



Heat and mass transfer in MHD micropolar fluid flow with Soret and Dufour effects in a Non-Darcian Porous Medium

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Abstract

This research investigates the influence of an inclined magnetic field, Soret and Dufour effects, and activation energy on the two-dimensional hydromagnetic mixed convection of heat and mass in a micropolar fluid flow past a stretching sheet within a non-Darcian porous medium, incorporating thermal radiation. The complex, nonlinear governing equations are reduced to a system of first-order ordinary differential equations. These equations are solved as a boundary value problem using the shooting method in combination with the fourth-order Adams–Moulton method, coded in FORTRAN. The accuracy of the FORTRAN implementation is validated through comparison with existing published results. Numerical simulations are conducted for a range of physical parameter values, and the findings are benchmarked against previous studies. The effects of key parameters on the fluid velocity, microrotation, temperature, and concentration fields are explored to gain insights into the flow characteristics.

Keywords. MHD, Micropolar fluid, Stretching sheet, Heat source/sink parameter, Radiation parameter, Shooting technique.

2010 Mathematics Subject Classification. 35Q40, 76D05.

1. INTRODUCTION

Heat and mass transfer processes are integral to numerous industrial applications. Mass transfer, in particular, plays a crucial role in petrochemical industries, especially in operations like fractional distillation used to separate components of crude oil. In fact, all refining activities rely on mass transfer mechanisms. Similarly, heat transfer is essential in several sectors such as food processing (e.g., meat and snack manufacturing), chemical production (e.g., batch and continuous reactors), asphalt and concrete industries (e.g., heating concrete or mixing hot asphalt), and commercial laundry systems (e.g., steam ironing and generation).

The analysis of heat and mass transfer over stretching surfaces is also widely applied in industries like glass fiber drawing, the extrusion of plastic sheets, and glass blowing. In these processes, the efficiency and quality of the final product are often dependent on the rate of heat and mass exchange at the stretching surface. Research by Bhatti [6], Devi [9] and Raju et al. [30] has examined the industrial relevance of such phenomena.

Micropolar fluids, comprising randomly oriented micro-sized rigid particles suspended in a viscous medium, have become an area of growing scientific interest. These fluids are relevant in applications involving polymer suspensions, liquid crystals, and chemically active fluids. Extensive research has been conducted on micropolar fluids and their applications, as documented in [3, 4, 12–14, 18, 26, 36, 37].

In the realm of magnetohydrodynamics (MHD), heat and mass transfer have been widely studied. For instance, Mansour et al. [22] provided insights into MHD flows, while the impact of viscous dissipation in free convection over

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a non-uniform temperature surface in a micropolar fluid was addressed in [10, 11]. Further investigations into Joule heating and mass transfer within MHD micropolar flows have been conducted by researchers like Deepa and Murali [8].

The influence of thermal radiation on heat and mass transfer has also attracted considerable attention due to its relevance in high-temperature industrial processes such as internal combustion, rocket propulsion, and energy conversion systems. The work of Abo-Eldohad and Ghonaim [1], Rahman [28], and others [7, 16, 20, 31, 33] has contributed significantly to this field.

Internal and external heat sources or sinks can notably influence heat transfer behaviors. These mechanisms are important for both experimental and practical applications, as they help regulate the thermal environment. Researchers such as Pal and Chatterjee [27], Subhas and Mahesha [35], and Rahman et al. [29] have studied these effects extensively.

The Soret and Dufour effects describe cross-diffusion phenomena in fluid systems. The Soret effect, also known as thermal diffusion, involves mass flux due to temperature gradients. Conversely, the Dufour effect refers to energy flux induced by concentration gradients. Though less frequently analyzed, the Dufour effect can significantly influence heat transfer, especially in gases and liquid mixtures at ambient conditions. Investigations into these effects by Bataller [5], Khan et al. [17], Mortimer and Eyring [24], Ali et al. [2], and Hayat et al. [15] have demonstrated their importance in various fluid dynamic contexts.

Magnetic fields have a significant impact on the behavior of electrically conducting fluids. Fluids tend to flow more easily in the direction of the magnetic field and face resistance when moving across it. This behavior is critical in MHD applications where field configurations influence flow stability and confinement. The effect of inclined magnetic fields has been explored in studies by Singh et al. [34] and Seth et al. [32], offering valuable insights into magneto-fluid interactions.

Activation energy, representing the threshold energy required to initiate molecular motion or chemical reactions, is vital in analyzing fluid dynamics. It is especially important in fields such as chemical process engineering, food preservation, and modeling geothermal reservoirs. Key studies in this domain include those by Monkos [23], Maleque [21], and Mustafa et al. [25], all of whom have investigated the role of activation energy in fluid flows.

In the current work, we first replicate and analyze the findings presented by Mabood [19]. We then extend his model by incorporating the effects of an inclined magnetic field, Dufour number, and activation energy factors not previously discussed in his study. The boundary value problem formed by the resulting system of ordinary differential equations is solved using a combination of the shooting method and the fourth-order Adams–Moulton technique.

The paper is organized as follows: Section 2 details the problem formulation and mathematical modeling. Section 3 outlines the non-dimensionalization of the governing equations. Section 4 presents the numerical approach using the shooting method and the Adams–Moulton method. Section 5 discusses the results in detail, while Section 6 concludes the study with key findings and implications.

2. MATHEMATICAL MODELLING

A two-dimensional, steady mixed convection flow of an incompressible and electrically conducting micropolar fluid is examined. The fluid is assumed to move above and along the chosen x-axis, with the perpendicular y-axis oriented normal to the direction of motion. The flow is initiated by stretching a sheet through the application of two equal and opposite forces. This stretching causes the sheet's velocity to vary proportionally with the distance from the point of origin ($x=0$). The fluid motion is further influenced by an external magnetic field. Additionally, the effects of viscous dissipation, Ohmic heating, and the Soret mechanism are incorporated into the analysis.

The governing partial differential equations describing the MHD convective flow over a stretching sheet in a micropolar fluid, incorporating thermal radiation, activation energy, the Dufour effect, and an inclined magnetic field, are formulated as follows (refer to [19]).

Continuity equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0. \quad (2.1)$$



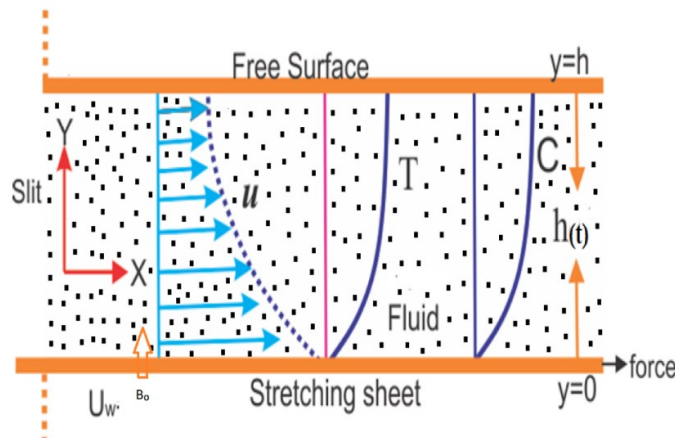


FIGURE 1. Geometry for the flow.

