



New concepts of locally SNC-boundary, SNC-nowhere dense and domain sets in stable neutrosophic Crisp topological spaces

Dheargham Ali Abdulsada^{1,*}, L. A. A. Jabar², and Yasen Rajihy³

¹Department of Mathematics, College of Education, University of Sumer, Iraq.

²Department of Mathematics, College of Education for Pure Sciences, University of Babylon, Iraq.

³Department Networks, College of Information Technology, University of Babylon, Iraq.

Abstract

Based on the concept of SNC-nowhere dense sets, this paper develops a systematic study of several related local and structural notions in stable Neutrosophic crisp topological spaces. These notions provide a refined framework for analyzing the behavior of sets with respect to the different types of Neutrosophic crisp points, which distinguish this setting from classical topology. In particular, we introduce and investigate locally SNC-boundary sets and locally SNC-nowhere dense sets, and we establish their fundamental properties using closure and interior operators in the sense of stable Neutrosophic crisp topology. Moreover, we define SNC-closed domain sets and SNC-open domain sets as natural extensions of domain-type concepts to the Neutrosophic crisp context. We derive several characterizations, inclusion relations, and stability properties under finite unions and intersections, and we examine their connections with frontier-type operators. The results obtained clarify the interplay between global and local structures in stable Neutrosophic crisp spaces and contribute to a deeper understanding of their internal topological behavior, with potential relevance to further theoretical developments in generalized topological frameworks.

Keywords. SNC-nowhere dense, Locally SNC-boundary, SNC-closed and SNC-open domain, Stable topological space.

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1. INTRODUCTION

The main approach to any study in Neutrosophic Crisp set theory is to define the empty set and the universal set of such sets, because they have more than one type. The same applies to intersection, union, complements, and other operations. Therefore, our choice of these will be as follows [3, 8]:

$$\tilde{X} = (X, \emptyset, \emptyset), \quad \tilde{\emptyset} = (\emptyset, X, X), \quad \tilde{K} = (K_1, K_2, K_3), \quad (1.1)$$

$$K_1 \cap K_2 = \emptyset, \quad K_1 \cap K_3 = \emptyset, \quad K_2 \cap K_3 = \emptyset, \quad (1.2)$$

$$\tilde{L} \cap \tilde{K} = (L_1 \cap K_1, L_2 \cup K_2, L_3 \cup K_3), \quad (1.3)$$

$$\tilde{L} \cup \tilde{K} = (L_1 \cup K_1, L_2 \cap K_2, L_3 \cap K_3), \quad (1.4)$$

$$\tilde{L} \subseteq \tilde{K} \text{ iff } L_1 \subseteq K_1, L_2 \subseteq K_2 \text{ and } L_3 \subseteq K_3, \quad (1.5)$$

$$(\tilde{K})^c = (K_1^c, K_2^c, K_3^c). \quad (1.6)$$

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* Corresponding author. Email: d.ali@uos.edu.iq.

The Neutrosophic crisp point (NC-points) are:

$$\tilde{p}^{(1)} = (\{q\}, \emptyset, \{q\}^c) \in {}_1\tilde{L} \text{ iff } q \in L_1, q \notin L_3, \quad (1.7)$$

$$\tilde{p}^{(2)} = (\emptyset, \{q\}, \{q\}^c) \in {}_2\tilde{L} \text{ iff } q \in L_2, q \notin L_3, \quad (1.8)$$

$$\tilde{p}^{(3)} = (\{q\}, \emptyset, \emptyset) \in {}_3\tilde{L} \text{ iff } q \in L_1, \quad (1.9)$$

$$\tilde{p}^{(4)} = (\emptyset, \{q\}, \emptyset) \in {}_4\tilde{L} \text{ iff } q \in L_2. \quad (1.10)$$

As for topology, it is as [4] defined it and called it stable Neutrosophic crisp topological space (SNCT-Space). We reformulate this definition as a collection of axioms.

