_{p\times n}]$$

$$\widetilde{\boldsymbol{M}}_{2i} = [\boldsymbol{D}_{2i} \quad \mathbf{0}_{p\times m} \quad \mathbf{0}_{p\times m}] \tag{29}$$

The robust stability of the closed-loop system (26) and (28) with an H_∞ performance index is guaranteed by the following Theorem.

Theorem 2. The closed-loop system (26) and (28) is asymptotically stable with an H_∞ performance index $\|\boldsymbol{y}(t)\|_2 \le \gamma_2\|\boldsymbol{\mu}\|_2$, where γ_2 is a scalar, if there exists symmetric positive definite matrices $\mathbf{X},\mathbf{Y} \in \mathbb{R}^{n\times n}$, matrices $\mathbf{Q}_{1ij\ell} \in \mathbb{R}^{n\times n}$, $\mathbf{Q}_{2i} \in \mathbb{R}^{n\times p}$, $\mathbf{Q}_{3i\ell} \in \mathbb{R}^{m\times n}$, and $\mathbf{Q}_{4j} \in \mathbb{R}^{m\times p}$, and scalars $\lambda_{1ij\ell},\lambda_{2ij\ell},\lambda_{3ij\ell} > 0$, $(i,j,\ell = 1,\dots,r)$ that solve the following optimization problem

minimize γ_2, subject to:

$$\begin{gathered} \boldsymbol{\Pi}_{ijj} < 0, \qquad i = 1,\dots,r \\ \frac{1}{r-1}\boldsymbol{\Pi}_{ijj} + \frac{1}{2}\left(\boldsymbol{\Pi}_{ij\ell} + \boldsymbol{\Pi}_{i\ell j}\right) < 0, \\ i = 1,\cdots,r, 1 \le j < \ell \le r \end{gathered} \tag{30}$$

where $\mathbf{\Pi}_{ij\ell}$ is shown in (31) at the top of next page. If the above conditions are feasible, then the FTC gain matrices in (24) are given by

$$\begin{aligned} \mathbf{K}_{2j} &= \mathbf{Q}_{4j} \\ \mathbf{N}_{2i} &= \mathbf{N}^{-1}\mathbf{Q}_{2i} \\ \mathbf{K}_{1j\ell} &= \left(\mathbf{Q}_{3j\ell} - \mathbf{Q}_{4j}\mathbf{C}_\ell\mathbf{X}\right)\mathbf{M}^{-T} \\ \boldsymbol{N}_{1ij\ell} &= \boldsymbol{N}^{-1}\left(\boldsymbol{Q}_{1ij\ell} - \boldsymbol{Y}\boldsymbol{A}_i\boldsymbol{X} - \boldsymbol{Q}_{2i}\boldsymbol{C}_\ell\boldsymbol{X}\right)\boldsymbol{M}^{-T} \end{aligned} \tag{32}$$

where $\mathbf{M},\mathbf{N} \in \mathbb{R}^{n\times n}$ satisfy $\mathbf{M}\mathbf{N}^T = \mathbf{I}_n - \mathbf{X}\mathbf{Y}$.

Proof. Consider the candidate Lyapunov function

$$V_2 = \widetilde{\boldsymbol{x}}^T\widetilde{\boldsymbol{P}}\widetilde{\boldsymbol{x}} \tag{33}$$

its time derivative is given by

$$\begin{aligned} \dot{V}_2 = \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{\ell=1}^{r} h_i(\boldsymbol{z})h_i(\boldsymbol{z})h_\ell(\boldsymbol{z})\{&\widetilde{\boldsymbol{x}}^T\big(\widetilde{\boldsymbol{A}}_{ij\ell}^T\widetilde{\boldsymbol{P}} \\ &+ \widetilde{\boldsymbol{P}}\widetilde{\boldsymbol{A}}_{ij\ell}\big)\widetilde{\boldsymbol{x}} \\ &+ 2\widetilde{\boldsymbol{x}}^T\widetilde{\boldsymbol{P}}\widetilde{\boldsymbol{B}}_i\widetilde{\boldsymbol{F}}\widetilde{\boldsymbol{K}}_{1j\ell}\widetilde{\boldsymbol{C}}_\ell\widetilde{\boldsymbol{x}} \\ &+ 2\widetilde{\boldsymbol{x}}^T\widetilde{\boldsymbol{P}}\widetilde{\boldsymbol{D}}_{i\ell}\boldsymbol{\mu} \\ &+ 2\widetilde{\boldsymbol{x}}^T\widetilde{\boldsymbol{P}}\widetilde{\boldsymbol{B}}_i\widetilde{\boldsymbol{F}}\widetilde{\boldsymbol{K}}_{2j}\widetilde{\boldsymbol{E}}_\ell\boldsymbol{\mu}\} \end{aligned} \tag{34}$$

According to Lemma 2, we have

$$\begin{aligned} 2\widetilde{\boldsymbol{x}}^T\widetilde{\boldsymbol{P}}\widetilde{\boldsymbol{B}}_i\widetilde{\boldsymbol{F}}\widetilde{\boldsymbol{K}}_{1j\ell}\widetilde{\boldsymbol{C}}_\ell\widetilde{\boldsymbol{x}} &\le \varepsilon_{1ij\ell}^{-1}\alpha^2\widetilde{\boldsymbol{x}}^T\widetilde{\boldsymbol{P}}\widetilde{\boldsymbol{B}}_i\widetilde{\boldsymbol{B}}_i^T\widetilde{\boldsymbol{P}}\widetilde{\boldsymbol{x}} \\ &+ \varepsilon_{1ij\ell}\widetilde{\boldsymbol{x}}^T\widetilde{\boldsymbol{C}}_\ell^T\widetilde{\boldsymbol{K}}_{1j\ell}^T\widetilde{\boldsymbol{K}}_{1j\ell}\widetilde{\boldsymbol{C}}_\ell\widetilde{\boldsymbol{x}} \end{aligned} \tag{35}$$

