Research Paper
Computational Methods for Differential Equations
http://cmde.tabrizu.ac.ir
Vol. 10, No. 1, 2022, pp. 158-167
DOI:10.22034/cmde.2020.38121.1677

New optical soliton solutions for the thin-film ferroelectric materials equation instead of the numerical solution

Ahmet Bekir[1,*], Maha S. M. Shehata[2], and Emad H. M. Zahran[3]
[1]Neighbourhood of Akcaglan, Imarli Street, Number:28/4, 26030, Eskisehir, Turkey.
[2]Departments of Mathematics, Zagazig University, Faculty of Science, Zagazig, Egypt.
[3]Departments of Mathematical and Physical Engineering, Benha University, Faculty of Engineering, Shubra, Egypt.

Abstract

In this article, we will implement the$(G\prime/G)$-expansion method which is used for the first time to obtain new optical soliton solutions of the thin-film ferroelectric materials equation (TFFME). Also, the numerical solutions of the suggested equation according to the variational iteration method (VIM) are demonstrated effectively. A comparison between the achieved exact and numerical solutions has been established successfully.

Keywords. The Thin-Film ferroelectric materials equation, The $(G\prime/G)$-expansion method, The variational iteration method, Travelling wave solutions.

2010 Mathematics Subject Classification. 35C08, 35Q55, 83C15.

1. Introduction

Recently, the propagation of waves in thin-film ferroelectric materials plays a vital role in many branches of physics and hydrodynamics. The equation that represents these phenomena with the aid of soliton science can be written as [2, 15].

$$\left(\frac{m_d c^2}{Q_d^2} - K\right)\varphi^{''} + \left[(g_2 - 2\beta)\,\varphi + g_4\varphi^3 + g_6\varphi^5\right] = 0, \tag{1.1}$$

where g_2, g_4,and g_6 are the unknowns that classified the temperature and the pressure, while m_d is the charge density.

There are three principal axes to get the exact solutions to NLPDEs, namely the reduction methods, Lie symmetry group, and the ansatze approaches methods. The well-known ansatze approaches methods are the $(G\prime/G)$-expansion method, the extended Jacobian Elliptic function expansion method, the Modified Decomposition Method, the Riccati-Bernoulli Sub-ODE Method, The Modified Extended Tanh-Function Method, The Modified Simple Equation Method, The Exp$(-\varphi(\zeta))$-Method, The Modified Exp $(-\varphi(\zeta))$-Expansion Method, The Extended Trial Equation Method, The First Integral Method [4–6, 8, 12, 13, 19–22, 24, 26, 31–33]. The majority of these methods is built on the balance rule in its preparing. One of these methods which is effective and realized the accurate solutions for the many nonlinear physical problems is $(G\prime/G)-$expansion method. Recently the application of the $(G\prime/G)$-expansion method have been established, sea for example Younis [28] who studied the dynamics of optical solitons in $(n+1)-$dimensional nonlinear Schrödinger equation with Kerr and power law nonlinearities that describe the propagation of light pulses in optical fibers using $(G\prime/G)-$expansion method, Farah et al [11] who studied multiple soliton interactions with the help of Hirota bilinear method and also they study dromions for MTs model with the help of the extended $(G\prime/G)-$expansion method.

Received: 25 January 2020 ; Accepted: 23 December 2020.
* Corresponding author. Email: bekirahmet@gmail.com .

Furthermore, recently several trials are implemented through other authoress to construct soliton solution of important models in NLPDE, sea for example Younis et al [29] who constructed new families of exact traveling wave solutions with the modified nonlinear Schrödinger equation using the extended Fan sub-equation method with five parameters, Younis et al [30] who studied the optical solitons with coupled nonlinear Schrödinger system (CNLSS) that describes the propagation of waves in birefringence polarization-preserving fibers with four-wave mixing effect, Ali and Younis [1] who studied the propagation of rogue waves with a nonautonomous NLSE in the presence of external potential, Younas and Younis [27] who studied the extraction of chirped soliton to Chen–Lee–Liu equation (CLLE) with the group velocity dispersion (GVD) and self-steeping coefficients that describe pulse transmission through optical monomode fibres, Baskonus and Eskitascioglu [3] who applied the sine-Gordon expansion method to extract some complex optical soliton solutions to the $(2+1)-$dimensional extended shallow water wave model, Demiray and Bulut [7] who applied the extended trial equation method to find the exact solutions of $(1+1)$ dimensional nonlinear Ostrovsky equation and Manafian and Lakestani [16] who used the improved $\tan\left(\frac{\varphi(\zeta)}{2}\right)-$expansion method to construct different types of solutions to Gerdjikov–Ivanov model.