Definition 1.1. Let $\tilde{X}$ be a nonempty Neutrosophic crisp set. A family $\tau_{\tilde{X}}$ of Neutrosophic crisp subsets of $\tilde{X}$ is called a *stable Neutrosophic crisp topology* on $\tilde{X}$ if it satisfies the following axioms:

$$(A1) \quad \emptyset \in \tau_{\tilde{X}} \text{ and } \tilde{X} \in \tau_{\tilde{X}}.$$

$$(A2) \quad \text{For any } \tilde{L}_1, \tilde{L}_2 \in \tau_{\tilde{X}}, \text{ there exists } \tilde{L}_3 \in \tau_{\tilde{X}} \text{ such that } \tilde{L}_3 \subseteq \tilde{L}_1 \cap \tilde{L}_2.$$

$$(A3) \quad \text{For any indexed family } \{\tilde{L}_\lambda\}_{\lambda \in \Lambda} \subseteq \tau_{\tilde{X}}, \text{ there exists } \tilde{L} \in \tau_{\tilde{X}} \text{ such that } \tilde{L} \subseteq \cup_{\lambda \in \Lambda} \tilde{L}_\lambda.$$

The pair $(\tilde{X}, \tau_{\tilde{X}})$ is called a *stable Neutrosophic crisp topological space* (SNCT-space).

It is hoped that it is topological, but it is not, because it is not closed under the intersection and union [7, 14]. But if it achieves it, we say that it has the properties of advance and soon to arrive, respectively. The operators of closure and interior associated with $\tilde{L}$ are given by:

$$S_{cl}(\tilde{L}) = [\cup \tilde{U}]^c, \quad \tilde{L} \cap \tilde{U} = \emptyset, \quad \forall \tilde{U} \in \tau_{\tilde{X}}, \quad (1.11)$$

$$S_{int}(\tilde{L}) = [\cap \tilde{U}]^c, \quad (\tilde{L})^c \cap \tilde{U} = \emptyset, \quad \forall \tilde{U} \in \tau_{\tilde{X}}. \quad (1.12)$$

It should be noted that neither $S_{cl}(\tilde{L})$ nor $S_{int}(\tilde{L})$ necessarily preserves the corresponding SNC-closed or SNC-open structure. Several studies have contributed to the development of Neutrosophic topology and its generalizations. Karatas and Kuru [8] introduced the basic notion of Neutrosophic topology and established its fundamental properties. Salama and Alblowi [13] studied Neutrosophic topological spaces and initiated several structural directions in the theory. Smarandache and collaborators further developed generalized Neutrosophic frameworks and their applications [17, 18]. More recently, attention has shifted toward generalized open and closed sets, as well as new local structures in Neutrosophic environments [10, 12, 16]. These developments motivate the study of local boundary-type and domain-type concepts in stable Neutrosophic crisp topological spaces. Nonlinear models that arise in natural models of life and some applications from various sciences were used in numerous fields. The common numerical methods include cubic B-splines method [1]. Moreover, the researchers have developed several approaches for constructing closed-form solutions to nonlinear differential equations, such as the Hirota's bilinear operator [15], the generalized trial equation scheme and enhance modified extended Tanh function method [11], the plasma physics [6], and optics [9].

In this paper, we study locally SNC-boundary and locally SNC-nowhere dense sets in stable Neutrosophic crisp topological spaces. We introduce SNC-closed domain and SNC-open domain sets and establish their main properties and characterizations.

2. SNC-BOUNDARY SET AND SNC-NOWHERE DENSE

In this section, we introduce the notions of locally SNC-boundary sets and locally SNC-nowhere dense sets at Neutrosophic crisp points, and we investigate their fundamental properties and interrelations within stable Neutrosophic crisp topological spaces.

Definition 2.1. Let $\tilde{M}$ be a Neutrosophic crisp set.

- (1) $\tilde{M}$ is a *SNC-boundary set* (SNC_b -set) if $S_{int}(\tilde{M}) = \emptyset$.
- (2) $\tilde{M}$ is a *SNC-nowhere dense set* (SNC_{nd} -set) if $S_{int}(S_{cl}(\tilde{M})) = \emptyset$.



Definition 2.2. Let $\tilde{K}$ be a Neutrosophic crisp set and $\tilde{p}^{(i)}$ be an NC-point of type i , where $i = 1, 2, 3, 4$.

- (1) $\tilde{K}$ is a *SNC-boundary set* (SNC_b -set) at the NC-point $\tilde{p}^{(i)}$ if there exists $\tilde{G} \in \tilde{\tau}$ such that $\tilde{p}^{(i)} \in_i \tilde{G}$ and $\tilde{K} \cap \tilde{G}$ is a SNC_b -set.
- (2) $\tilde{K}$ is a *SNC-nowhere dense set* (SNC_{nd} -set) at the NC-point $\tilde{p}^{(i)}$ if there exists $\tilde{G} \in \tilde{\tau}$ such that $\tilde{p}^{(i)} \in_i \tilde{G}$ and $\tilde{K} \cap \tilde{G}$ is a SNC_{nd} -set.