and

$$\begin{aligned} 2\widetilde{\boldsymbol{x}}^T\widetilde{\boldsymbol{P}}\widetilde{\boldsymbol{B}}_i\widetilde{\boldsymbol{F}}\widetilde{\boldsymbol{K}}_{2j}\widetilde{\boldsymbol{E}}_\ell\boldsymbol{\mu} &\le \varepsilon_{2ij\ell}^{-1}\alpha^2\widetilde{\boldsymbol{x}}^T\widetilde{\boldsymbol{P}}\widetilde{\boldsymbol{B}}_i\widetilde{\boldsymbol{B}}_i^T\widetilde{\boldsymbol{P}}\widetilde{\boldsymbol{x}} \\ &+ \varepsilon_{2ij\ell}\boldsymbol{\mu}^T\widetilde{\boldsymbol{E}}_\ell^T\widetilde{\boldsymbol{K}}_{2j}^T\widetilde{\boldsymbol{K}}_{2j}\widetilde{\boldsymbol{E}}_\ell\boldsymbol{\mu} \end{aligned} \tag{36}$$

where $\alpha = \|\widetilde{\mathbf{F}}\|_2$ and according to the condition (6), we have $\alpha \le \frac{\delta_2}{\delta_1}$. Defining the cost function as

$$J_2 = \dot{V}_2 + \gamma_2^{-1}\boldsymbol{y}^T\boldsymbol{y} - \gamma_2\boldsymbol{\mu}^T\boldsymbol{\mu} \tag{37}$$

then we have

$$J_2 \le \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{\ell=1}^{r} h_i(\boldsymbol{z})h_i(\boldsymbol{z})h_\ell(\boldsymbol{z})\{\breve{\boldsymbol{x}}^T\boldsymbol{\Xi}_{ij\ell}\breve{\boldsymbol{x}}\} \tag{38}$$

where

$$\breve{\boldsymbol{x}} = \begin{bmatrix} \widetilde{\boldsymbol{x}} \\ \boldsymbol{\mu} \end{bmatrix}, \qquad \boldsymbol{\Xi}_{ij\ell} = \begin{bmatrix} \boldsymbol{\xi}_{11} & \boldsymbol{\xi}_{12} \\ * & \boldsymbol{\xi}_{22} \end{bmatrix}$$

$$\begin{aligned} \boldsymbol{\xi}_{11} &= \widetilde{\boldsymbol{A}}_{ij\ell}^T\widetilde{\boldsymbol{P}} + \widetilde{\boldsymbol{P}}\widetilde{\boldsymbol{A}}_{ij\ell} + \gamma_2^{-1}\widetilde{\boldsymbol{M}}_{1i}^T\widetilde{\boldsymbol{M}}_{1i} \\ &+ \left(\varepsilon_{1ij\ell}^{-1} + \varepsilon_{2ij\ell}^{-1}\right)\alpha^2\widetilde{\boldsymbol{P}}\widetilde{\boldsymbol{B}}_i\widetilde{\boldsymbol{B}}_i^T\widetilde{\boldsymbol{P}} \\ &+ \varepsilon_{1ij\ell}\widetilde{\boldsymbol{C}}_\ell^T\widetilde{\boldsymbol{K}}_{1j\ell}^T\widetilde{\boldsymbol{K}}_{1j\ell}\widetilde{\boldsymbol{C}}_\ell \\ \boldsymbol{\xi}_{12} &= \widetilde{\boldsymbol{P}}\widetilde{\boldsymbol{D}}_{i\ell} + \gamma_2^{-1}\widetilde{\boldsymbol{M}}_{1i}^T\widetilde{\boldsymbol{M}}_{2i} \\ \boldsymbol{\xi}_{22} &= \varepsilon_{2ij\ell}\widetilde{\boldsymbol{E}}_\ell^T\widetilde{\boldsymbol{K}}_{2j}^T\widetilde{\boldsymbol{K}}_{2j}\widetilde{\boldsymbol{E}}_\ell + \gamma_2^{-1}\widetilde{\boldsymbol{M}}_{2i}^T\widetilde{\boldsymbol{M}}_{2i} \\ &- \gamma_1\boldsymbol{I}_{2m+w} \end{aligned} \tag{39}$$

$$\mathbf{\Pi}_{ij\ell} = \begin{bmatrix} \mathbf{A}_i\mathbf{X} + \mathbf{X}\mathbf{A}_i^T & \mathbf{A}_i + \mathbf{Q}_{1ij\ell}^T & \mathbf{D}_{1i} & \mathbf{B}_i & \mathbf{0}_{n\times m} & \mathbf{0}_{n\times m} & \mathbf{X}\mathbf{C}_i^T & \mathbf{B}_i & \mathbf{Q}_{3j\ell}^T \\ * & \boldsymbol{\pi}_{22} & \mathbf{Y}\mathbf{D}_{1i} + \mathbf{Q}_{2i}\mathbf{D}_{2\ell} & \mathbf{Y}\mathbf{B}_i & \mathbf{0}_{n\times m} & \mathbf{0}_{n\times m} & \mathbf{C}_i^T & \mathbf{Y}\mathbf{B}_i & \mathbf{C}_\ell^T\mathbf{Q}_{4j}^T \\ * & * & -\gamma_2\mathbf{I}_w & \mathbf{0}_{w\times m} & \mathbf{0}_{w\times m} & \mathbf{D}_{2\ell}^T\mathbf{Q}_{4j}^T & \mathbf{D}_{2i}^T & \mathbf{0}_{w\times m} & \mathbf{0}_{w\times m} \\ * & * & * & -\gamma_2\mathbf{I}_m & \mathbf{0}_{m\times m} & -\mathbf{I}_m & \mathbf{0}_{m\times p} & \mathbf{0}_{m\times m} & \mathbf{0}_{m\times m} \\ * & * & * & * & -\gamma_2\mathbf{I}_m & -\mathbf{I}_m & \mathbf{0}_{m\times p} & \mathbf{0}_{m\times m} & \mathbf{0}_{m\times m} \\ * & * & * & * & * & -\lambda_{1ij\ell}\mathbf{I}_m & \mathbf{0}_{m\times p} & \mathbf{0}_{m\times m} & \mathbf{0}_{m\times m} \\ * & * & * & * & * & * & -\gamma_2\mathbf{I}_p & \mathbf{0}_{p\times m} & \mathbf{0}_{p\times m} \\ * & * & * & * & * & * & * & -\lambda_{2ij\ell}\mathbf{I}_m & \mathbf{0}_{m\times m} \\ * & * & * & * & * & * & * & * & -\lambda_{3ij\ell}\mathbf{I}_m \end{bmatrix} \tag{31}$$