Moreover, a series of numerical methods are invited to find the approximate solutions for the nonlinear phenomena arising in physics and mathematics such as the Abdominal Decomposition Method, Badi- Approximation Method, Finite Element Method, Boundary Element Method and the VIM,…etc., to achieve the approximate solutions of these problems.

2. Technique description of the (G'/G)-expansion method [3,4]

To propose the general formalism of the nonlinear evolution equation, let us introduce R as a function of $H(x,t)$ and its partial derivatives as,

$$R\left(H, H_x, H_{xx}, H_{tt}, ...\right) = 0, \tag{2.1}$$

that involves the highest order derivatives and nonlinear terms.

By using the transformation $H(x,t) = H(\zeta), \zeta = x - Ct$ Equation (2.1) can be reduced to the following ODE:

$$S\left(H, H', H'', H''', ...\right) = 0, \tag{2.2}$$

where S is a function in $H(\zeta)$ and its total derivatives, while $' = \frac{d}{d\zeta}$.

According to the constructed method [2], the solution is,

$$H(\zeta) = A_0 + \sum_{k=0}^{m} A_k \left[G'/G\right]^k, A_m \neq 0, \tag{2.3}$$

where the positive integer m in Eq. (2.3) can be calculated by balancing the highest order derivative term and the nonlinear term, while $G(\zeta)$ satisfies the second order different equation $G'' + \mu G' + \lambda G = 0$. The solution of this equation admits three forms of solutions according to these three cases $\mu^2 - 4\lambda \succ 0$, $\mu^2 - 4\lambda \prec 0$, and $\mu^2 - 4\lambda = 0$.

2.1. **Cas 1.** When $\mu^2 - 4\lambda \succ 0$, the solution is

$$(G'/G) = \frac{\sqrt[2]{\mu^2 - 4\lambda}}{2}\left[\frac{l_1 \sinh\left(\frac{\sqrt[2]{\mu^2-4\lambda}}{2}\right)\zeta + l_2 \cosh\left(\frac{\sqrt[2]{\mu^2-4\lambda}}{2}\right)\zeta}{l_2 \sinh\left(\frac{\sqrt[2]{\mu^2-4\lambda}}{2}\right)\zeta + l_1 \cosh\left(\frac{\sqrt[2]{\mu^2-4\lambda}}{2}\right)\zeta}\right] - \frac{\mu}{2}. \tag{2.4}$$

2.2. **Cas 2.** When $\mu^2 - 4\lambda \prec 0$, the solution is

$$(G'/G) = \frac{\sqrt[2]{\mu^2 - 4\lambda}}{2}\left[\frac{-l_1 \sin\left(\frac{\sqrt[2]{\mu^2-4\lambda}}{2}\right)\zeta + l_2 \cos\left(\frac{\sqrt[2]{\mu^2-4\lambda}}{2}\right)\zeta}{l_2 \sin\left(\frac{\sqrt[2]{\mu^2-4\lambda}}{2}\right)\zeta + l_1 \cos\left(\frac{\sqrt[2]{\mu^2-4\lambda}}{2}\right)\zeta}\right] - \frac{\mu}{2}. \tag{2.5}$$

2.3. **Cas 3.** When $\mu^2 - 4\lambda = 0$, the solution is

$$(G^{'}/G) = \left(\frac{l_2}{l_1 + l_2\zeta}\right) - \frac{\mu}{2}. \tag{2.6}$$

By Substituting the function $H(\zeta)$ and its partial derivative in the given problem we get a polynomial of $(G^{'}/G)^k$, $(k = 0, 1, 2, ...)$ .In this polynomial, by collecting all terms of the same power of $(G^{'}/G)^k$ and equating the coefficients of different exponential of $(G^{'}/G)^k$ to zero, we get a system of algebraic equations which can be solved by any computer program to find the constants A_k.