Conservations of momentum:

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \left(v + \frac{k}{\rho} \right) \frac{\partial^2 u}{\partial y^2} - \frac{k_1}{\rho} \frac{\partial N}{\partial y} - \frac{v\varphi}{k} u - \frac{C_b \varphi}{\sqrt{k}} u^2 - \frac{\sigma B_0^2}{\rho} u \sin^2(\beta) + g\beta' (T - T_\infty) + g\beta^* (C - C_\infty). \quad (2.2)$$

Angular momentum:

$$\rho j \left(u \frac{\partial N}{\partial x} + v \frac{\partial N}{\partial y} \right) = \gamma \frac{\partial^2 N}{\partial y^2} - k_1 \left(2N + \frac{\partial u}{\partial y} \right). \quad (2.3)$$

Energy:

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho C_p} \frac{\partial q_r}{\partial y} + \frac{\sigma B_0^2}{\rho C_p} u^2 \sin^2(\beta) + \frac{q'''}{\rho C_p} + \left(\frac{\mu}{\rho C_p} \right) \left(\frac{\partial u}{\partial y} \right)^2 + \frac{D_m k_T}{C_\infty C_p} \left(\frac{\partial^2 C}{\partial y^2} \right). \quad (2.4)$$

Mass diffusion:

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_m \left(\frac{\partial^2 C}{\partial y^2} \right) + \frac{D_m k_r}{T_m} \left(\frac{\partial^2 T}{\partial y^2} \right) - K_r^2 (e^{-\frac{E_a}{kT}}) (C - C_\infty) \left(\frac{T}{T_\infty} \right)^n. \quad (2.5)$$

The term $K_r^2 (e^{-\frac{E_a}{kT}}) (C - C_\infty) \left(\frac{T}{T_\infty} \right)^n$ represents the Arrhenius equation, where E_a is activation energy, K_r is reaction rate and n is the fitted rate constant. β is the inclination angle of magnetic field. The non-uniform heat generation/absorption term, q''' , is expressed as $q''' = \frac{K u_w}{x \nu} (Q_0 (T_w - T_\infty) e^{-\eta} + Q_1 (T - T_\infty))$ where (Q_0) and (Q_1) represent the coefficients associated with the space-dependent and temperature-dependent heat source/sink, respectively. Positive values of both (Q_0) and (Q_1) indicate the presence of internal heat generation, whereas negative values of these parameters signify internal heat absorption within the fluid. The thermal conductivity, denoted by κ , is considered to vary linearly with temperature and is expressed as $\kappa = \kappa_\infty (1 + \varepsilon \theta(\eta))$, where $\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}$ represents the dimensionless temperature. The parameter $\varepsilon = \frac{\kappa_w - \kappa_\infty}{\kappa_\infty}$ characterizes the relative variation in thermal conductivity and depends on the type of fluid. Since ε is generally small, its value is positive for fluids such as air and water, whereas it becomes negative for lubricating oils and similar fluids.

The corresponding BCs are [19]

$$\begin{cases} u = u_w = bx, v = 0, N = -m \frac{\partial u}{\partial y}, T = T_w = T + A_0 \left(\frac{x}{l} \right)^2, C = C_w = C + A_1 \left(\frac{x}{l} \right)^2 & \text{at } y = 0, \\ u \rightarrow 0, N \rightarrow 0, T \rightarrow T_\infty, C \rightarrow C_\infty, & \text{as } y \rightarrow \infty. \end{cases} \quad (2.6)$$



3. DIMENSIONLESS FORM OF THE MODEL

We will now describe the set of transformations to change the governing equations:

$$\begin{cases} \eta = \sqrt{\frac{b}{v}}y, & N = bx\sqrt{\frac{b}{v}}g(\eta), & u = vx f'(\eta), & v = \sqrt{bv}f(\eta), \\ \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, & \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}, & T - T_\infty = A_0 \left(\frac{x}{l}\right)^2 \theta(\eta), \\ T_w - T_\infty = A_0 \left(\frac{x}{l}\right)^2, & C - C_\infty = A_1 \left(\frac{x}{l}\right)^2 \phi(\eta), & C_w - C_\infty = A_1 \left(\frac{x}{l}\right)^2, \end{cases} \quad (3.1)$$

where f and g are the stream and microrotation functions respectively and both are dimensionless, η is the similarity variable, l is the characteristic length. The above Equation (2.1) is satisfied identically. Remaining governing Eqs. (2.2)–(2.5) are transformed into the following form:

$$(1 + K)f'''' + ff'' - (f')^2 - Da^{-1}f' - \alpha(f')^2 + kg' - Mf' \sin^2(\beta) + Gr\theta + Gm\phi = 0, \quad (3.2)$$

$$\left(1 + \frac{K}{2}\right)g'' + fg' - f'g - K(2g + f'') = 0, \quad (3.3)$$

$$(1 + R + \varepsilon\theta)\theta'' + \text{Pr}(f\theta' - 2f'\theta) + \varepsilon(\theta')^2 + \text{Pr}MEc(f')^2 + \text{Pr}Ec(f'')^2 + (1 + \varepsilon\theta)(Q_0f' + Q_1\theta) + P_rDf\phi'' = 0, \quad (3.4)$$

$$\phi'' + \text{Sc}(\phi'f - 2\phi f') + \text{Sc}S_0\theta'' - \text{Sc}\sigma(1 + \delta\theta)^n e^{\frac{-E}{1+\delta\theta}} = 0. \quad (3.5)$$

The BCs, now, become:

$$\begin{cases} f'(\eta) = 1, & f(\eta) = 0, & g(\eta) = -m_0 f''(\eta), & \theta(\eta) = 1, & \phi(\eta) = 1, & \text{at } \eta = 0, \\ f'(\eta) \rightarrow 0, & g(\eta) \rightarrow 0, & \theta(\eta) \rightarrow 0, & \phi(\eta) \rightarrow 0 & \text{as } \eta \rightarrow \infty. \end{cases} \quad (3.6)$$

In addition, the system involves the following dimensionless physical parameters governing the flow and transport processes:

- Material parameter: $K = \frac{k_1}{\mu}$ representing the strength of coupling between fluid motion and microrotation.
- Magnetic parameter: $M = \frac{\sigma B_0^2}{\rho b}$, representing the ratio of the Lorentz force to the viscous force.
- Inverse Darcy parameter: $Da^{-1} = \frac{\phi v}{kb}$, representing the resistance offered by the porous medium.
- Grashof number: $Gr = \frac{g\beta(T - T_\infty)}{b^2 l}$, representing the ratio of buoyancy force to viscous force.
- Modified Grashof number: $G_m = \frac{g\beta^*(C - C_\infty)}{b^2 l}$, representing the relative contribution of solutal buoyancy to viscous forces.
- Prandtl number: $\text{Pr} = \frac{\mu C_p}{K_\infty}$, representing the ratio of momentum diffusivity to thermal diffusivity.
- Radiation parameter: $R = \frac{16T_\infty^3 \sigma^*}{3k^* k_\infty}$, representing the relative contribution of radiative to conductive heat transfer.
- Eckert number: $Ec = \frac{b^2 l}{A_0 C_p}$, representing the ratio of kinetic energy to thermal energy.
- Schmidt number: $\text{Sc} = \frac{v}{D_m}$, representing the ratio of momentum diffusivity to mass diffusivity.
- Soret number: $S_0 = \frac{D_m K_T (T_w - T_\infty)}{T_m (C_w - C_\infty)}$, representing the mass diffusion induced by temperature gradients.
- Dufour number: $Df = \frac{D_m K_T (C_w - C_\infty)}{C_s C_p (T_w - T_\infty)}$, representing the heat flux induced by concentration gradients.
- Activation energy parameter: $E = \frac{E_a}{kT_\infty}$, representing the normalized activation energy governing the chemical reaction rate.
- Chemical reaction parameter: $\sigma = \frac{K_r^2}{a}$, representing the relative strength of the chemical reaction.