$$\boldsymbol{\pi}_{22} = \mathbf{Y}\mathbf{A}_i + \mathbf{A}_i^T\mathbf{Y} + \mathbf{Q}_{2i}\mathbf{C}_\ell + \mathbf{C}_\ell^T\mathbf{Q}_{2i}^T$$

Defining $\lambda_{1ij\ell}^{-1} = \varepsilon_{2ij\ell}$, $\lambda_{2ij\ell}^{-1} = \left(\varepsilon_{1ij\ell}^{-1} + \varepsilon_{2ij\ell}^{-1}\right)\alpha^2$, and $\lambda_{3ij\ell}^{-1} = \varepsilon_{1ij\ell}$, and using Schur complement, the inequality $\Xi_{ij\ell} \leq 0$ can be rewritten as

$$\begin{bmatrix} \boldsymbol{\rho}_{11} & \widetilde{\boldsymbol{P}}\widetilde{\boldsymbol{D}}_{i\ell} & \mathbf{0}_{2n\times m} & \widetilde{\boldsymbol{M}}_{1i}^T & \widetilde{\boldsymbol{P}}\widetilde{\boldsymbol{B}}_i & \boldsymbol{\rho}_{16} \\ * & \boldsymbol{\rho}_{22} & \boldsymbol{\rho}_{23} & \widetilde{\boldsymbol{M}}_{2i}^T & \boldsymbol{\rho}_{25} & \boldsymbol{\rho}_{26} \\ * & * & \boldsymbol{\rho}_{33} & \mathbf{0}_{m\times p} & \boldsymbol{\rho}_{35} & \boldsymbol{\rho}_{36} \\ * & * & * & -\gamma_2\boldsymbol{I}_p & \boldsymbol{\rho}_{45} & \boldsymbol{\rho}_{46} \\ * & * & * & * & \boldsymbol{\rho}_{55} & \boldsymbol{\rho}_{56} \\ * & * & * & * & * & \boldsymbol{\rho}_{66} \end{bmatrix} < 0 \tag{40}$$

$$\boldsymbol{\rho}_{11} = \widetilde{\boldsymbol{A}}_{ij\ell}^T\widetilde{\boldsymbol{P}} + \widetilde{\boldsymbol{P}}\widetilde{\boldsymbol{A}}_{ij\ell}, \qquad \boldsymbol{\rho}_{16} = \widetilde{\boldsymbol{C}}_\ell^T\widetilde{\boldsymbol{K}}_{1j\ell}^T$$

$$\boldsymbol{\rho}_{22} = -\gamma_2\boldsymbol{I}_{2m+w}, \qquad \boldsymbol{\rho}_{23} = \widetilde{\boldsymbol{E}}_\ell^T\widetilde{\boldsymbol{K}}_{2j}^T$$

$$\boldsymbol{\rho}_{25} = \boldsymbol{\rho}_{26} = \mathbf{0}_{(2m+w)\times m}$$

$$\boldsymbol{\rho}_{33} = -\lambda_{1ij\ell}\boldsymbol{I}_m, \qquad \boldsymbol{\rho}_{35} = \boldsymbol{\rho}_{36} = \boldsymbol{\rho}_{56} = \mathbf{0}_{m\times m}$$

$$\boldsymbol{\rho}_{45} = \boldsymbol{\rho}_{46} = \mathbf{0}_{p\times m}$$

$$\boldsymbol{\rho}_{55} = -\lambda_{2ij\ell}\boldsymbol{I}_m, \qquad \boldsymbol{\rho}_{66} = -\lambda_{3ij\ell}\boldsymbol{I}_m$$

Now, supposing the symmetric positive definite matrix $\widetilde{\mathbf{P}}$ and its inverse have the following form

$$\widetilde{\boldsymbol{P}} = \begin{bmatrix} \boldsymbol{Y} & \boldsymbol{N} \\ \boldsymbol{N}^T & \boldsymbol{W} \end{bmatrix}, \qquad \widetilde{\boldsymbol{P}}^{-1} = \begin{bmatrix} \boldsymbol{X} & \boldsymbol{M} \\ \boldsymbol{M}^T & \boldsymbol{Z} \end{bmatrix} \tag{41}$$

due to $\widetilde{\mathbf{P}}\widetilde{\mathbf{P}}^{-1} = \mathbf{I}_n$, one could get

$$\widetilde{\boldsymbol{P}}\begin{bmatrix} \boldsymbol{X} \\ \boldsymbol{M}^T \end{bmatrix} = \begin{bmatrix} \boldsymbol{I}_n \\ \mathbf{0}_{n\times n} \end{bmatrix}, \widetilde{\boldsymbol{P}}\begin{bmatrix} \boldsymbol{X} & \boldsymbol{I}_n \\ \boldsymbol{M}^T & \mathbf{0}_{n\times n} \end{bmatrix} = \begin{bmatrix} \boldsymbol{I}_n & \boldsymbol{Y} \\ \mathbf{0}_{n\times n} & \boldsymbol{N}^T \end{bmatrix} \tag{42}$$