3. Application

In this section, we will apply the (G'/G)-expansion as a new technique to achieve the exact solutions for the TFFME in terms of some variables. Hence, when these variables take specific values, the traveling wave solutions could be constructed. By applying the homogeneous balance rule between φ'', φ^5 for the suggested problem equation (1.1) we get $m = \frac{1}{2}$nwhich is a fraction. Consequently take the transformation $\varphi = \sqrt[2]{H}$ which will transform the given equation to,

$$\rho_1\left(\frac{-1}{4}H'^2 + \frac{1}{2}HH''\right) - \rho_2 H^2 - g_4 H^3 - g_6 H^4 = 0, \tag{3.1}$$

where $\rho_1 = \left(\frac{m_d c^2}{Q_d^2} - K\right)$, $\rho_2 = (g_2 - 2\beta)$.

Now, by applying the balance rule between HH'', $H^4 \Rightarrow m = 1$, hence according to the proposed method the solution is,

$$H(\zeta) = A_0 + A_1(G'/G). \tag{3.2}$$

By substituting about the functions H, H^2, H^3, H^4, and H'' at Eq. (3.1) and equating the coefficients of different powers of G^{-k} to zero, we will obtain this system of equations,

$$\begin{aligned}
(G'/G)^4 &\Rightarrow \frac{3}{4}\left(\frac{m_d c^2}{Q_d^2} - K\right) - g_6 A_1^2 = 0,\\
(G'/G)^3 &\Rightarrow (\lambda A_1 + A_0)\left(\frac{m_d c^2}{Q_d^2} - K\right) - A_1^2\left(g_4 + 4g_6 A_0\right) = 0,\\
(G'/G)^2 &\Rightarrow \left(\frac{-\lambda^2}{4} + \frac{3\lambda A_0}{2} + \frac{A_1\lambda^2}{2} + \frac{A_1\mu}{2}\right)\left(\frac{m_d c^2}{Q_d^2} - K\right) - A_1\left(g_2 - 2\beta - 3g_4 A_0 - 6g_6 A_0^2\right) = 0,\\
(G'/G) &\Rightarrow \left(\frac{\lambda^2}{2} + \mu\right)\left(\frac{m_d c^2}{Q_d^2} - K\right) - 2\left(g_2 - 2\beta\right) - 3g_4 A_0 - 3g_6 A_0^2 = 0,\\
constant &\Rightarrow \left(\frac{-A_1^2\mu^2}{4} + \frac{\lambda A_1 A_0 \mu}{2}\right)\left(\frac{m_d c^2}{Q_d^2} - K\right) - A_0^2\left(g_2 - 2\beta - g_4 A_0 - g_6 A_0^2\right) = 0.
\end{aligned} \tag{3.3}$$

By solving this system of equations using any computer program, we will get the following results,

$$\begin{aligned}
A_1 &= \pm\sqrt[2]{\frac{3\left(\frac{m_d c^2}{Q_d^2} - K\right)}{4g_6}},\\
A_0 &= \frac{\left(\frac{g_4}{g_6}\right) \pm \sqrt[2]{\left(\frac{g_4}{g_6}\right)^2 + \frac{4(g_2 - 2\beta)}{g_6}}}{2}.
\end{aligned} \tag{3.4}$$

By substituting at equation (3.2) we obtain,

$$H(\zeta)=\frac{1}{2}\left\{\left(\frac{g_4}{g_6}\right)\pm\left(\sqrt[2]{\left(\frac{g_4}{g_6}\right)^2+\frac{4(g_2-2\beta)}{g_6}}+(G'/G)\sqrt[2]{\frac{3\left(\frac{m_d c^2}{Q_d^2}-K\right)}{4g_6}}\right)\right\}. \tag{3.5}$$

Hence according to the proposed method the solutions are;