Example 2.3. Let $X = \{a, b, c\}$ and define $\tau_{\tilde{X}} = \{\emptyset, \tilde{X}, \tilde{H}\}$ where $\tilde{H} = (\{a\}, \emptyset, \{b, c\})$. Let $\tilde{M} = (\{b\}, \emptyset, \{a, c\})$. Then $S_{cl}(\tilde{M}) = \tilde{M}$. Now $S_{int}(S_{cl}(\tilde{M})) = S_{int}(\tilde{M}) = \emptyset$. Thus, $\tilde{M}$ is a SNC-nowhere dense set.

Remark 2.4. Let $\tilde{K}, \tilde{K}_1, \tilde{K}_2, \tilde{N} \subseteq \tilde{X}$ and let $\tilde{p}^{(i)}$ be an NC-point of type i , where $i = 1, 2, 3, 4$. Then:

- (1) $\tilde{K}$ is SNC-nowhere dense at $\tilde{p}^{(i)}$ if and only if $S_{cl}(\tilde{K})$ is a SNC-boundary set at $\tilde{p}^{(i)}$.
- (2) If $\tilde{K}_1$ and $\tilde{K}_2$ are SNC-nowhere dense at $\tilde{p}^{(i)}$, then $\tilde{K}_1 \cup \tilde{K}_2$ is SNC-nowhere dense at $\tilde{p}^{(i)}$.
- (3) For each $\tilde{G} \in \tau_{\tilde{X}}$ and $\tilde{p}^{(i)} \in_i \tilde{G}$, if $\tilde{K}$ is a SNC-set, then $\tilde{K}$ is SNC-nowhere dense at $\tilde{p}^{(i)}$.
- (4) For any SNC-nowhere dense set $\tilde{N}$, if $\tilde{K}$ is SNC-nowhere dense at $\tilde{p}^{(i)}$, then both $\tilde{K} \cup \tilde{N}$ and $\tilde{K} \cap (\tilde{N})^c$ are also SNC-nowhere dense at $\tilde{p}^{(i)}$.
- (5) $S_{cl}(S_{int}(\tilde{K}))$ is the NC-set of SN-points at which $\tilde{K}$ is not locally SNC-boundary.
- (6) $S_{cl}(S_{int}(S_{cl}(\tilde{K})))$ is the NC-set of SN-points at which $\tilde{K}$ is not locally SNC-nowhere dense.

Lemma 2.5. If a Neutrosophic crisp set $\tilde{K}$ is locally SNC-boundary at an NC-point $\tilde{p}^{(i)}$, then $\tilde{p}^{(i)} \notin_i S_{int}(\tilde{K})$.

Proof. Assume that $\tilde{K}$ is locally SNC-boundary at the NC-point $\tilde{p}^{(i)}$. Then there exists an open set $\tilde{G} \in \tau_{\tilde{X}}$ such that $\tilde{p}^{(i)} \in_i \tilde{G}$ and $\tilde{K} \cap \tilde{G}$ is a SNC-boundary set. By definition of a SNC-boundary set, $S_{int}(\tilde{K} \cap \tilde{G}) = \emptyset$. Suppose, for contradiction, that $\tilde{p}^{(i)} \in_i S_{int}(\tilde{K})$. Then there exists an open set $\tilde{U} \in \tau_{\tilde{X}}$ such that $\tilde{p}^{(i)} \in_i \tilde{U} \subseteq \tilde{K}$. Since $\tilde{p}^{(i)} \in_i \tilde{G}$, we have $\tilde{p}^{(i)} \in_i (\tilde{U} \cap \tilde{G})$. Moreover, $\tilde{U} \cap \tilde{G} \subseteq \tilde{K} \cap \tilde{G}$. This implies that $S_{int}(\tilde{K} \cap \tilde{G}) \neq \emptyset$, which contradicts the assumption that $\tilde{K} \cap \tilde{G}$ is a SNC-boundary set. Therefore, $\tilde{p}^{(i)} \notin_i S_{int}(\tilde{K})$. $\square$

Proposition 2.6. Every SNC-boundary set is locally SNC-boundary.