Define

$$\boldsymbol{H}_1 = \begin{bmatrix} \boldsymbol{X} & \boldsymbol{I}_n \\ \boldsymbol{M}^T & \mathbf{0}_{n\times n} \end{bmatrix}, \qquad \boldsymbol{H}_2 = \begin{bmatrix} \boldsymbol{I}_n & \boldsymbol{Y} \\ \mathbf{0}_{n\times n} & \boldsymbol{N}^T \end{bmatrix} \tag{43}$$

it follows that $\widetilde{\mathbf{P}}\mathbf{H}_1 = \mathbf{H}_2$. Premultiplying and postmultiplying by $\mathrm{diag}\left(\mathbf{H}_1^T, \mathbf{I}_{2m+w}, \mathbf{I}_m, \mathbf{I}_p, \mathbf{I}_m, \mathbf{I}_m\right)$ and its transpose in (40) respectively, and denoting $\mathbf{Q}_{1ij\ell} = \mathbf{Y}\mathbf{A}_i\mathbf{X} + \mathbf{N}\mathbf{N}_{2i}\mathbf{C}_\ell\mathbf{X} + \mathbf{N}\mathbf{N}_{1ij}\mathbf{M}^T$, $\mathbf{Q}_{2i} = \mathbf{N}\mathbf{N}_{2i}$, $\mathbf{Q}_{3i\ell} = \mathbf{K}_{2j}\mathbf{C}_\ell\mathbf{X} + \mathbf{K}_{1j\ell}\mathbf{M}^T$, and $\mathbf{Q}_{4j} = \mathbf{K}_{2j}$, and taking Lemma 1 into account, if (30) hold, then the closed-loop system (26) and (28) is asymptotically stable with H_∞ performance index $\|\boldsymbol{y}(t)\|_2 \leq \gamma_2\|\boldsymbol{\mu}\|_2$. This completes the proof. ■

At the following, the obtained results in this paper which is a solution to Problem 1 is summarized in the form of an algorithm.

Algorithm 1: Given the GT-S fuzzy model (4), design dynamic output feedback FTC system by performing following steps:

(1) Solve LMIs (9) and obtain state/fault observer gains in (10).
(2) Construct the state/fault observer in Figure 1.
(3) Solve LMIs (15) and obtain gain matrices (16).
(4) Construct the dynamic output feedback FTC system in (11).
(5) Construct the closed-loop system in Figure 1.

This algorithm is constructive and could be executed using standard scientific numerical software such as Matlab [51].

4. Simulation Results

To show the effectiveness of the obtained results, we use the problem of balancing an inverted pendulum on a moving cart [46]. The dynamic equations of the system are given by

$$\dot{x}_1(t) = x_2(t) \tag{44}$$

$$\dot{x}_2(t) = \frac{g\sin x_1(t) - \frac{1}{2}amlx_2^2(t)\sin\left(2x_1(t)\right)}{\frac{4l}{3} - aml\cos^2 x_1(t)} - \frac{a\cos x_1(t)}{\frac{4l}{3} - aml\cos^2 x_1(t)}u(t)$$

$$y(t) = x_1(t)$$

where $x_1(t)$ and $x_2(t)$ are the angle of the pendulum from the vertical line in radians, and the angular velocity in rad/s, respectively. $g = 9.8\ m/s$ is the gravity constant, $2l = 1\ m$ is the pendulum length, and $a = \frac{1}{(m+M)}$ where $m = 2\ kg$ and $M = 2\ kg$ are the pendulum and the cart mass, respectively. This system is modelled with 3 local rules in some works like [36] and with 2 rules in some other works like [46]. The GT-S fuzzy model for this system including additive actuator fault and unknown disturbance has 2 local nonlinear rules, as follows

Rule i: IF $x_1(t)$ is M_i, THEN

$$\begin{aligned} \dot{\boldsymbol{x}}(t) &= \boldsymbol{A}_i\boldsymbol{x}(t) + \boldsymbol{B}_i\boldsymbol{F}\left(\boldsymbol{x}(t)\right)[\boldsymbol{u}(t) + \boldsymbol{f}(t)] + \boldsymbol{D}_{1i}\boldsymbol{\omega}(t) \\ \boldsymbol{y}(t) &= \boldsymbol{C}_i\boldsymbol{x}(t) + \boldsymbol{D}_{2i}\boldsymbol{\omega}(t), \qquad i = 1,2 \end{aligned} \tag{45}$$

where

$$\boldsymbol{A}_1 = \begin{bmatrix} 0 & 1 \\ \frac{g}{\frac{4l}{3} - aml} & 0 \end{bmatrix}, \qquad \boldsymbol{B}_1 = \boldsymbol{B}_2 = \begin{bmatrix} 0 \\ -a \end{bmatrix}$$

$$\boldsymbol{A}_2 = \begin{bmatrix} 0 & 1 \\ \frac{2g}{\pi\left(\frac{4l}{3} - aml\beta^2\right)} & 0 \end{bmatrix}, \boldsymbol{C}_1 = \boldsymbol{C}_2 = [0 \quad 1] \tag{46}$$

$$\boldsymbol{D}_{11} = \boldsymbol{D}_{12} = \begin{bmatrix} 0.01 \\ 0.01 \end{bmatrix}, \boldsymbol{D}_{21} = \boldsymbol{D}_{22} = [0.001]$$

$$\boldsymbol{F}\left(\boldsymbol{x}(t)\right) = \frac{\cos x_1(t)}{\frac{4l}{3} - aml\cos^2 x_1(t)}$$

and $\beta = \cos(88^\circ)$ and the membership functions M_1 and M_2 are shown in Fig. 2.