3.1. **Case 1.** $\mu^2-4\lambda\succ 0$, the solution is

$$H(\zeta)=\left\{\left(\frac{g_4}{2g_6}\right)\pm\left(\begin{array}{c}\frac{1}{2}\sqrt[2]{\left(\frac{g_4}{g_6}\right)^2+\frac{4(g_2-2\beta)}{g_6}}+\frac{\sqrt[2]{\mu^2-4\lambda}}{4}\sqrt[2]{\frac{3\left(\frac{m_d c^2}{Q_d^2}-K\right)}{4g_6}}\\ \frac{-l_1\sinh\left(\frac{\sqrt[2]{\mu^2-4\lambda}}{2}\right)\zeta+l_2\cosh\left(\frac{\sqrt[2]{\mu^2-4\lambda}}{2}\right)\zeta}{l_2\sinh\left(\frac{\sqrt[2]{\mu^2-4\lambda}}{2}\right)\zeta+l_1\cosh\left(\frac{\sqrt[2]{\mu^2-4\lambda}}{2}\right)\zeta}-\frac{\mu}{2}.\end{array}\right)\right\} \tag{3.6}$$

$$\varphi(\zeta)=\left\{\left(\frac{g_4}{2g_6}\right)\pm\left(\begin{array}{c}\frac{1}{2}\sqrt[2]{\left(\frac{g_4}{g_6}\right)^2+\frac{4(g_2-2\beta)}{g_6}}+\frac{\sqrt[2]{\mu^2-4\lambda}}{4}\sqrt[2]{\frac{3\left(\frac{m_d c^2}{Q_d^2}-K\right)}{4g_6}}\\ \frac{-l_1\sinh\left(\frac{\sqrt[2]{\mu^2-4\lambda}}{2}\right)\zeta+l_2\cosh\left(\frac{\sqrt[2]{\mu^2-4\lambda}}{2}\right)\zeta}{l_2\sinh\left(\frac{\sqrt[2]{\mu^2-4\lambda}}{2}\right)\zeta+l_1\cosh\left(\frac{\sqrt[2]{\mu^2-4\lambda}}{2}\right)\zeta}-\frac{\mu}{4}\sqrt[2]{\frac{3\left(\frac{m_d c^2}{Q_d^2}-K\right)}{4g_6}}.\end{array}\right)\right\}^{\frac{1}{2}} \tag{3.7}$$

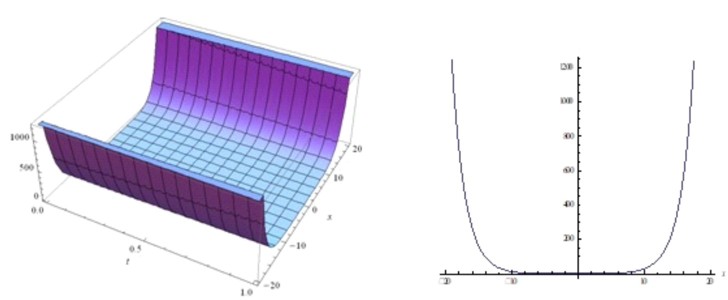

FIGURE 1. The plot of Eq.(3.7) in 2D and 3D with values: $l_1=2, l_2=3, \rho_1=1, \rho_2=1.5, \lambda=2, \mu=3, w=2, c=0.1, T_c=369, T=T_c+10^{-8}, Q_d=2\times10^5, m_d=6.02\times10^{-3}, \alpha_0=10.48\times10^4, g_2=\alpha_0(T-T_c), g_4=4\times10^{-3}, g_6=6\times10^{-3}$

