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On traveling wave solutions: the decoupled nonlinear Schrödinger equations with inter modal dispersion

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Abstract In this article, the decoupled nonlinear Schrdingers equations have been considered that describe the model of dual-core fibers with group velocity mismatch, group velocity dispersion, and spatio-temporal dispersion. These equations are analyzed using two different integrations schemes, namely, extended tanh-function and sine-cosine schemes. The different kind of traveling wave solutions: solitary, topological, periodic and rational, fall out as by-product of these schemes. Finally, the existence of the solutions for the constraint conditions is also shown.

Keywords. Traveling wave solutions, Dual-core model, Decoupled nonlinear Schrödinger's equation, Integration schemes.

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1. Introduction

Traveling wave and soliton solutions are one of the most interesting and fascinating areas of research in different fields of engineering and physical sciences. These molecules are basic ingredients for information transfer, through optical fibers for trans-continental and trans-oceanic distances [1, 2, 3, 4, 5, 6, 7, 8, 9, 16, 17, 18, 19, 30, 33, 34, 36, 37, 38, 39]. Consequently, it is imperative to address the dynamics of these soliton pulses from a mathematical aspect. This will lead to a deeper understanding of the engineering perspective of these solutions. In this paper, we will study the different kinds of traveling wave solutions in dual-core optical fibers from a purely mathematical viewpoint. Therefore, the importance of this paper will be to extract exact traveling wave solution for the nonlinear model. This model is described by the decoupled nonlinear Schrdinger's equation (NLSE) with group velocity mismatch, group velocity dispersion and spatio-temporal dispersion. There are several integration tools available to solve the model. Many nonlinear Schrdinger equations have been examined with regards to soliton theory, where complete integrability was emphasized by various analytical techniques. A few methods of them are: homotopy analysis method, variation principle, Kudryashov method, simplest equation method and several others [10, 11, 12, 13, 14, 15, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 31, 32, 35, 40, 41, 42, 43, 44, 45, 46].

The structures of this paper as follows, the nonlinear model has been summarized in section 2. In section 3, an overview of the integration scheme is given along with the analysis of the model. The next section gives the travelling wave solutions with sine-cosine method. In the last section, the conclusions have been given.

2. The governing equation

Pulse propagation in a decoupled two-core fibers has a distinction from continuous wave propagation. In a conventional two core fibers, pulse propagation has been studied extensively by solving the decoupled model equations; where the light coupling between the two cores is characterized by a structure dependent parameter called the coupling coefficients. The model for decoupled NLSE read as [14, 32]:

$$i\left(\frac{\partial\psi_1}{\partial t}+a_1\frac{\partial\psi_2}{\partial x}\right)+b_1\frac{\partial^2\psi_1}{\partial x^2}+c_1\frac{\partial^2\psi_1}{\partial x\partial t}+d_1F\left(|\psi_1|^2\right)\psi_1+k_1\psi_2=0, \tag{2.1}$$

$$i\left(\frac{\partial\psi_2}{\partial t}+a_2\frac{\partial\psi_1}{\partial x}\right)+b_2\frac{\partial^2\psi_2}{\partial x^2}+c_2\frac{\partial^2\psi_2}{\partial x\partial t}+d_2F\left(|\psi_2|^2\right)\psi_2+k_2\psi_1=0, \tag{2.2}$$

where ψ_1 and ψ_2 are the field envelopes, while x is the propagation co-ordinate and $1/a_j$ are group velocity mismatch, b_j is group velocity dispersion, c_j represents spatio-temporal dispersion and k_j is linear coupling coefficients, for $j = 1, 2$. It may also be noted that d_j is defined by $2\pi n_2/\vartheta A_{eff}$, where n_2, ϑ and A_{eff} are nonlinear refractive index, the wavelength and effective mode area of each wavelength, respectively. For more details, see [1, 2, 46].

The following section described the integration scheme that is used to investigate the soliton solutions.