Proof. Let $\tilde{K}$ be a SNC-boundary set. Then, by definition, $S_{int}(\tilde{K}) = \emptyset$. Let $\tilde{p}^{(i)}$ be any NC-point in X . Choose $\tilde{G} = \tilde{X}$, where $\tilde{X} \in \tau_{\tilde{X}}$. Then $\tilde{K} \cap \tilde{G} = \tilde{K}$. Since $\tilde{K}$ is SNC-boundary, it follows that $\tilde{K} \cap \tilde{G}$ is also SNC-boundary. Hence, $\tilde{K}$ is locally SNC-boundary at every NC-point. $\square$

Example 2.7. Let $X = \{a, b\}$. Define $\tau_{\tilde{X}} = \{\emptyset, \tilde{X}, \tilde{G}\}$ where $\tilde{G} = (\{a\}, \emptyset, \{b\})$. Consider the Neutrosophic crisp set $\tilde{K} = (\{a\}, \emptyset, \emptyset)$. Then $S_{int}(\tilde{K}) = \tilde{G} \neq \emptyset$. Thus, $\tilde{K}$ is not a SNC-boundary set. Now consider the NC-point corresponding to a , denoted $\tilde{a}^{(1)}$. Take $\tilde{G} \in \tau_{\tilde{X}}$ such that $\tilde{a}^{(1)} \in_1 \tilde{G}$. Then $\tilde{K} \cap \tilde{G}$ has empty interior relative to $\tilde{G}$. Hence, $\tilde{K}$ is locally SNC-boundary at $\tilde{a}^{(1)}$.

Proposition 2.8. (i) The NC-set of all NC-points where $\tilde{K}$ is locally SNC_b -set is a SNC_b -set.

(ii) The NC-set of all NC-points where $S_{cl}(\tilde{K})$ is locally SNC_b -set is a SNC_{nd} -set.

Proof. (i) Let $\tilde{E} = \{\tilde{p}^{(i)} \in_i \tilde{X} : \tilde{p}^{(i)} \in_i \tilde{K} \text{ and there exists } \tilde{G} \in \tau_{\tilde{X}}, \tilde{p}^{(i)} \in_i \tilde{G}, S_{int}(\tilde{K} \cap \tilde{G}) = \emptyset\}$. Assume $S_{int}(\tilde{E}) \neq \emptyset$. Then there exists $\tilde{U} \in \tau_{\tilde{X}}, \tilde{U} \neq \emptyset, \tilde{U} \subseteq \tilde{E}$. Pick $\tilde{p}^{(i)} \in_i \tilde{U}$. Then $\tilde{p}^{(i)} \in_i \tilde{E}$, so there exists $\tilde{G} \in \tau_{\tilde{X}}$ with $\tilde{p}^{(i)} \in_i \tilde{G}$, $S_{int}(\tilde{K} \cap \tilde{G}) = \emptyset$. But $\tilde{p}^{(i)} \in_i (\tilde{U} \cap \tilde{G})$, $\tilde{U} \cap \tilde{G} \in \tau_{\tilde{X}}, \tilde{U} \cap \tilde{G} \neq \emptyset$, and since $\tilde{U} \subseteq \tilde{E} \subseteq \tilde{K}$, $\tilde{U} \cap \tilde{G} \subseteq \tilde{K} \cap \tilde{G}$, so $S_{int}(\tilde{K} \cap \tilde{G}) \neq \emptyset$, a contradiction. Hence $S_{int}(\tilde{E}) = \emptyset$, so $\tilde{E}$ is a SNC_b -set.

(ii) Let $\tilde{E} = \{\tilde{p}^{(i)} \in_i \tilde{X} : \text{there exists } \tilde{G} \in \tau_{\tilde{X}}, \tilde{p}^{(i)} \in_i \tilde{G}, S_{int}(S_{cl}(\tilde{K}) \cap \tilde{G}) = \emptyset\}$. Assume $S_{int}(S_{cl}(\tilde{E})) \neq \emptyset$. Then there exists $\tilde{U} \in \tau_{\tilde{X}}, \tilde{U} \neq \emptyset, \tilde{U} \subseteq S_{cl}(\tilde{E})$. Choose $\tilde{p}^{(i)} \in_i (\tilde{U} \cap \tilde{E})$. Then $\tilde{p}^{(i)} \in_i \tilde{E}$, so there exists $\tilde{G} \in \tau_{\tilde{X}}$ with $\tilde{p}^{(i)} \in_i \tilde{G}$, $S_{int}(S_{cl}(\tilde{K}) \cap \tilde{G}) = \emptyset$. But $\tilde{p}^{(i)} \in_i (\tilde{U} \cap \tilde{G})$, $\tilde{U} \cap \tilde{G} \in \tau_{\tilde{X}}, \tilde{U} \cap \tilde{G} \neq \emptyset$, and $\tilde{U} \subseteq S_{cl}(\tilde{E})$ implies $\tilde{U} \cap \tilde{G} \subseteq S_{cl}(\tilde{E}) \cap \tilde{G}$. In particular, $\tilde{U} \cap \tilde{G}$ is a nonempty open subset of $S_{cl}(\tilde{K}) \cap \tilde{G}$, hence $S_{int}(S_{cl}(\tilde{K}) \cap \tilde{G}) \neq \emptyset$, a contradiction. Therefore $S_{int}(S_{cl}(\tilde{E})) = \emptyset$, so $\tilde{E}$ is a SNC_{nd} -set. $\square$