3.2. **Case 2.** When $\mu^2-4\lambda\prec 0$, the solution is

$$H(\zeta)=\frac{1}{2}\left\{\left(\frac{g_4}{g_6}\right)\pm\left(\begin{array}{c}\sqrt[2]{\left(\frac{g_4}{g_6}\right)^2+\frac{4(g_2-2\beta)}{g_6}}+\frac{\sqrt[2]{\mu^2-4\lambda}}{2}\sqrt[2]{\frac{3\left(\frac{m_d c^2}{Q_d^2}-K\right)}{g_6}}\\ \frac{-l_1\sin\left(\frac{\sqrt[2]{\mu^2-4\lambda}}{2}\right)\zeta+l_2\cos\left(\frac{\sqrt[2]{\mu^2-4\lambda}}{2}\right)\zeta}{l_1\sin\left(\frac{\sqrt[2]{\mu^2-4\lambda}}{2}\right)\zeta+l_2\cos\left(\frac{\sqrt[2]{\mu^2-4\lambda}}{2}\right)\zeta}-\frac{\mu}{2}.\end{array}\right)\right\} \tag{3.8}$$

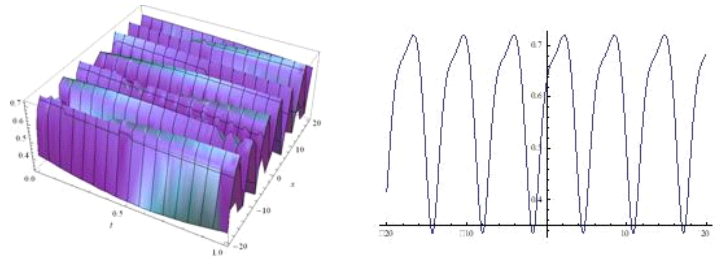

FIGURE 2. The plot of Eq.(3.9) in 2D and 3D with values: $l_1 = 2, l_2 = 3, \rho_1 = 1, \rho_2 = 1.5, \lambda = 2, \mu = 3, w = 2, c = 0.1, T_c = 369, T = T_c + 10^{-8}, Q_d = 2 \times 10^5, m_d = 6.02 \times 10^{-3}, \alpha_0 = 10.48 \times 10^4, g_2 = \alpha_0 (T - T_c), g_4 = 4 \times 10^{-3}, g_6 = 6 \times 10^{-3}$

$$\varphi(\zeta) = \left\{ \left(\frac{g_4}{2g_6}\right) \pm \left(\begin{array}{c} \frac{1}{2}\sqrt[2]{\left(\frac{g_4}{g_6}\right)^2 + \frac{4(g_2-2\beta)}{g_6}} + \frac{\sqrt[2]{\mu^2-4\lambda}}{4}\sqrt[2]{\frac{3\left(\frac{m_d c^2}{Q_d^2}-K\right)}{g_6}} \\ \frac{-l_1 \sin\left(\frac{\sqrt[2]{\mu^2-4\lambda}}{2}\right)\zeta + l_2 \cos\left(\frac{\sqrt[2]{\mu^2-4\lambda}}{2}\right)\zeta}{l_1 \sin\left(\frac{\sqrt[2]{\mu^2-4\lambda}}{2}\right)\zeta + l_2 \cos\left(\frac{\sqrt[2]{\mu^2-4\lambda}}{2}\right)\zeta} - \frac{\mu}{4}\sqrt[2]{\frac{3\left(\frac{m_d c^2}{Q_d^2}-K\right)}{g_6}} \end{array} \right) \right\}^{\frac{1}{2}}. \tag{3.9}$$

3.3. **Cas 3.** When $\mu^2 - 4\lambda = 0$, the solution is

$$H(\zeta) = \frac{1}{2}\left\{ \left(\frac{g_4}{g_6}\right) \pm \left(\begin{array}{c} \sqrt[2]{\left(\frac{g_4}{g_6}\right)^2 + \frac{4(g_2-2\beta)}{g_6}} + \\ \left(\frac{l_2}{l_1+l_2\zeta} - \frac{\mu}{4}\right)\sqrt[2]{\frac{3\left(\frac{m_d c^2}{Q_d^2}-K\right)}{g_6}} \end{array} \right) \right\} \tag{3.10}$$

$$\varphi(\zeta) = \left\{ \left(\frac{g_4}{2g_6}\right) \pm \left(\begin{array}{c} \frac{1}{2}\sqrt[2]{\left(\frac{g_4}{g_6}\right)^2 + \frac{4(g_2-2\beta)}{g_6}} + \\ \left(\frac{l_2}{l_1+l_2\zeta} - \frac{\mu}{4}\right)\sqrt[2]{\frac{3\left(\frac{m_d c^2}{Q_d^2}-K\right)}{g_6}} \end{array} \right) \right\}^{\frac{1}{2}}. \tag{3.11}$$