The skin-friction coefficient C_{fx} , Nusselt number Nu_x and Sherwood number Sh_x are defined as Mathematical forms of local skin-friction coefficient, Nusselt and Sherwood numbers are

$$C_{fx} = \frac{\tau_w}{\rho (u_w)^2}, \quad Nu_x = \frac{xq_w}{k(T_w - T_\infty)}, \quad Sh_x = \frac{xm_w}{D_m(C_w - C_\infty)}. \quad (3.7)$$

Here, the heat flux q_w , the skin-friction on at plate τ_w , and the mass transfer rate m_w are given by

$$\begin{aligned} \tau_w &= \left[(\mu + k_1) \frac{\partial u}{\partial y} \right]_{y=0}, \quad q_w = -K \left[\frac{\partial T}{\partial y} \right]_{y=0}, \quad m_w = -D_m \left[\frac{\partial C}{\partial y} \right]_{y=0}, \\ C_f Re_x^{1/2} &= [1 + K] f'(0), \quad Nu_x Re_x^{-1/2} = -\theta'(0), \quad Sh_x Re_x^{-1/2} = -\phi'(0). \end{aligned} \quad (3.8)$$

Here $Re_x = \frac{u_w x}{\nu}$ is the local Reynolds number.

4. NUMERICAL TREATMENT

This section outlines the application of the shooting method to solve the transformed ordinary differential equations (3.2)-(3.5), along with the associated boundary conditions (3.6). To proceed, the system is first reduced to a set of first-order ordinary differential equations using the following substitutions:

$$f = y_1, \quad f' = y_2, \quad f'' = y_3, \quad g = y_4, \quad g' = y_5, \quad \theta = y_6, \quad \theta' = y_7, \quad \phi = y_8, \quad \phi' = y_9.$$

By applying the above substitutions, the resulting system of first-order ordinary differential equations can be expressed as follows:

$$\begin{cases} y_1' = y_2, & y_1(0) = 0, \\ y_2' = y_3, & y_2(0) = 1, \\ y_3' = \frac{1}{(1+K)} [y_2^2 - y_1 y_3 - Da^{-1} y_2 + \alpha y_2^2 - K y_5 + M y_2 - Gr y_6 - G_m y_8], & y_3(0) = r, \\ y_4' = y_5, & y_4(0) = -m_0 r, \\ y_5' = \frac{2}{(2+K)} [-y_1 y_5 + y_2 y_4 + K(2y_4 + y_3)], & y_5(0) = s, \\ y_6' = y_7, & y_6(0) = 1, \\ y_7' = \frac{1}{[1+R+\epsilon y_6]} [-(y_1 y_7 - 2y_2 y_6) - \epsilon y_7^2 - P_r E_c y_3^2 - (1+\epsilon y_6)(Q_0 y_2 + Q_1 y_6)], & y_7(0) = t, \\ y_8' = y_9, & y_8(0) = 1, \\ y_9' = -Sc(y_1 y_9 - 2y_8 y_2 + S_0 y_7' \frac{Nt}{Nb} y_7'), & y_9(0) = u. \end{cases} \quad (4.1)$$

The given initial value problem (IVP) is numerically solved using the shooting technique in combination with the Adams–Moulton method. For the application of the Adams–Moulton approach, the unknown initial condition is selected as $y_3(0) = r$, $y_5(0) = s$, $y_7(0) = t$, $y_9(0) = u$.

In the system of equations given above, the unspecified conditions should be determined so that

$$(y_2(\eta_\infty, r, s, t, u))_{\eta=\eta_\infty} = 0,$$

$$(y_4(\eta_\infty, r, s, t, u))_{\eta=\eta_\infty} = 0,$$

$$(y_6(\eta_\infty, r, s, t, u))_{\eta=\eta_\infty} = 0,$$

$$(y_8(\eta_\infty, r, s, t, u))_{\eta=\eta_\infty} = 0.$$



Newton's iterative scheme gets the form

$$\begin{bmatrix} r^{k+1} \\ s^{k+1} \\ t^{k+1} \\ u^{k+1} \end{bmatrix} = \begin{bmatrix} r^k \\ s^k \\ t^k \\ u^k \end{bmatrix} - \begin{bmatrix} \frac{\partial y_2}{\partial r} & \frac{\partial y_4}{\partial r} & \frac{\partial y_6}{\partial r} & \frac{\partial y_8}{\partial r} \\ \frac{\partial y_2}{\partial s} & \frac{\partial y_4}{\partial s} & \frac{\partial y_6}{\partial s} & \frac{\partial y_8}{\partial s} \\ \frac{\partial y_2}{\partial t} & \frac{\partial y_4}{\partial t} & \frac{\partial y_6}{\partial t} & \frac{\partial y_8}{\partial t} \\ \frac{\partial y_2}{\partial u} & \frac{\partial y_4}{\partial u} & \frac{\partial y_6}{\partial u} & \frac{\partial y_8}{\partial u} \end{bmatrix}_{(r^k, s^k, t^k, u^k, \eta)}^{-1} \begin{bmatrix} y_2 \\ y_4 \\ y_6 \\ y_8 \end{bmatrix}_{(r^k, s^k, t^k, u^k, \eta)}. \quad (4.2)$$

The completion of the shooting technique requires the following steps;

- (i) Selecting the values $r = r^{(0)}$, $s = s^{(0)}$, $t = t^{(0)}$ and $u = u^{(0)}$.
- (ii) Choose $\varepsilon > 0$, $\max\{|y_2(\eta_{max})|, |y_4(\eta_{max})|, |y_6(\eta_{max})|, |y_8(\eta_{max})|\} < \varepsilon$ stop, or else go to (iii).
- (iii) Calculate $r^{(k+1)}$, $s^{(k+1)}$, $t^{(k+1)}$ and $u^{(k+1)}$, $k=0,1,2,\dots$, using (4.2).
- (iv) Do again(i) in addition (ii).

The convergence criteria are chosen to be successive value agree up to 5 significant digits. The choice of $\eta_{max} = 10$ was more than enough for end condition.