3. Quick review of the extended tanh-function method

In this section, the extended tanh method [13, 41] has been summarized to obtain the solutions of nonlinear partial differential equations (NPDEs). Hence, we consider the NPDEs of in the following way:

$$P\left(u, u_t, u_x, ..., u_{tt}, u_{xt}, u_{xx}, ...\right) = 0, \tag{3.1}$$

where P is a polynomial of u and its partial derivatives in which the relationship of higher order derivatives and nonlinear terms.

To find the traveling wave solutions, we outline the following sequence of steps towards the extended tanh method:

Step 1: Firstly, by using traveling wave transformation

$$u(x,t) = U(\xi)e^{i\phi}, \text{ where } \xi = B(x - \nu t). \tag{3.2}$$

where B and ν are non-zero arbitrary constants, permits to reduce equation (3.1) to an ODE of $u = u(\xi)$ in the following form

$$Q(u, i\kappa c u^{'}, -i\kappa c u^{'}, -\kappa^2 u^{''}, ...) = 0. \tag{3.3}$$

Step 2: Assuming that the solution of equation (3.1) can be expressed by the following expression:

$$u(\xi) = \sum_{i=0}^{m} A_i \varphi^i, \tag{3.4}$$
$$\varphi^{'} = b + \varphi^2,$$

where b is a parameter to be determined, $\varphi = \varphi(\xi)$, $\varphi^{'} = \frac{d\varphi}{d\xi}$.

Step 3: To determine the positive integer m, we usually balance linear terms of the highest order in the resulting equation with the highest order nonlinear terms appearing in (3.3).

Step 4: We collect all the terms with the same order of φ^j together. Equate each coefficient of the polynomials of φ^i to zero, yields the set of algebraic equations for κ, c and A_i, B_i $(i = 1, 2, ..., m)$ with the aid of the Maple.

Step 5: κ, c and A_i, B_i $(i = 1, 2, ..., m)$.

(i) If $b < 0$

$$\varphi = -\sqrt{-b}\tanh(\sqrt{-b}\xi), \text{ or } \varphi = -\sqrt{-b}\coth(\sqrt{-b}\xi), \tag{3.5}$$

it depends on initial conditions.
(ii) If $b > 0$

$$\varphi = \sqrt{b}\tan(\sqrt{b}\xi), \text{ or } \varphi = -\sqrt{b}\cot(\sqrt{b}\xi), \tag{3.6}$$

it depends on initial conditions.
(iii) If $b = 0$

$$\varphi = \frac{-1}{\xi}. \tag{3.7}$$

After substituting the above results into equation (3.3), the optical soliton solutions of equation (3.1) can be obtained.

3.1. **Applications of the extended tanh-function method.** In order to study, this decoupled system is being split into

$$\psi_j(x,t) = P_j(\xi)e^{i\phi}, \tag{3.8}$$

where

$$\xi = B(x - \nu t).$$

Here, $P_j(\xi)$, for $j = 1, 2$ are the amplitude components of the wave profiles, while ϕ is the phase component of the profiles where $\phi = -\kappa x + \omega t + \theta$. The parameters κ, ω and θ are the wave number, frequency and the phase constant, respectively. While B represents the width of the soliton and ν is the velocity of soliton.

Substitute equation (3.8) and its derivatives into equations (2.1) and (2.2), and decomposed into real and imaginary parts. The real part and imaginary part equations for the two components are given below, for $n = 3 - j$ and $j = 1, 2$.

$$(-\omega + b_j\kappa^2 + c_j\kappa\omega)P_j + (a_j\kappa + k_1)P_n + B^2(b_j - c_j\nu)P_j^{''} + d_jP_j^3 = 0, \tag{3.9}$$

$$B\left[\nu - 2\kappa b_j + c_j(\nu\kappa - B)\right]P_j^{'} + a_jBP_n^{'} = 0. \tag{3.10}$$