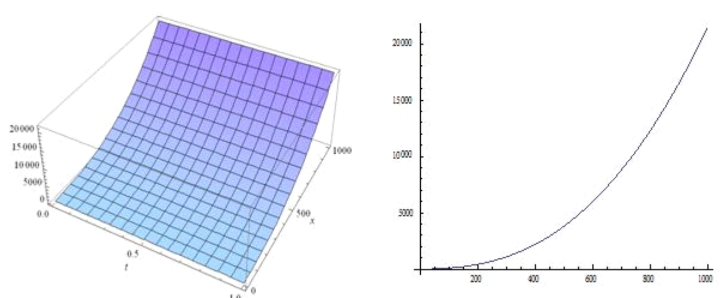

FIGURE 3. The plot of Eq.(3.11) in 2D and 3D with values: $l_1 = 2, l_2 = 3, \rho_1 = 1, \rho_2 = 1.5, \lambda = 2, \mu = 3, w = 2, c = 0.1, T_c = 369, T = T_c + 10^{-8}, Q_d = 2 \times 10^5, m_d = 6.02 \times 10^{-3}, \alpha_0 = 10.48 \times 10^4, g_2 = \alpha_0 (T - T_c), g_4 = 4 \times 10^{-3}, g_6 = 6 \times 10^{-3}$

4. The Variational iteration method

Consider the differential equation with inhomogeneous term $f(\zeta)$ and R, S the linear and the nonlinear operators respectively as:

$$LH + NH = f(\zeta). \tag{4.1}$$

The VIM [25] proposes a correction functional for equation (4.1) to be

$$H_{m+1}(\zeta) = H_m(\zeta) + \int_0^{\zeta} \lambda(t)\left(LH_m(t) + N\widetilde{H}(t) - g(t)\right)dt. \tag{4.2}$$

where λ is a general Lagrange's multiplier, which can be identified optimally via the variational theory, and $\widetilde{H}_m$ as a restricted variation which means$\delta\ \widetilde{H}_m$. The Lagrange multiplier λ is crucial and critical in the method, and it can be a constant or a function. Having λ determined, an iteration formula should be used for the determination of the successive approximations $H_{m+1}(\zeta); n \geq 0$ of the solution $H(\zeta)$. The zeros approximation H_0 can be any selective function. However, using the initial values H_0; H_0' are preferably used for the selective zeros approximation u_0 as will be seen later. Consequently, the solution is given by

$$H(\zeta) = \lim_{\zeta \to \infty} H_m(\zeta).$$

It is interesting to point out that we formally derived the distinct forms of the Lagrange multipliers in [4.2], hence we skip details. We only set a summary of the obtained results.

It is important to give briefly the significant forms of Eq. (20) according to the Lagrange multipliers λ in these results.

For the 1-st order ODE in the form,

$$H' + q(\zeta)H = P(\zeta), H(0) = \rho, \tag{4.3}$$

we find that $\lambda = -1$, and the correction function give the iteration formula;

$$H_{m+1}(\zeta) = H_m(\zeta) - \int_0^{\zeta} \left(H_m'(t) + q(t)H_m(t) - P(t)\right)dt. \tag{4.4}$$

For the 2-nd order ODE in the form,

$$H''(\zeta) + CH'(\zeta) + dH(\zeta) = g(\zeta), H(0) = \rho, H'(0) = \eta, \tag{4.5}$$

we find that $\lambda = t - x$, and the correction function give the iteration formula

$$H_{m+1}(\zeta) = H_m(\zeta) + \int_0^{\zeta} (t - x)\left(H_m''(t) + cH_m'(t) + dH_m(t) - g(t)\right)dt. \tag{4.6}$$