5. RESULTS INTERPRETATION

This section thoroughly examines how flow-controlling parameters influence the corresponding profile distribution. The governing flow equations are transformed into a dimensionless system using appropriate physical variables, which are then solved numerically using the shooting technique with Adams-Moulton method of order four. The significance and interpretation of each parameter's impact are discussed in detail, and the study's findings are comprehensively presented. Throughout this investigation, the parameter values used to calculate the outcomes are fixed as $Da^{-1} = 0.1$, $\beta = \frac{\pi}{6}$, $Gr = Ec = \alpha = S_0 = Gm = R = 0.1$, $Pr=0.71$, $\varepsilon=0.01$, $Q_0 = 0.02$, $\delta = K = M = 1$, $\sigma = 0.01$, $m_0 = E = 0.1$, $Q_1 = 0.01$, $Sc = 0.22$, $Df = 0.1$, $n = 0.2$ unless otherwise mentioned.

To validate the accuracy of the results, the currently obtained outcomes are compared with solutions available in literature. From Tables 1 and 2, it can be concluded that the results show excellent agreement with the existing literature.

TABLE 1. Comparison of $C_f Re_x^{1/2}$ for different values of K keeping all other parameters zero.

K	Ishak, A. [16]	Mabood, F. et al. [19]	Present Results
0	-1.000000	-1.000008	-1.00000800
1	-1.367872	-1.367996	-1.36799700
2	-1.621225	-1.621575	-1.62157500
3	—	-1.827382	-1.82738300
4	-2.004133	-2.005420	-2.00542000
5	—	-2.164823	-2.16482200

TABLE 2. Comparison of Nu_x for different values of Pr keeping all other parameters zero.

Pr	Ishak, A. [16]	Pal, D., & Chatterjee, S. [27]	Mabood, F. et al. [19]	Present Results
1	1.3333	1.333333	1.33333334	1.3333330000
3	2.5097	2.509725	2.50972157	2.5097220000
10	4.7969	4.796873	4.79687059	4.7968610000



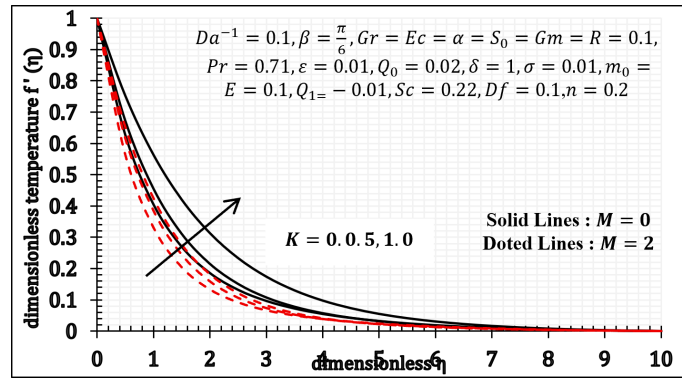
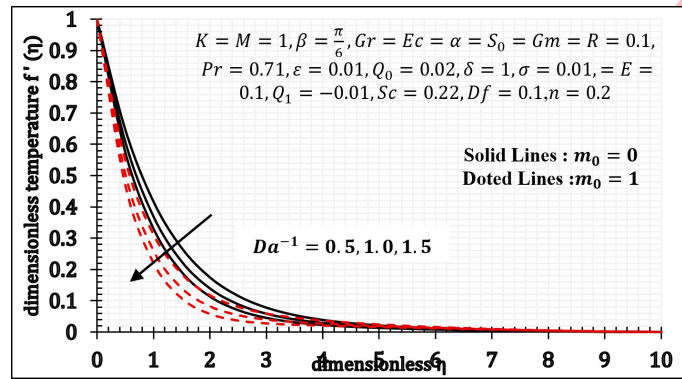
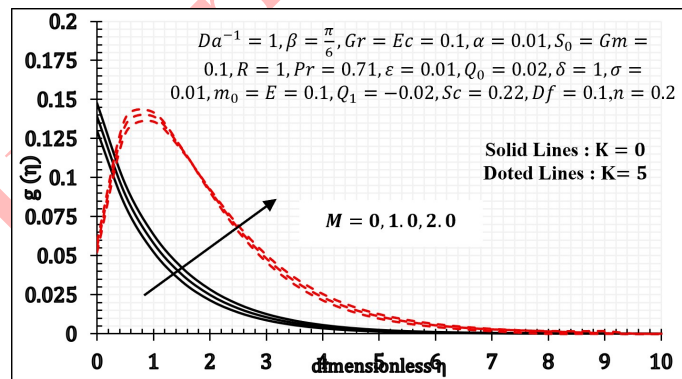
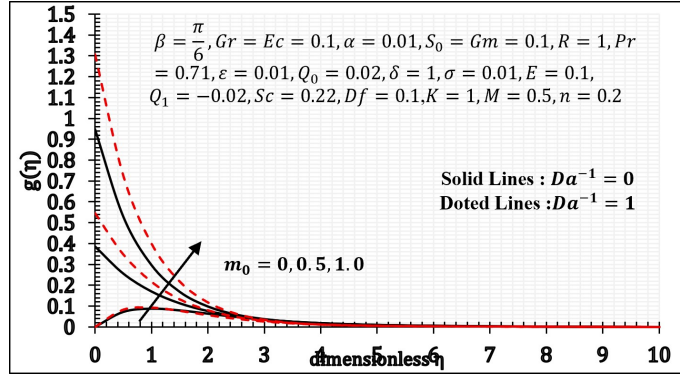
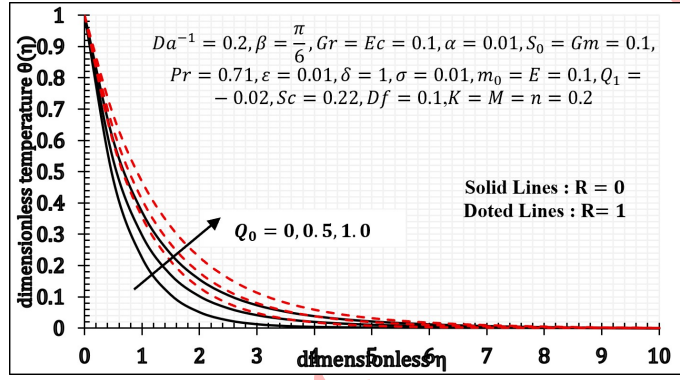
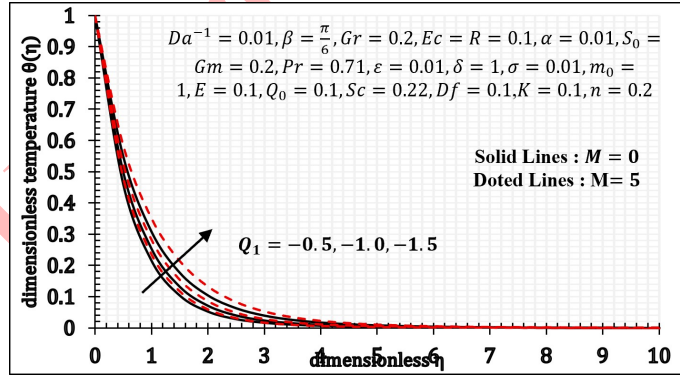
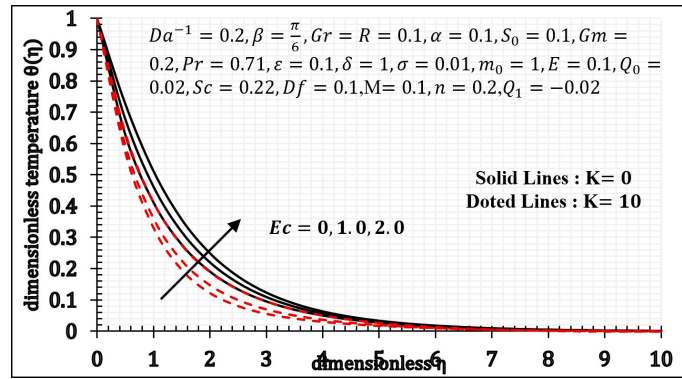
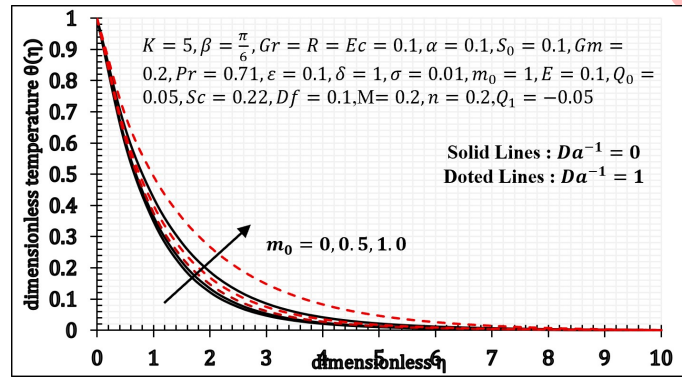
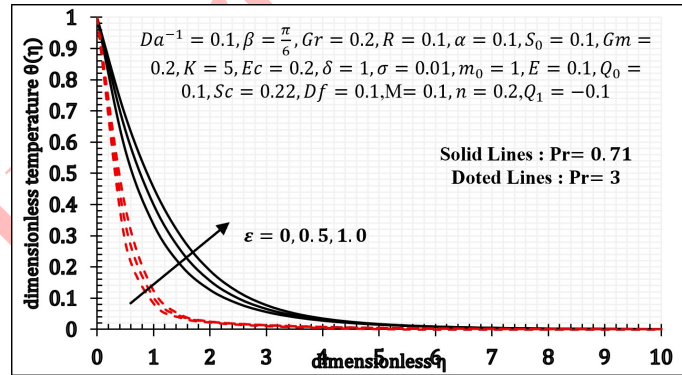
FIGURE 2. Impact of K and M on velocity $f'(\eta)$.FIGURE 3. Impact of Da^{-1} and m_0 on velocity $f'(\eta)$.FIGURE 4. Impact of M and K on angular velocity $g(\eta)$.