By balancing $P^{''}$ with P^3 in equation (3.9) using homogenous balance method, give

$$m + 2 = 3m \Rightarrow m = 1.$$

Therefore, equation (3.5) takes the form

$$P_j(\xi) = A_j + B_j\varphi, \text{ for } j = 1, 2. \tag{3.11}$$

By substituting the vales of $P_j, P_j^{'}, P_j^{''}, P^3$ and P_n into equations (3.9) and (3.10), which yield the equations

$$\begin{aligned}(-\omega + b_j\kappa^2 + c_j\kappa\omega)\left(A_j + B_j\varphi\right) + \left(a_j\kappa + k_1\right)\left(A_n + B_n\varphi\right)\\ +B^2(b_j - c_j\nu)\left(2B_jb\varphi + 2B_j\varphi^3\right) + d_j\left(A_j + B_j\varphi\right)^3 = 0,\end{aligned} \tag{3.12}$$

$$B\left[\nu - 2\kappa b_j + c_j(\nu\kappa - B)\right]\left(B_jb + B_j\varphi^2\right) + a_jB\left(B_nb + B_n\varphi^2\right) = 0. \tag{3.13}$$

Algebraic equations set can be obtained after equating the coefficients of φ^p for $p = 0, 1, 2, 3$, and setting equal to zero. Then, we have

$$2B_j B^2(b_j - \nu c_j) + d_j B_j^3 = 0, \tag{3.14}$$

$$(-\omega + b_j\kappa^2 + c_j\kappa\omega)A_j + (a_j\kappa + k_j)A_n + d_j A_j^3 = 0, \tag{3.15}$$

$$(-\omega + b_j\kappa^2 + c_j\kappa\omega)B_j + (a_j\kappa + k_j)B_n$$
$$+2B_j bB^2(b_j - c_j\nu) + 3d_j B_j A_j^2 = 0, \tag{3.16}$$

$$B_j(-\nu - 2\kappa b_j + c_j\nu\kappa - c_j B) + a_j bB_n = 0, \tag{3.17}$$

$$3d_j A_j B_j^2 = 0. \tag{3.18}$$

After solving the equation (3.15), the following value of B_j can be obtained:

$$B_j = \pm B\left(\frac{2(c_j\nu - b_j)}{d_j}\right)^{\frac{1}{2}}, \tag{3.19}$$

with the constraint condition

$$d_j B^2(c_j\nu - b_j) > 0. \tag{3.20}$$

The following ratio can be obtained by equating the coefficients in the following manner, for $n = 3 - j$ and $j = 1, 2$,

$$\frac{B_j}{B_n} = \left(\frac{(c_j\nu - b_j)d_n}{(c_n\nu - b_n)d_j}\right)^{\frac{1}{2}}, \tag{3.21}$$

which introduces the naturally restriction

$$\prod_{j=1}^{2} d_j(c_j\nu - b_j) > 0. \tag{3.22}$$

For $b < 0$ and $A_j = 0$, the following topological and singular wave solutions can be constructed:

$$\psi_{j_1}(x,t) = \mp B\left(\frac{2b(b_j - c_j\nu)}{d_j}\right)^{\frac{1}{2}} \times \left[\tanh\left(-\sqrt{-b}B(x - \nu t)\right)\right]e^{i(-\kappa x + \omega t + \theta)}, \tag{3.23}$$

and

$$\psi_{j_2}(x,t) = \mp B\left(\frac{2b(b_j - c_j\nu)}{d_j}\right)^{\frac{1}{2}} \times \left[\coth\left(-\sqrt{-b}B(x - \nu t)\right)\right]e^{i(-\kappa x + \omega t + \theta)}. \tag{3.24}$$

For $b > 0$ and $A_j = 0$, the following periodic wave solutions can be obtained:

$$\psi_{j_3}(x,t) = \pm B\left(\frac{2b(c_j\nu - b_j)}{d_j}\right)^{\frac{1}{2}} \times \left[\tan\left(\sqrt{b}B(x - \nu t)\right)\right]e^{i(-\kappa x + \omega t + \theta)}, \tag{3.25}$$

and

$$\psi_{j_4}(x,t) = \mp B\left(\frac{2b(c_j\nu - b_j)}{d_j}\right)^{\frac{1}{2}} \times \left[\cot\left(\sqrt{b}B(x - \nu t)\right)\right]e^{i(-\kappa x + \omega t + \theta)}. \tag{3.26}$$