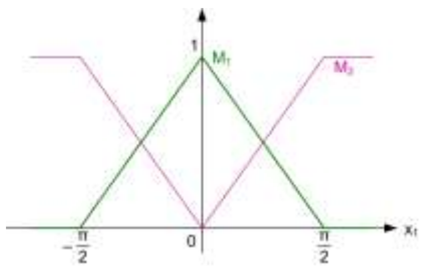

Fig. 2. Membership functions.

This example has feasible LMI conditions and the results of performing Algorithm 1 on this problem using YALMIP parser in the Matlab software and and SeDuMi version 1.1 as solver [52] are as follows:

$$\mathbf{L}_1 = \begin{bmatrix} 37.8444 \\ 275.4664 \end{bmatrix}, \quad \mathbf{L}_2 = \begin{bmatrix} 38.1722 \\ 263.1176 \end{bmatrix}$$
$$\mathbf{G}_1 = -1292.0301, \quad \mathbf{G}_2 = -1300.8683$$
$$\mathbf{K}_{111} = 10^{-8} \times [3.1685 \quad 15.8322]$$
$$\mathbf{K}_{112} = 10^{-8} \times [3.1498 \quad 16.8236]$$
$$\mathbf{K}_{121} = 10^{-7} \times [-2.2365 \quad -8.8956]$$
$$\mathbf{K}_{122} = 10^{-8} \times [-1.3205 \quad 0.2946]$$
$$\mathbf{K}_{21} = 814.2532, \quad \mathbf{K}_{22} = 333.8132$$
$$\mathbf{N}_{1111} = \begin{bmatrix} -1.2587 & 24.0408 \\ 0.9471 & 1.5031 \end{bmatrix}$$
$$\mathbf{N}_{1112} = \begin{bmatrix} -0.9704 & 24.0745 \\ -7.6898 & 0.9186 \end{bmatrix}$$
$$\mathbf{N}_{1121} = \begin{bmatrix} -1.5461 & 23.9864 \\ 9.5841 & 2.0801 \end{bmatrix}$$
$$\mathbf{N}_{1122} = \begin{bmatrix} -1.2585 & 24.0376 \\ 0.9471 & 1.5018 \end{bmatrix}$$
$$\mathbf{N}_{1211} = \begin{bmatrix} -0.4840 & 9.6417 \\ 0.9892 & 0.6319 \end{bmatrix}$$
$$\mathbf{N}_{1212} = \begin{bmatrix} -1.6067 & 8.7531 \\ -5.9343 & -0.4389 \end{bmatrix}$$
$$\mathbf{N}_{1221} = \begin{bmatrix} 0.6385 & 10.5174 \\ 7.9126 & 1.6969 \end{bmatrix}$$
$$\mathbf{N}_{1222} = \begin{bmatrix} -0.4839 & 9.6397 \\ 0.9892 & 0.6309 \end{bmatrix}$$
$$\boldsymbol{N}_{21} = 10^7 \times \begin{bmatrix} -5.5956 \\ -26.2000 \end{bmatrix}$$
$$\boldsymbol{N}_{22} = 10^7 \times \begin{bmatrix} 3.5732 \\ -18.9800 \end{bmatrix}$$

and the minimization results for γ_1 and γ_2 obtained by several methods are summarized in Table II.

Table II. Minimization values of γ_1 and $\gamma_2$obtained from several methods.

	Proposed	[46]	[47]
γ_{1min}	1.5125	3.6104	2.9457
γ_{2min}	0.0098	0.0388	0.0314

Suppose a constant fault as

$$\boldsymbol{f}(t) = \begin{cases} 0 & , \quad 0 \le t < 5 \\ 20\left(1 - e^{-(t-5)}\right) & , \quad 5 \le t \le 20 \end{cases} \tag{47}$$

which is almost constant after $t = 8\ sec$. Consider $\boldsymbol{\omega}(t)$ as a white Gaussian noise with power of $-10\ dB$, and suppose the initial conditions are $\boldsymbol{x}(0) = [20^\circ \quad 0^\circ]^T$. Simulation has been run with Matlab and the results along with that of [46] and [47] are shown in Figs. 3 and 4.

It can be seen that the proposed method stabilizes the closed-loop system and attenuates the effects of actuator fault and disturbance. The dashed and dotted lines are related to the results of [46] and [47], respectively, and show some oscillations in the beginning of the simulation, which do not exist in the results of proposed method. On the other hand, deviation of the vertical line when the actuator fault occurs is bigger in the proposed method. That is due to big amplitude of the fault signal and the existence of nonlinear term in the rules, but the proposed method estimates the fault better than [46] and [47] without oscillations and stabilizes the system in a proper time. Zooming the pictures show that in the steady-state, the proposed method completely rejects unknown disturbance, while there are some oscillations around the equilibrium point in other methods' results.

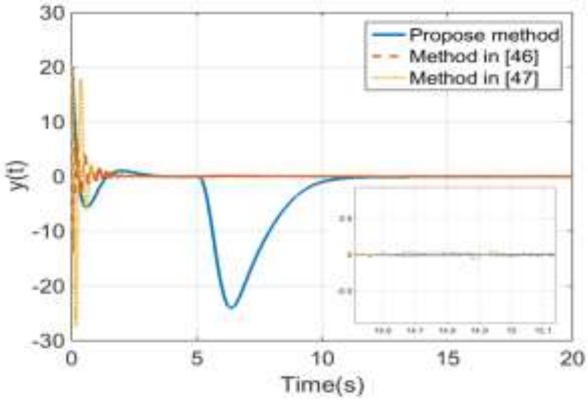

Fig. 3. Output responses under the constant fault.