Figure 2 depicts the effects of the material parameter (K) and magnetic parameter (M) on the velocity distribution. An increase in the magnetic parameter reduces the velocity throughout the boundary layer. This behavior is attributed to the Lorentz force generated by the interaction between the applied magnetic field and the electrically conducting fluid. The Lorentz force acts opposite to the flow direction and suppresses the fluid motion, thereby reducing the

FIGURE 5. Impact of Da^{-1} and m_0 on angular velocity $g(\eta)$.FIGURE 6. Impact of R and Q_0 on temperature $\theta(\eta)$.FIGURE 7. Impact of Q_1 and M on temperature $\theta(\eta)$.

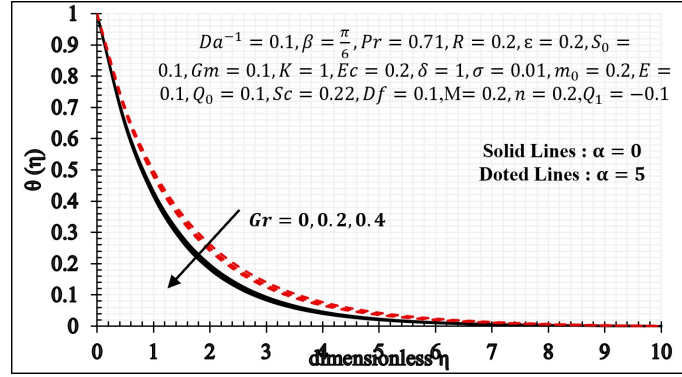
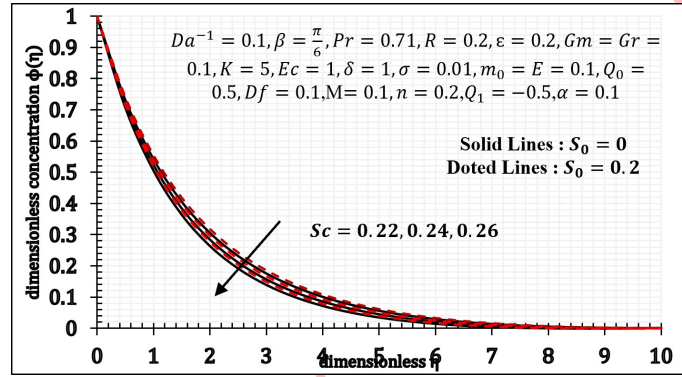
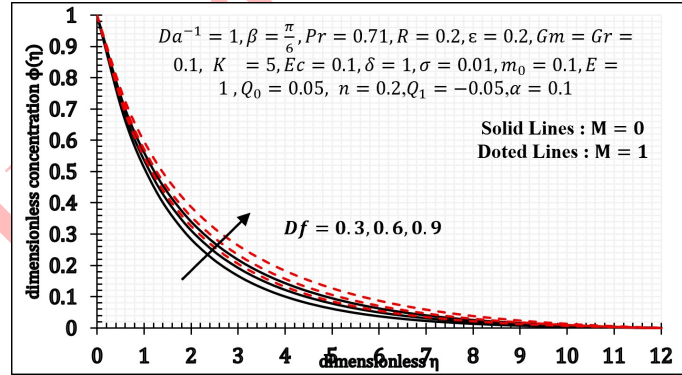
momentum boundary-layer thickness. Conversely, larger values of the material parameter enhance the velocity profile. Physically, stronger micropolar interactions improve momentum transport through particle microrotation, which weakens the effective resistance to fluid motion and promotes fluid acceleration near the surface.

Figure 3 illustrates the influence of the inverse Darcy number Da^{-1} and the micropolar parameter m_0 on the velocity profile. Increasing Da^{-1} corresponds to a decrease in medium permeability, which intensifies the drag force exerted

FIGURE 8. Impact of Ec and K on temperature $\theta(\eta)$.FIGURE 9. Impact of m_0 and Da^{-1} on temperature $\theta(\eta)$.FIGURE 10. Impact of ε and Pr on temperature $\theta(\eta)$.