For $b = 0$ and $A_j = 0$, we have the following rational form of the solution:

$$\psi_{j_5}(x,t) = \mp\left(\frac{2(c_j\nu - b_j)}{d_j}\right)^{\frac{1}{2}} \times \left[(x-\nu t)^{-1}\right] e^{i(-\kappa x+\omega t+\theta)}. \tag{3.27}$$

Solving the system of nonlinear equations (3.17)-(3.21), the following value of B_j can also be obtained:

$$B_j = \pm B\frac{\sqrt{2(c_n\nu - b_n)}\,[\nu(1-c_n\kappa)+(2\kappa b_n + c_n B)]}{ba_n\sqrt{d_n}}. \tag{3.28}$$

Thus, another class of the traveling wave solutions can be obtained in the following manner:

For $b < 0$ and $A_j = 0$, an other class of topological and singular solutions can also be obtained:

$$\begin{aligned}\psi_{j_6}(x,t) &= \mp B\frac{\sqrt{2b(c_n\nu - b_n)}\,[\nu(1-c_n\kappa)+(2\kappa b_n + c_n B)]}{a_n b\sqrt{d_n}}\\ &\times \left[\tanh\left(-\sqrt{-b}B(x-\nu t)\right)\right]e^{i(-\kappa x+\omega t+\theta)},\end{aligned} \tag{3.29}$$

and

$$\begin{aligned}\psi_{j_7}(x,t) &= \mp B\frac{\sqrt{2b(c_n\nu - b_n)}\,[\nu(1-c_n\kappa)+(2\kappa b_n + c_n B)]}{a_n b\sqrt{d_n}}\\ &\times \left[\tanh\left(-\sqrt{-b}B(x-\nu t)\right)\right]e^{i(-\kappa x+\omega t+\theta)}.\end{aligned} \tag{3.30}$$

For $b > 0$ and $A_j = 0$, another class of periodic wave solutions can be constructed as follows:

$$\begin{aligned}\psi_{j_8}(x,t) &= \pm B\frac{\sqrt{2(c_n\nu - b_n)}\,[\nu(1-c_n\kappa)+(2\kappa b_n + c_n B)]}{a_n\sqrt{bd_n}}\\ &\times \left[\tan\left(\sqrt{b}B(x-\nu t)\right)\right]e^{i(-\kappa x+\omega t+\theta)}\end{aligned} \tag{3.31}$$

and

$$\begin{aligned}\psi_{j_9}(x,t) &= \mp B\frac{\sqrt{2(c_n\nu - b_n)}\,[\nu(1-c_n\kappa)+(2\kappa b_n + c_n B)]}{a_n\sqrt{bd_n}}\\ &\times \left[\tan\left(\sqrt{b}B(x-\nu t)\right)\right]e^{i(-\kappa x+\omega t+\theta)}.\end{aligned} \tag{3.32}$$

For $b = 0$ and $A_j = 0$, the rational solution can also be obtained:

$$\psi_{j_{10}}(x,t) = \mp\frac{\sqrt{2(c_n\nu - b_n)}\,[\nu(1-c_n\kappa)+(2\kappa b_n + c_n B)]}{ba_n\sqrt{d_n}(x-\nu t)}e^{i(-\kappa x+\omega t+\theta)}. \tag{3.33}$$

In the following section, the sine-cosine method has been discussed.