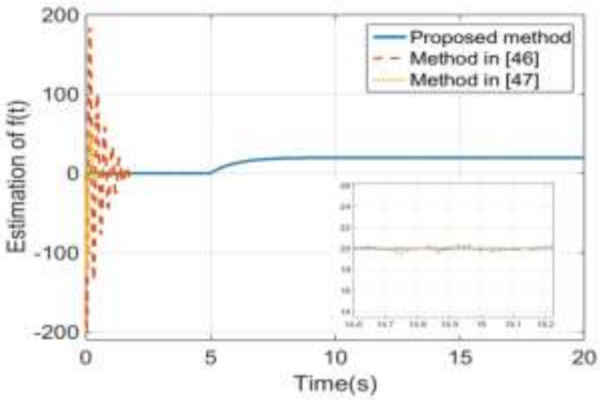

Fig. 4. Estimation of the constant fault.

For the intermittent fault of the form

$$\boldsymbol{f}(t) = \begin{cases} 0 & , \quad 0 \le t < 5 \\ 10\, sin(t-5) & , \quad 5 \le t \le 20 \end{cases} \tag{48}$$

the simulation results are shown in Figs. 5 and 6. Here, the fault information performance is better than that of other methods without big oscillations and the stability is preserved as well. Again, the robustness of the proposed method against disturbance could be seen in the zoomed pictures.

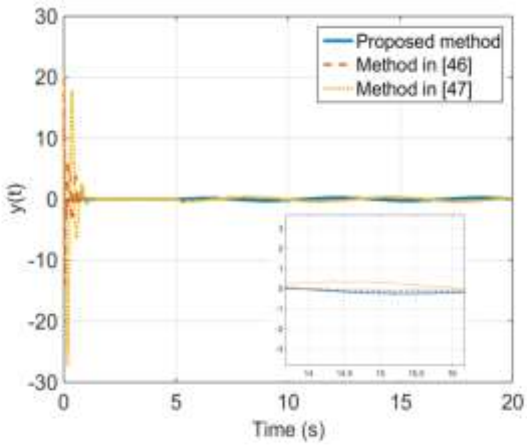

Fig. 5. Output responses under intermittent fault.

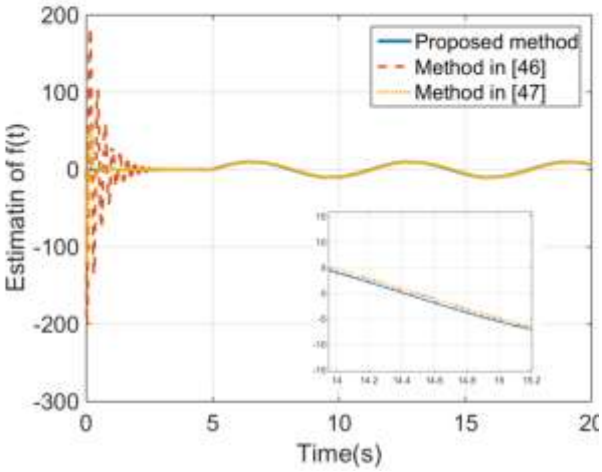

Fig. 6. Estimation of the intermittent fault.

As a concluding remark, although the number of LMIs is more than that of [46], as the simulation results show, the performance of our proposed dynamic output feedback FTC scheme is better than that of the existing ones.

5. Conclusion

In this paper, a special generalized form of T-S fuzzy model with nonlinear local rules has been utilized and a wide class of nonlinear systems are modelled with this model. A dynamic output feedback FTC law is designed based on this GT-S model by solving an LMI feasibility problem and the sufficient conditions for stabilizing the closed-loop system in the presence of unknown disturbances and additive actuator faults are derived. In general, the number of fuzzy rules in the utilized fuzzy model is fewer than that of conventional T-S fuzzy model; also, the problem of uncontrollable local rules is solved using GT-S fuzzy model. The proposed design approach could be applied to a large class of nonlinear systems with unmeasurable states and is robust to different types of unwanted signals. Extending the proposed algorithm to some class of uncertain nonlinear systems with actuator and sensor faults and applying them to the real systems with time-delays and considering some free parameters to decrease conservatism of the solutions are interesting issues for future works.