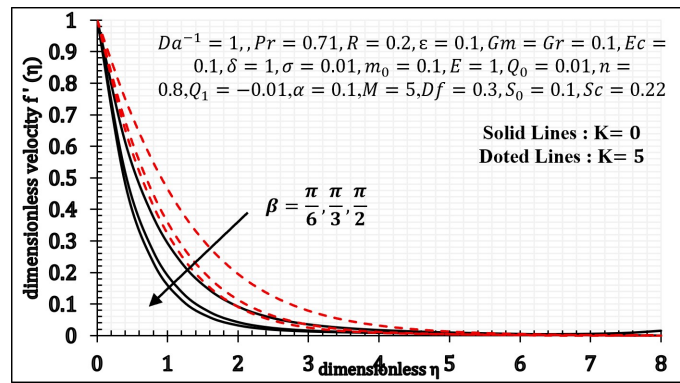
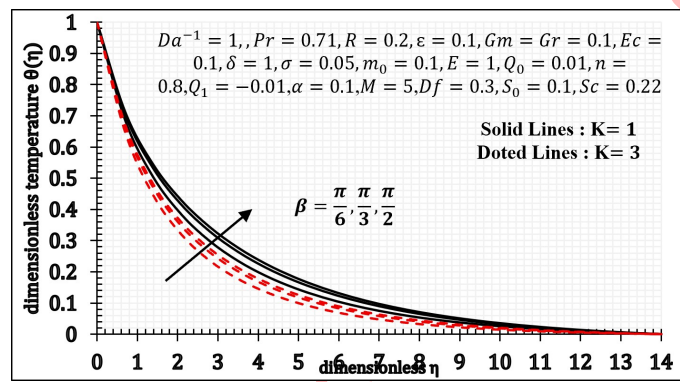
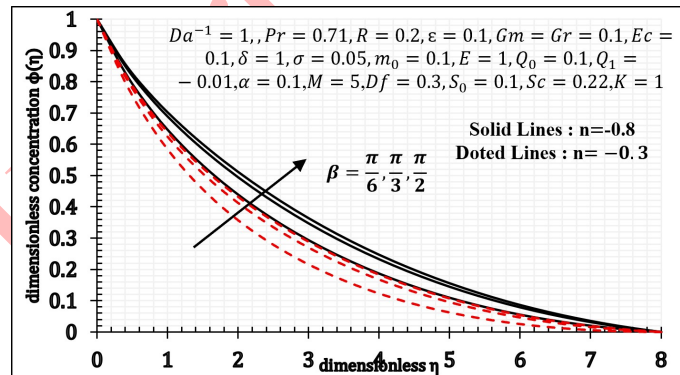
by the porous matrix. Consequently, fluid motion is hindered and the velocity decreases. In contrast, larger values of m_0 strengthen the coupling between linear and angular momentum transport, facilitating momentum diffusion and leading to an increase in velocity within the boundary layer.

Figure 4 presents the variation of angular velocity with the magnetic parameter (M) and material parameter (K). The angular velocity decreases with increasing magnetic parameter because the Lorentz force suppresses both

FIGURE 11. Impact of Gr and α on temperature $\theta(\eta)$.FIGURE 12. Impact of Sc and S_0 on the concentration $\phi(\eta)$.FIGURE 13. Impact of Df and M on the concentration $\phi(\eta)$.

translational and rotational motions of the fluid particles. On the other hand, higher values of K significantly enhance the microrotation field. This enhancement arises from stronger coupling effects between the velocity and spin fields, allowing fluid microelements to rotate more actively within the flow region.

Figure 5 shows the effects of m_0 and Da^{-1} on angular velocity. The microrotation profile increases with increasing m_0 due to intensified rotational interactions among fluid microstructures. However, increasing Da^{-1} suppresses

FIGURE 14. Impact of β and K on velocity $f'(\eta)$.FIGURE 15. Impact of β and K on temperature $\theta(\eta)$.FIGURE 16. Impact of β and n on concentration $\phi(\eta)$.

angular velocity because the porous matrix restricts particle movement and dissipates rotational kinetic energy, thereby weakening the spin motion of the micropolar fluid.

Figure 6 demonstrates the effects of the thermal radiation parameter (R) and heat generation parameter Q_0 on the temperature distribution. Both parameters contribute to an increase in temperature. Enhanced thermal radiation introduces additional thermal energy into the fluid, which thickens the thermal boundary layer. Similarly, positive

heat generation acts as an internal energy source, continuously supplying heat to the system and elevating the fluid temperature throughout the boundary layer.

Figure 7 illustrates the influence of the heat source parameter Q_1 and magnetic parameter (M) on temperature. Increasing Q_1 raises the temperature because additional thermal energy is generated within the fluid domain. Furthermore, higher magnetic field strength also increases the temperature. The magnetic field induces resistive heating through electromagnetic dissipation, converting kinetic energy into thermal energy and thereby thickening the thermal boundary layer.

Figure 8 displays the effects of the Eckert number (Ec) and material parameter (K) on temperature. The temperature increases significantly with increasing Ec. Physically, the Eckert number quantifies the conversion of kinetic energy into internal energy through viscous dissipation. Therefore, stronger viscous heating elevates the fluid temperature. The material parameter K also enhances temperature due to increased rotational activity and momentum exchange, which contribute additional energy to the thermal field.

Figure 9 presents the effects of m_0 and Da^{-1} on temperature. Increasing m_0 raises the temperature because stronger micropolar interactions generate additional rotational energy that is converted into thermal energy. Likewise, larger values of Da^{-1} tend to increase temperature since the porous medium resistance slows the fluid motion, causing more mechanical energy to dissipate as heat and enhancing thermal diffusion within the boundary layer.

Figure 10 illustrates the influence of the thermal relaxation parameter ε and Prandtl number (Pr) on temperature. Increasing ε suppresses the temperature distribution because thermal relaxation delays heat propagation within the fluid, reducing thermal energy transport. Similarly, larger Prandtl numbers reduce temperature since fluids with high Pr possess lower thermal diffusivity, restricting heat penetration and resulting in a thinner thermal boundary layer.

Figure 11 shows the effects of the thermal Grashof number (Gr) and the heat transfer parameter α on temperature. An increase in Gr enhances buoyancy-driven transport, which intensifies fluid motion and promotes thermal energy distribution throughout the flow field. Consequently, the temperature profile increases. Likewise, larger values of α strengthen heat transfer mechanisms, leading to an elevation in fluid temperature and thermal boundary-layer thickness.

Figure 12 depicts the influence of the Schmidt number (Sc) and source parameter S_0 on concentration. Increasing the Schmidt number reduces the concentration profile because larger Sc corresponds to lower mass diffusivity, restricting species transport from the surface into the fluid. In contrast, increasing S_0 enhances concentration due to the additional generation of species within the flow domain, which thickens the concentration boundary layer.

Figure 13 demonstrates the effect of different Dufour numbers and magnetic field values on the concentration profile, with all other parameters maintained at constant levels. It is evident that an increase in the Dufour number leads to a corresponding rise in the concentration distribution. Similarly, higher values of magnetic field also result in elevated concentration levels. This trend indicates that the concentration gradient becomes more pronounced with increasing Dufour number. Physically, this occurs because the Dufour effect represents the generation of energy flux due to concentration gradient. As the Dufour number increases, the combination between heat and mass transfer becomes stronger, thereby enhancing mass diffusion and raising concentration throughout the flow region.