4. Quick review of the Sine-Cosine method

The main steps of sine-cosine method has been presented as follows:

Step 1: We describe the wave variable ξ=x-ct into the NPDE Eq. (3.1), we get the NODE. Where $u = u(x,t)$ is the traveling wave solution. This allows us to use the following changes

$$\frac{\partial}{\partial t} = -c\frac{\partial}{\partial \xi}, \frac{\partial^2}{\partial t^2} = c^2\frac{\partial^2}{\partial \xi^2}, \frac{\partial}{\partial x} = \frac{\partial}{\partial \xi}, \frac{\partial^2}{\partial x^2} = \frac{\partial^2}{\partial \xi^2}, \tag{4.1}$$

$$\frac{\partial}{\partial y} = \frac{\partial}{\partial \xi}, \frac{\partial^2}{\partial y^2} = \frac{\partial^2}{\partial \xi^2}, \frac{\partial}{\partial z} = \frac{\partial}{\partial \xi}, \frac{\partial^2}{\partial z^2} = \frac{\partial^2}{\partial \xi^2}. \tag{4.2}$$

Step 2: The solutions of nonlinear partial differential equations can be expressed in the form

$$u(x,t) = \{\lambda sin^{\beta}(\mu\xi)\}, |\xi| < \frac{\pi}{\mu}, \tag{4.3}$$

or in the form

$$u(x,t) = \{\lambda cos^{\beta}(\mu\xi)\}, |\xi| < \frac{\pi}{\mu}, \tag{4.4}$$

where λ, μ and β are parameters that will be determined, μ and c are the wave number and the wave speed, respectively, we use

$$u(\xi) = \lambda sin^{\beta}(\mu\xi), \tag{4.5}$$

$$u''(\xi) = \lambda'' sin^{n\beta}(\mu\xi), \tag{4.6}$$

$$u^n(\xi) = n\mu\beta\lambda^n cos(\mu\xi) sin^{n\beta-1}(\mu\xi), \tag{4.7}$$

$$u^n(\xi) = -n^2\mu^2\beta^2\lambda^n sin^{n\beta}(\mu\xi) + n\mu^2\lambda^n\beta(n\beta-1)sin^{n\beta-2}(\mu\xi) \tag{4.8}$$

and the derivatives of Eq. (4.3) become

$$u(\xi) = \lambda cos^{\beta}(\mu\xi), \tag{4.9}$$

$$u''(\xi) = \lambda'' cos^{n\beta}(\mu\xi), \tag{4.10}$$

$$u^n(\xi) = n\mu\beta\lambda^n sin(\mu\xi) cos^{n\beta-1}(\mu\xi), \tag{4.11}$$

$$u^n(\xi) = -n^2\mu^2\beta^2\lambda^n cos^{n\beta}(\mu\xi) + n\mu^2\lambda^n\beta(n\beta-1)cos^{n\beta-2}(\mu\xi) \tag{4.12}$$

and other derivatives.

Step 3: We substitute Eq. (4.4) or Eq. (4.7) into the reduced equation obtained above in Eq. (4.1), balance the terms of the cosine function when Eq. (4.4) is used, or balance the terms of the sine functions when Eq. (4.3) is used, and solving the resulting system of algebraic equations by using the computerized symbolic calculations. We next collect all terms with the same power in $cos^k(\mu\xi)$ or $sin^k(\mu\xi)$ and set to zero their coefficients to get a system of algebraic equations among the unknowns λ, μ and β. We obtain all possible values of the parameters λ, μ and β.

4.1. **Applications of the Sine-Cosine method.** We solve Eqs. (3.9) and (3.10) by using this method. Hence, we substitute Eq. (4.7) into Eqs. (3.9) and (3.10) we get,

$$\begin{aligned}(-\omega + b_j\kappa^2 + c_j\kappa\omega)\alpha_j \sin^{\beta_j}(\mu_j\xi) + (a_j\kappa + k_1)\alpha_n \sin^{\beta_n}(\mu_n\xi)\\ +d_j\alpha_j \sin^{3\beta_j}(\mu_j\xi) + B^2(b_j - c_j\nu)\alpha_j\beta_j(\beta_j - 1)\mu_j^2 \sin^{\beta_j-2}(\mu_j\xi)\\ -\alpha_j\beta_j^2\mu_j^2 B^2(b_j - c_j\nu)\sin^{\beta_j}(\mu_j\xi) = 0,\end{aligned} \tag{4.13}$$

$$\begin{aligned}B[\nu - 2\kappa b_j + c_j(\nu\kappa - B)]\alpha_j\beta_j\mu_j \sin^{\beta_j-1}(\mu_j\xi)\cos(\mu_j\xi)\\ +a_j\alpha_n\beta_n B\mu_n \sin^{\beta_n-1}\cos(\mu_n\xi) = 0.\end{aligned} \tag{4.14}$$