6. References

[1] Y. Zhang, J. Jiang, “Bibliographical review on reconfigurable fault-tolerant control systems”, *Annual Reviews in Control*, vol. 32, no. 2, pp. 229-252, 2008.
[2] P. M. Frank, “Analytical and qualitative model-based fault diagnosis–a survey and some new results”, *European Journal of control*, vol. 2, no. 1, pp. 6-28, 1996.
[3] S. X. Ding, “Model-based fault diagnosis techniques: design schemes, algorithms, and tools”, Springer Science & Business Media, 2008.
[4] M. Witczak, “Fault diagnosis and fault-tolerant control strategies for non-linear systems: Analytical and soft computing approaches”, Springer International Publishing, 2014.
[5] J. Gertler, “Fault detection and diagnosis in engineering systems”, Routledge, 2017.
[6] R. Isermann, “Supervision, fault-detection and fault-diagnosis methods—a short introduction”, In Combustion Engine Diagnosis, pp. 25-47, Springer Vieweg, Berlin, Heidelberg, 2017.
[7] Z. Gao, C. Cecati, S. X. Ding, “A survey of fault diagnosis and fault-tolerant techniques—Part I: Fault diagnosis with model-based and signal-based approaches”, *IEEE Transactions on Industrial Electronics*, vol. 62, no. 6, pp. 3757-3767, 2015.
[8] Z. Gao, C. Cecati, S. X. Ding, “A survey of fault diagnosis and fault-tolerant techniques—Part II: Fault diagnosis with knowledge-based and hybrid/active approaches”, *IEEE Transactions on Industrial Electronics*, vol. 62, no. 6, pp. 3768-3774, 2015.
[9] H. Kargar, J. Zarei, R. Razavi-Far, “Robust fault detection filter design for nonlinear networked control systems with time-varying delays and packet dropout”, *Circuits, Systems, and Signal Processing*, vol. 38, no. 1, pp. 63-84, 2019.
[10] M. Li, Z. Zuo, H. Liu, C. Liu, B. Zhu, “Adaptive fault tolerant control for trajectory tracking of a quadrotor helicopter”, *Transactions of the Institute of Measurement and Control*, vol. 40, no. 12, pp. 3560-3569, 2018.
[11] M. Witczak, V. Puig, S. M. de Oca, “A fault-tolerant control strategy for non-linear discrete-time systems: application to the twin-rotor system”, *International Journal of Control*, vol. 86, no. 10, pp. 1788-1799, 2013.
[12] X. Jin, “Adaptive fault tolerant tracking control for a class of stochastic nonlinear systems with output constraint and actuator faults”, *Systems & Control Letters*, vol. 107, pp. 100-109, 2017.
[13] X. Yin, Z. Li, L. Zhang, M. Han, “Distributed state estimation of sensor-network systems subject to Markovian channel switching with application to a chemical process”, *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 48, no. 6, pp. 864-874, 2018.
[14] L. Liu, Y. J. Liu, S. Tong, “Neural networks-based adaptive finite-time fault-tolerant control for a class of strict-feedback switched nonlinear systems”, *IEEE Transactions on Cybernetics*, vol. 49, no. 7, pp. 2536-2545, 2018.
[15] X. Yun, L. Wu, Y. Xu, “Adaptive fault-tolerant control for a class of uncertain lower-triangular nonlinear systems with actuator failures”, In 2018 Chinese Control and Decision Conference (CCDC), June 2018, Shenyang, China, pp. 795-800.
[16] A. Moradvandi, S. A. Malek, M. Shahrokhi, “Adaptive finite-time fault-tolerant controller for a class of uncertain MIMO nonlinear switched systems subject to output constraints and unknown input nonlinearities”, *Nonlinear Analysis: Hybrid Systems*, vol. 35, February 2020. DOI: 10.1016/j.nahs.2019.100821
[17] X. Jin, X. Zhao, J. Yu, X. Wu, J. Chi, “Adaptive fault-tolerant consensus for a class of leader-following systems using neural network learning strategy”, *Neural Networks*, vol. 121, pp. 474-483, 2020.
[18] A. Khodadadi, M. Shahriari-kahkeshi, A. Chatraei, “A novel scheme for actuator fault tolerant controller design based on the fault identification”, *Tabriz Journal*