Figure 14 illustrates the influence of the inclination angle $\beta = \frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2}$ on the dimensionless velocity profile $f'(\eta)$ for two values of the porous medium parameter ($K = 0$ and $K = 5$). In all cases, the velocity decreases smoothly from its maximum value at the wall and gradually approaches zero with increasing η , demonstrating the development of the momentum boundary layer. It is evident that the velocity profile shifts upward as the inclination angle increases, indicating that a larger β enhances the fluid motion by increasing the effective buoyancy force acting along the flow direction. Moreover, the dashed curves corresponding to $K = 5$ remain above the solid curves for $K = 0$, showing that a higher porous medium parameter promotes fluid acceleration and increases the momentum boundary-layer thickness by reducing the resistance to flow. Therefore, the combined effect of increasing both β and K is to strengthen the fluid velocity and delay its decay away from the surface, resulting in enhanced momentum transport within the boundary layer.

Figure 15 demonstrates the influence of the inclination angle $\beta = \frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2}$ on the dimensionless temperature profile $\theta(\eta)$ for two values of the porous medium parameter ($K = 1$ and $K = 3$). The temperature decreases continuously



from the heated surface and gradually approaches the ambient condition as the similarity variable η increases, confirming the development of the thermal boundary layer. It is observed that the temperature profiles corresponding to $K = 3$ (dashed lines) lie below those for $K = 1$ (solid lines), indicating that increasing the porous medium parameter enhances heat dissipation and reduces the thermal boundary-layer thickness. In addition, increasing the inclination angle slightly raises the temperature distribution for both values of K , suggesting that a stronger buoyancy effect weakens the rate of cooling and allows the fluid to retain more thermal energy within the boundary layer.

Figure 16 depicts the variation of the dimensionless concentration profile $\phi(\eta)$ with the inclination angle $\beta = \frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2}$ for two values of the power-law index ($n = -0.8$ and $n = -0.3$). The concentration decreases monotonically from the wall to the free stream, indicating the formation of a concentration boundary layer. The solid curves corresponding to $n = -0.8$ remain above the dashed curves for $n = -0.3$, demonstrating that a more negative power-law index enhances species concentration and increases the concentration boundary-layer thickness by slowing mass diffusion away from the surface. Moreover, increasing the inclination angle shifts the concentration profiles upward for both values of n , implying that stronger buoyancy-induced flow suppresses mass transfer and maintains a higher concentration within the boundary layer over a longer distance.

6. CONCLUSION

The current work developed a comprehensive mathematical model to investigate the dimensional hydromagnetic mixed convection of heat and mass nanofluid flow past a stretching sheet within a non-Darcian porous medium, incorporating thermal radiation. The model analysis incorporates a non-uniform heat source, chemical reaction, viscous dissipation and variable thermal conductivity in the thermal field, which are critical in capturing the complexities of the system. The model equations are transformed from the initial partial derivatives to ordinary derivatives and solved using numerical approach of shooting with Adams-Moulton method. The numerical outcomes are displayed using figures and tables with appropriate deliberations of results relevant to various applications. The solutions to the model equations compared favorably with existing studies. Some key points obtained from this study are that.

- The concentration profile declines by the escalation of the fitted rate constant n , the chemical reaction rate σ and the temperature difference parameter δ .
- A higher concentration is observed on increasing the Dufour number. Also, escalation of D_f escalating the temperature profile.
- The angular velocity escalates with the escalation in m_0 , the material parameter K , the inverse Darcy number Da^{-1} and the magnetic field parameter M .
- In the presence of differently oriented magnetic fields the velocity decreases but the temperature and the concentration increase.
- The rate of mass transfer escalates with Soret number and declines with the escalation of Schmidt number.
- The velocity and the concentration escalate by the escalation in the modified Grashof number Gm and the thermal Grashof number Gr and a reverse scenario is observed in the temperature field.



NOMENCLATURE

A_0, A_1	Constants	M	Magnetic field parameter
$\vec{B}$	Transverse magnetic field	m_0	A constant
B_0	Uniform transverse magnetic field	m_w	Rate of mass transfer
b	Stretching parameter	N	Components of microrotation
C	Concentration of the species	Nu_x	Local Nusselt number
C_b	Drag coefficient	T	Fluid temperature
C_f	Skin friction coefficient	T_m	Fluid temperature
C_∞	Concentration of the solute far from the sheet	T_w	Wall temperature of the fluid
C_w	Wall concentration of the solute	R	Thermal radiation parameter
C_p	Specific heat at constant pressure	Re_x	Local Reynolds number
D_m	Mass diffusivity coefficient	Sc	Schmidt number
Da^{-1}	Inverse Darcy number	S_0	Soret number
Ec	Eckert number	Q_0	Space-dependent heat source/sink coefficient
f	Dimensionless stream function	Q_1	Temperature-dependent heat source/sink coefficient
G_m	Modified Grashof number	q_r	Radiative heat flux
G_r	Grashof number	q_w	Rate of heat transfer
g	Dimensionless angular velocity	q'''	Non-uniform heat generation/absorption
j	Micro-inertia per unit mass	u	Fluid velocity in the x -direction
K	Material parameter	v	Fluid velocity in the y -direction
K_T	Thermal diffusion ratio	x	Flow-direction coordinate along the stretching sheet
k_1	Vortex viscosity	y	Distance normal to the stretching sheet
k^*	Absorption coefficient	l	Characteristic length

Greek letters

β	Inclination of magnetic field
β^*	Coefficient of concentration expansion
θ	Non-dimensional temperature parameter
ψ	Non-dimensional concentration parameter
ϕ	Porosity of the porous medium
η	Similarity variable
σ	Electrical conductivity
ρ	Density of the fluid
ν	Kinematic viscosity
σ^*	Stefan–Boltzmann constant
γ	Spin gradient viscosity

REFERENCES

- [1] E. M. Abo Eldohad and A. F. Ghonaim, *Radiation effect on heat transfer of a micropolar fluid through a porous medium*, Applied Mathematics and Computation, 169 (2005), 500–516.
- [2] N. Ali, S. U. Khan, Z. Abbas, and M. Sajid, *Soret and Dufour effects on hydromagnetic flow of viscoelastic fluid over porous oscillatory stretching sheet with thermal radiation*, Journal of the Brazilian Society of Mechanical Sciences and Engineering, 38(8) (2016), 2533–2546.
- [3] T. Ariman, N. D. Sylvester, and M. A. Turk, *Application of microcontinuum fluid mechanics*, International Journal of Engineering Science, 12(4) (1974), 273–293.
- [4] T. Ariman, M. A. Turk, and N. D. Sylvester, *Microcontinuum fluid mechanics: A review*, International Journal of Engineering Science, 11(8) (1973), 905–930.