For the 3−th order ODE in the form,

$$H'''(\zeta) + cH''(\zeta) + dH'(\zeta) + eH(\zeta) = g(\zeta), H(0) = \rho, H'(0) = \eta, H''(0) = \sigma, \tag{4.7}$$

we find that $\lambda = \frac{-1}{2!}(t - x)^2$, and the correction function give the iteration formula

$$H_{m+1}(\zeta) = H_m(\zeta) - \frac{1}{2!}\int_0^{\zeta} (t - x)^2\left(H_m'''(t) + cH_m''(t) + dH_m'(t) + eH_m - g(t)\right)dt. \tag{4.8}$$

Consecountly, for the general form of ODE

$$H^{(m)} + f\left(H', H'', H''', ..., H^{(m-1)}\right) = g(\zeta), \tag{4.9}$$

$$H(0) = \rho_0, H'(0) = \rho_1, \tag{4.10}$$

$$H''(0) = \rho_2, ..., H^{(m-1)}(0) = \rho_{m-1}, \tag{4.11}$$

the lagrange multiplier take the general form $\lambda = \frac{(-1)^m}{(m-1)!}(t-x)^{m-1}$, while the general form of iteration rule become,

$$H_{m+1}(\zeta) = H_m(\zeta) + \frac{(-1)^m}{(m-1)!}\int_0^\zeta (t-x)^{m-1}\left(H^m(t) + f\left(H', H'', H''', ..., H^{(m-1)}\right) - g(t)\right)dt. \tag{4.12}$$

Furthermore the zeros approximation $h_0(\zeta)$ can be selected perfectly to be,

$$H_0(\zeta) = H_0(0) + H'(0)\zeta + \frac{1}{2!}H''(0)\zeta^2 + \frac{1}{3!}H'''(0)\zeta^3 ... + \frac{1}{(m-1)}H^{(m-1)}(0)\zeta^{(m-1)} \tag{4.13}$$

where m is the order of the ODE.

5. Application

For simplicity, we will implement the numerical solution corresponding to the third exact solution only. From the second order differential equation (3.1) and the third case of the constructed method mentioned above, we get

$$H(\zeta) = A_0 + A_1\left[\frac{l_2}{l_1 + l_2\zeta} - \frac{\mu}{2}\right], \tag{5.1}$$

$$H(\zeta) = \frac{g_4}{2g_6} \pm \frac{1}{2}\sqrt[2]{\left(\frac{g_4}{g_6}\right)^2 + \frac{4(g_2 - 2\beta)}{g_6}} + \sqrt[2]{\frac{3\left(\frac{m_d c^2}{Q_d^2} - K\right)}{4g_6}}\left[\frac{l_2}{l_1 + l_2\zeta} - \frac{\mu}{2}\right], \tag{5.2}$$

with the initial condition tends to

$$H(0) = \frac{g_4}{2g_6} \pm \frac{1}{2}\sqrt[2]{\left(\frac{g_4}{g_6}\right)^2 + \frac{4(g_2 - 2\beta)}{g_6}} + \sqrt[2]{\frac{3\left(\frac{m_d c^2}{Q_d^2} - K\right)}{4g_6}}\left[\frac{l_2}{l_1} - \frac{\mu}{2}\right]. \tag{5.3}$$

$$H'(\zeta) = \sqrt[2]{\frac{3\left(\frac{m_d c^2}{Q_d^2} - K\right)}{4g_6}}\left[\frac{-l_2^2}{l_1 + l_2\zeta}\right], \tag{5.4}$$

$$H'(0) = \sqrt[2]{\frac{3\left(\frac{m_d c^2}{Q_d^2} - K\right)}{4g_6}}\left[\frac{-l_2^2}{l_1}\right], \tag{5.5}$$

$$H_0(\zeta) = H(0) + \zeta H'(0), \tag{5.6}$$

According to the VIM, the first iteration is,

$$H_0(\zeta) = 1.326 - 0.141\zeta, \tag{5.7}$$

$$H_1(\zeta) = H_0(\zeta) - \int_0^\zeta \left(\begin{array}{c} -0.25H_0'^2(t) + 0.5H_0(t)H_0''(t) - 1.5H_0^2(t) \\ -\left(4\times10^{-3}\right)H_0^3(t) - \left(6\times10^{-3}\right)H_0^4(t) \end{array}\right)dt,$$