of Electrical Engineering, vol. 48, no. 2, pp. 595-608, 2018 (in persian).
[19] J. L. Castro, M. Delgado, "Fuzzy systems with defuzzification are universal approximators", *IEEE Transactions on Systems, Man, and Cybernetics, Part B: Cybernetics*, vol. 26, no. 1, pp. 149-152, 1996.
[20] C. W. De Silva, "Intelligent control: fuzzy logic applications", CRC press, 2018.
[21] K. Tanaka, H. O. Wang, "Fuzzy control systems design and analysis: A linear matrix inequality approach", John Wiley & Sons, 2004.
[22] J. Mrazgua, M. Ouahi, "Fuzzy fault-tolerant H_∞ control approach for nonlinear active suspension systems with actuator failure", *Procedia Computer Science*, vol. 148, pp. 465-474, 2019.
[23] X. K. Du, H. Zhao, X. H. Chang, "Unknown input observer design for fuzzy systems with uncertainties", *Applied Mathematics and Computation*, vol. 266, pp. 108-118, 2015.
[24] J. Dong, G. H. Yang, "Observer-based output feedback control for discrete-time T-S fuzzy systems with partly immeasurable premise variables", *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 47, no.(1), pp. 98-110, 2017.
[25] D. Ye, X. Li, "Event-triggered fault detection for continuous-time networked polynomial-fuzzy-model-based systems", *Applied Mathematics and Computation*, vol. 366, February 2020. DOI: 10.1016/j.amc.2019.124729
[26] Y. Gao, F. Xiao, J. Liu, R. Wang, "Distributed soft fault detection for interval type-2 fuzzy-model-based stochastic systems with wireless sensor networks", *IEEE Transactions on Industrial Informatics*, vol. 15, no. 1, pp. 334-347, 2018.
[27] S. Zeghlache, A. Djerioui, L. Benyettou, T. Benslimane, H. Mekki, A. Bouguerra, "Fault tolerant control for modified quadrotor via adaptive type-2 fuzzy backstepping subject to actuator faults", *ISA transactions*, vol. 95, pp. 330-345, 2019.
[28] H. Patel and V. Shah, "Fault tolerant control using interval type-2 Takagi-Sugeno fuzzy controller for nonlinear system", In International Conference on Intelligent Systems Design and Applications, December 2018, Vellore, India, pp. 150-164.
[29] J. Tan, S. Dian, T. Zhao, "Further studies on stability and stabilization of T-S fuzzy systems with time-varying delays via fuzzy Lyapunov-Krasovskii functional method", *Asian Journal of Control*, vol. 20, no. 6, pp. 2207-2222, 2018.
[30] D. Kharrat, H. Gassara, A. El Hajjaji, M. Chaabane, "Adaptive fuzzy observer-based fault-tolerant control for Takagi-Sugeno descriptor nonlinear systems with time delay", *Circuits, Systems, and Signal Processing*, vol. 37, no. 4, pp. 1542-1561, 2018.
[31] S. Asadi, A. Khayatian, M. Dehghani, N. Vafamand, "Robust TS fuzzy-based sliding mode observer design for actuator fault reconstruction: Non-quadratic Lyapunov function approach", *Tabriz Journal of Electrical Engineering*, vol. 49, no. 1, pp. 25-35, 2019 (in persian).
[32] R. Rajesh, M. R. Kaimal, "T-S fuzzy model with nonlinear consequence and PDC controller for a class of nonlinear control systems", *Applied Soft Computing*, vol. 7, no. 3, pp. 772-782, 2007.
[33] J. Dong, Y. Wang, G. H. Yang, "Control synthesis of continuous-time TS fuzzy systems with local nonlinear models", *IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics)*, vol. 39, no. 5, pp. 1245-1258, 2009.
[34] J. Dong, Y. Wang, G. H. Yang, "Output feedback fuzzy controller design with local nonlinear feedback laws for discrete-time nonlinear systems", *IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics)*, vol. 40, no. 6, pp. 1447-1459, 2010.
[35] J. Dong, Y. Wang, G. H. Yang, "H_∞ and mixed H_2/ H_∞ control of discrete-time T-S fuzzy systems with local nonlinear models", *Fuzzy Sets and Systems*, vol. 164, no. 1, pp. 1-24, 2011.
[36] H. Moodi, M. Farrokhi, "Robust observer-based controller design for Takagi-Sugeno systems with nonlinear consequent parts", *Fuzzy Sets and Systems*, vol. 273, pp. 141-154, 2015.
[37] H. Moodi, A. Kazemy, "Robust controller design for Takagi-Sugeno systems with nonlinear consequent part and time delay", *International Journal of Fuzzy Systems*, vol. 21, no. 3, pp. 745-754, 2019.
[38] H. Moodi, D. Bustan, "Wind turbine control using TS systems with nonlinear consequent parts", *Energy*, vol. 172, pp. 922-931, 2019.
[39] H. Wang, D. Ye, G. H. Yang, "Actuator fault diagnosis for uncertain T-S fuzzy systems with local nonlinear models", *Nonlinear Dynamics*, vol. 76, no. 4, pp. 1977-1988, 2014.
[40] J. Han, H. Zhang, Y. Wang, Y. Liu, "Disturbance observer based fault estimation and dynamic output feedback fault tolerant control for fuzzy systems with local nonlinear models", *ISA transactions*, vol. 59, pp. 114-124, 2015.
[41] J. Han, H. Zhang, Y. Wang, X. Liu, "Robust fault estimation and accommodation for a class of T-S fuzzy systems with local nonlinear models", *Circuits, Systems, and Signal Processing*, vol. 35, no. 10, pp. 3506-3530, 2016.
[42] M. Klug, E. B. Castelan, V. J. Leite, L. F. Silva, "Fuzzy dynamic output feedback control through nonlinear Takagi-Sugeno models", *Fuzzy Sets and Systems*, vol. 263, pp. 92-111, 2015.
[43] S. Ochiai, J. Yoneyama, and Y. Uchida, "Guaranteed cost control design based on Takagi-Sugeno fuzzy systems with nonlinear subsystems", In 2013 IEEE International Conference on Systems, Man, and Cybernetics, October 2013, Manchester, UK, pp. 4712-4717.
[44] J. Yoneyama, "Output feedback control design for nonlinear systems based on a generalized Takagi-Sugeno fuzzy system", In 2014 World Automation Congress (WAC), August 2014, Waikoloa, HI, USA, pp. 313-318.
[45] J. Yoneyama, "Nonlinear control design based on generalized Takagi-Sugeno fuzzy systems", *Journal of the Franklin Institute*, vol. 351, no. 7, pp. 3524-3535, 2014.
[46] K. Zhang, B. Jiang, M. Staroswiecki, "Dynamic output feedback-fault tolerant controller design for Takagi-Sugeno fuzzy systems with actuator faults", *IEEE Transactions on Fuzzy Systems*, vol. 18, no. 1, pp. 194-201, 2010.

[47] J. Dong, J. Hou, "Output feedback fault-tolerant control by a set-theoretic description of T-S fuzzy systems", *Applied Mathematics and Computation*, vol. 301, pp. 117-134, 2017.
[48] A. Navarbaf, M. J. Khosrowjerdi, "Fault-tolerant controller design with fault estimation capability for a class of nonlinear systems using generalized Takagi-Sugeno fuzzy model", *Transactions of the Institute of Measurement and Control*, vol. 41, no. 15, pp. 4218-4229, 2019.
[49] H. D. Tuan, P. Apkarian, T. Narikiyo, Y. Yamamoto, "Parameterized linear matrix inequality techniques in fuzzy control system design", *IEEE Transactions on fuzzy systems*, vol. 9, no. 2, pp. 324-332, 2001.
[50] Y. Wang, L. Xie, C. E. de Souza, "Robust control of a class of uncertain nonlinear systems", *Systems & Control Letters*, vol. 19, no. 2, pp. 139-149, 1992.
[51] P. Gahinet, A. Nemirovskii, A. J. Laub, M. Chilali, "The LMI control toolbox", In Proceedings of the 33rd IEEE Conference on Decision and Control, December 1994, Lake Buena Vista, FL, USA, pp. 2038-2041.
[53] J. Lofberg, "YALMIP: a toolbox for modeling and optimization in MATLAB", In 2004 IEEE International Conference on Robotics and Automation (IEEE Cat. No.04CH37508), September 2004, New Orleans, LA, USA, pp. 284-289.