- [5] R. C. Bataller, *Viscoelastic fluid flow and heat transfer over a stretching sheet under the effects of a non-uniform heat source, viscous dissipation and thermal radiation*, International Journal of Heat and Mass Transfer, 50(6) (2007), 3152–3162.
- [6] M. M. Bhatti and M. M. Rashidi, *Study of heat and mass transfer with Joule heating on magnetohydrodynamic (MHD) peristaltic blood flow under the influence of Hall effect*, Propulsion and Power Research, 5 (2017), 177–185.
- [7] J. Chamkha, R. A. Mohamed, and S. E. Ahmed, *Unsteady MHD natural convection from a heated vertical porous plate in a micropolar fluid with Joule heating, chemical reaction and radiation effects*, Meccanica, 46 (2011), 399–411.
- [8] G. Deepa and G. Murali, *Effects of viscous dissipation on unsteady MHD free convective flow with thermophoresis past a radiate inclined permeable plate*, Iranian Journal of Science and Technology, 38 (2014), 379–388.
- [9] S. P. A. Devi and B. Ganga, *Effects of viscous and Joule dissipation on MHD flow, heat and mass transfer past a stretching porous surface embedded in a porous medium*, Nonlinear Analysis: Modelling and Control, 14(3) (2009), 303–314.
- [10] M. F. El-Amin, *Magnetohydrodynamic free convection and mass transfer flow in micropolar fluid with constant suction*, Journal of Magnetism and Magnetic Materials, 234 (2011), 567–574.
- [11] M. A. El-Hakiem, *Viscous dissipation effects on MHD free convection flow over a nonisothermal surface in a micropolar fluid*, International Communications in Heat and Mass Transfer, 27 (2000), 581–590.
- [12] C. Eringen, *Theory of micropolar fluids*, Journal of Mathematics and Mechanics, 16(1) (1966), 1–18.
- [13] C. Eringen, *Theory of micropolar fluids*, Journal of Mathematical Analysis and Applications, 38(2) (1972), 480–496.
- [14] C. Eringen, *Microcontinuum field theories II: Fluent media*, Springer (2001).
- [15] T. Hayat, B. Ashraf, A. Alsaedi, and M. S. Alhuthali, *Soret and Dufour effects in three-dimensional flow of Maxwell fluid with chemical reaction and convective condition*, International Journal of Numerical Methods for Heat and Fluid Flow, 25(1) (2015), 98–120.
- [16] A. Ishak, *Thermal boundary layer flow over a stretching sheet in a micropolar fluid with radiation effect*, Meccanica, 45 (2010), 367–373.
- [17] M. I. Khan, M. Waqas, T. Hayat, and A. Alsaedi, *Soret and Dufour effects in stretching flow of Jeffrey fluid subject to Newtonian heat and mass conditions*, Results in Physics, 7 (2017), 4183–4188.
- [18] G. Lukaszewicz, *Micropolar fluids: Theory and application*, Birkhäuser, (1999).
- [19] F. Mabood, S. M. Ibrahim, M. M. Rashidi, M. S. Shadloo and G. Lorenzini, *Non-uniform heat source/sink and Soret effects on MHD non-Darcian convective flow past a stretching sheet in a micropolar fluid with radiation*, International Journal of Heat and Mass Transfer, 93 (2016), 674–682.
- [20] M. A. A. Mahmoud, *Thermal radiation effects on MHD flow of a micropolar fluid over a stretching surface with variable thermal conductivity*, Physica A: Statistical Mechanics and its Applications, 375 (2007), 401–410.
- [21] K. A. Maleque, *Effects of binary chemical reaction and activation energy on MHD boundary layer heat and mass transfer flow with viscous dissipation and heat generation/absorption*, ISRN Thermodynamics, 2013 (2013), Article ID 284637.
- [22] M. A. Mansour, M. A. El-Hakiem, and S. M. El Kabeir, *Heat and mass transfer in magnetohydrodynamic flow of micropolar fluid on a circular cylinder with uniform heat and mass flux*, Journal of Magnetism and Magnetic Materials, 220 (2000), 259–270.
- [23] K. Momkos, *A comparison of the activation energy of viscous flow for hen egg-white lysozyme obtained on the basis of different models of viscosity for glass forming liquids*, Current Topics in Biophysics, 34(1) (2011), 1–9.
- [24] R. G. Mortimer and H. Eyring, *Elementary transition state theory of the Soret and Dufour effects*, Proceedings of the National Academy of Sciences of the United States of America, 77(4) (1980), 1728–1731.
- [25] M. Mustafa, J. A. Khan, T. Hayat, and A. Alsaedi, *Buoyancy effects on the MHD nanofluid flow past a vertical surface with chemical reaction and activation energy*, International Journal of Heat and Mass Transfer, 108 (2017), 1340–1346.
- [26] G. Narendar and K. Govardhan, *Viscous dissipation effect on micro nanofluid in the presence of inclined magnetic field*, International Journal of Nanoscience and Nanotechnology, 17(3) (2021), 181–190.



- [27] D. Pal and S. Chatterjee, *Effects of radiation on Darcy–Forchheimer convective flow over a stretching sheet in a micropolar fluid with non-uniform heat source/sink*, Journal of Applied Fluid Mechanics, 8 (2015), 207–212.
- [28] M. M. Rahman and T. Sultana, *Radiative heat transfer flow of micropolar fluid with variable heat flux in a porous medium*, Nonlinear Analysis: Modelling and Control, 13(1) (2008), 71–87.
- [29] M. M. Rahman, M. J. Uddin, and A. Aziz, *Effects of variable electric conductivity and non-uniform heat source (or sink) on convective micropolar fluid flow along an inclined flat plate with surface heat flux*, International Journal of Thermal Sciences, 48 (2009), 2331–2340.
- [30] C. S. K. Raju, N. Sandeep, V. Sugunamma, J. J. Babu, and J. V. R. Reddy, *Heat and mass transfer in magnetohydrodynamic Casson fluid over an exponentially permeable stretching surface*, Engineering Science and Technology, an International Journal, 19 (2006), 45–52.
- [31] M. M. Rashidi, S. A. Mohimani Pour, and S. Abbasbandy, *Analytic approximate solutions for heat transfer of a micropolar fluid through a porous medium with radiation*, Communications in Nonlinear Science and Numerical Simulation, 16 (2011), 1874–1889.
- [32] G. S. Seth, R. Nandkeolyar, and M. S. Ansari, *Effects of Hall current and rotation on unsteady MHD Couette flow in the presence of an inclined magnetic field*, Journal of Applied Fluid Mechanics, 5(2) (2012), 67–74.
- [33] M. S. Shadloo, A. Kimiaefar, and D. Bagheri, *Series solution for heat transfer of continuous stretching sheet immersed in a micropolar fluid in the existence of radiation*, International Journal of Numerical Methods for Heat and Fluid Flow, 23 (2013), 289–304.
- [34] N. K. Singh, V. Kumar, and G. K. Sharma, *The effect of inclined magnetic field on unsteady flow past a moving vertical plate with variable temperature*, International Journal of Latest Technology in Engineering, Management & Applied Science, 5(2) (2016), 2278–2540.
- [35] M. Subhas and N. Mahesha, *Heat transfer in MHD viscoelastic fluid flow over a stretching sheet with variable thermal conductivity, non-uniform heat source and radiation*, Applied Mathematical Modelling, 32 (2008), 1965–1983.
- [36] D. Swapna, K. Govardhan, G. Narender, and S. Misra, *MHD stagnation point flow of micropolar fluid over a stretching/shrinking sheet*, CFD Letters, 16(12) (2024), 113–127.
- [37] G. Thirupathi, K. Govardhan, G. Narender, S. Misra, and P. Kavitha, *Exploring the impact of thermal radiation, viscous dissipation, and magnetic field on micropolar fluid flow over a shrinking surface with velocity slip*, JP Journal of Heat and Mass Transfer, 37(1) (2024), 1–21.