$$\begin{aligned} H_1 \quad = \quad & 1.326 - 0141\zeta + \left(4.97\times10^{-3}\right)\zeta - \frac{1.5}{0.423}(1.326 - 0.141\zeta)^3 \\ & -\frac{1}{141}(1.326 - 0.141\zeta) - \frac{6}{705}(1.326 - 0.141\zeta)^5, \end{aligned}$$

$$H_2(\zeta) = H(\zeta) - \int_0^\zeta \left(\begin{array}{c} -0.25H_1'^2(t) + 0.5H_1(t)H_1''(t) - 1.5H_1^2(t) \\ -\left(4\times10^{-3}\right)H_1^3(t) - \left(6\times10^{-3}\right)H_1^4(t) \end{array}\right)dt,$$

$$H_3(\zeta) = H(\zeta) - \int_0^\zeta \left(\begin{array}{c} -0.25H_2'^2(t) + 0.5H_2(t)H_2''(t) - 1.5H_2^2(t) \\ -\left(4\times10^{-3}\right)H_2^3(t) - \left(6\times10^{-3}\right)H_2^4(t) \end{array}\right)dt,$$

....

$$H_{m+1}(\zeta) = H_m(\zeta) - \int_0^{\zeta} \begin{pmatrix} -0.25H_m'^2(t) + 0.5H_m(t)\,H_m''(t) - 1.5H_m^2(t) \\ -\left(4\times10^{-3}\right)H_m^3(t) - \left(6\times10^{-3}\right)H_m^4(t) \end{pmatrix} dt. \tag{5.8}$$

Using the fact that the exact solution is obtained by using $h(\zeta) = \lim_{\zeta\to\infty} h_m(\zeta)$, and we can easily obtained the solution in terms of the original function according to the fact that $\varphi = \sqrt[2]{H}$.

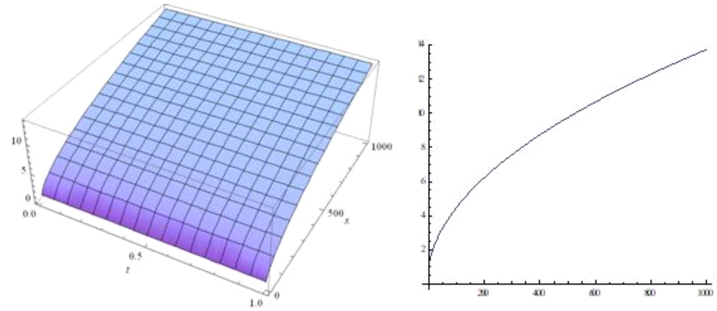

FIGURE 4. The plot of Eq.(5.8) in 2D and 3D with values: $l_1 = 2, l_2 = 3, \rho_1 = 1, \rho_2 = 1.5, \lambda = 2, \mu = 3, w = 2, c = 0.1, T_c = 369, T = T_c + 10^{-8}, Q_d = 2\times10^{5}, m_d = 6.02\times10^{-3}, \alpha_0 = 10.48\times10^{4}, g_2 = \alpha_0(T - T_c), g_4 = 4\times10^{-3}, g_6 = 6\times10^{-3}$

By the same steps, we can construct the numerical solutions corresponding to the other two forms of the achieved exact solutions.

6. Conclusion

In this work, the $(G\prime/G)$-expansion method has been applied successfully to achieve new exact and hence new solitary wave solutions for the thin-film ferroelectric materials equation in Figures 1, 2 and 3 with high accuracy compared with that realized by previous work [23]. Some of the obtained solutions are approximately isomorphic with that achieved by [23] and the others are new. Consequently, stretch positive forward studies of this equation are implemented. Also, the numerical solution corresponding to the achieved exact solution of this equation has been constructed only for the third exact solution using the VIM in Figure 4. Furthermore, a comparison between one of these new exact solutions with the numerical solution obtained by the VIM has been demonstrated. It is clear that there exista an agreement between the exact and the numerical solutions.

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