Equating the exponents and the coefficients of each pair of the sine function, we find the following system of algebraic equations:

$$\beta_j - 2 = 3\beta_j, \tag{4.15}$$

$$B^2(b_j - c_j\nu)(\beta_j - 1)\alpha_j\beta_j\mu_j^2 + d_j\alpha_j = 0, \tag{4.16}$$

$$(-\omega + b_j\kappa^2 + c_j\kappa\omega)\alpha_j + (a_j\kappa + k_1)\alpha_n - \alpha_j\beta_j^2\mu_j^2 B^2(b_j - c_j\nu) = 0. \tag{4.17}$$

By solving the above system of equations. One can get the solutions

$$\beta_j = -1,\ \mu_j = \frac{1}{B}\sqrt{\frac{2(b_j - c_j\nu)}{d_j}},\ \alpha_j = \frac{2\alpha_n(a_j\kappa + k_1)}{2\omega - 2c_j\omega\kappa - 2b_j\kappa^2 - d_j}. \tag{4.18}$$

We find the periodic solution as follows:

$$u_1(x,t) = \frac{2\alpha_n(a_j\kappa + k_1)}{2\omega - 2c_j\omega\kappa - 2b_j\kappa^2 - d_j} sec\left(\frac{1}{B}\sqrt{\frac{2(b_j - c_j\nu)}{d_j}}B(x-\nu t)\right)e^{i(-\kappa x+\omega t+\theta)}, \tag{4.19}$$

and

$$u_2(x,t) = \frac{2\alpha_n(a_j\kappa + k_1)}{2\omega - 2c_j\omega\kappa - 2b_j\kappa^2 - d_j} \csc\left(\frac{1}{B}\sqrt{\frac{2(b_j - c_j\nu)}{d_j}}B(x-\nu t)\right)e^{i(-\kappa x+\omega t+\theta)}. \tag{4.20}$$

However, for $c_j\nu - b_j < 0$, we obtain the solitary wave solution as follows:

$$u_3(x,t) = \frac{2\alpha_n(a_j\kappa + k_1)}{2\omega - 2c_j\omega\kappa - 2b_j\kappa^2 - d_j} sech\left(\frac{1}{B}\sqrt{\frac{2(b_j - c_j\nu)}{d_j}}B(x-\nu t)\right)e^{i(-\kappa x+\omega t+\theta)}, \tag{4.21}$$

and singular solution is in the following form:

$$u_4(x,t) = \frac{2\alpha_n(a_j\kappa + k_1)}{2\omega - 2c_j\omega\kappa - 2b_j\kappa^2 - d_j} \csc h\left(\frac{1}{B}\sqrt{\frac{2(b_j - c_j\nu)}{d_j}}B(x-\nu t)\right)e^{i(-\kappa x+\omega t+\theta)}. \tag{4.22}$$

5. Conclusion

The article obtains the traveling wave solutions of different kinds, which are solitary, topological, singular, periodic and rational solutions to the model for dual-core fibers. The integration mechanisms that are adopted, are extended tanh-function scheme and sine-cosine scheme. It is quite visible that these integration schemes have their limitations. Thus, this paper provides a lot of encouragement for future research in optics. Afterwards, extra solution methods will be applied to obtain optical and singular soliton solutions to the nonlinear model. Also, this model will be considered with other forms of nonlinear media. These are polynomial law, parabolic law, log law, saturable law, and several others. The constructed results may be helpful in explaining the physical meaning of the studied models and other related nonlinear phenomena models. Results are beneficial to the study of the wave propagation. All calculations in this paper have been made quickly with the aid of the Maple.

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