3. SNC-CLOSED DOMAIN SETS AND SNC-OPEN DOMAIN SETS

An important concept that depends on SNC_{nd} and closely related to SNC-regular open sets is the concept of SNC-closed domain (SNCCD-set) and SNC-open domain (SNCOD-set). These notions have a significant impact on functional analysis and dynamical systems.

Definition 3.1. Let $(\tilde{X}, \tau_{\tilde{X}})$ be a stable Neutrosophic crisp topological space. A Neutrosophic crisp set $\tilde{K} \subseteq \tilde{X}$ is called a *SNC-closed domain set* (SNCCD-set) if:

- (1) $\tilde{K}$ is SNC-closed; and
- (2) $\tilde{K}$ is not SNC-nowhere dense at any NC-point of $\tilde{K}$.

The complement of a SNCCD-set is called a *SNC-open domain set* (SNCOD-set).

Remark 3.2. Let $\tilde{K} \subseteq \tilde{X}$. Then:

$$(1) \tilde{K} \text{ is a SNCCD-set if and only if } \tilde{K} = S_{cl}(S_{int}(\tilde{K})), \quad (3.1)$$

$$(2) \tilde{K} \text{ is a SNCOD-set if and only if } \tilde{K} = S_{int}(S_{cl}(\tilde{K})), \quad (3.2)$$

$$(3) \text{ If } \tilde{K} \text{ is a SNCCD-set, then } (\tilde{K})^c \text{ is a SNCOD-set.} \quad (3.3)$$

Proposition 3.3. $\tilde{K}$ is a SNCCD-set iff $\tilde{K} = S_{cl}(\tilde{G})$ for some $\tilde{G} \in \tau_{\tilde{X}}$.

Proof. Assume that $\tilde{K}$ is a SNCCD-set. Then, by Definition 3.1, $\tilde{K} = S_{cl}(S_{int}(\tilde{K}))$. Let $\tilde{G} = S_{int}(\tilde{K})$. Since $\tilde{G}$ is open, we obtain $\tilde{K} = S_{cl}(\tilde{G})$.

Conversely, suppose that there exists an open set $\tilde{G} \in \tau_{\tilde{X}}$ such that $\tilde{K} = S_{cl}(\tilde{G})$. Then $S_{int}(\tilde{K}) \supseteq \tilde{G}$. Hence $S_{cl}(S_{int}(\tilde{K})) \supseteq S_{cl}(\tilde{G}) = \tilde{K}$. On the other hand, $S_{cl}(S_{int}(\tilde{K})) \subseteq S_{cl}(\tilde{K}) = \tilde{K}$. Therefore $\tilde{K} = S_{cl}(S_{int}(\tilde{K}))$, and so $\tilde{K}$ is a SNCCD-set. $\square$

Proposition 3.4. $\tilde{K}$ is a SNCCD-set iff $SNC F_r(\tilde{K}) \subseteq S_{cl}(S_{int}(\tilde{K}))$, for $\tilde{K}$ a SNCC-set.

Proof. Assume first that $\tilde{K}$ is a SNCCD-set. Then, by Definition 3.1, $\tilde{K} = S_{cl}(S_{int}(\tilde{K}))$. Since $SNC F_r(\tilde{K}) = S_{cl}(\tilde{K}) \cap S_{cl}((\tilde{K})^c)$, and $\tilde{K}$ is SNC-closed, we have $S_{cl}(\tilde{K}) = \tilde{K}$. Thus $SNC F_r(\tilde{K}) = \tilde{K} \cap S_{cl}((\tilde{K})^c) \subseteq \tilde{K} = S_{cl}(S_{int}(\tilde{K}))$.

Conversely, assume that $SNC F_r(\tilde{K}) \subseteq S_{cl}(S_{int}(\tilde{K}))$. Since $S_{int}(\tilde{K}) \cup SNC F_r(\tilde{K}) = S_{cl}(\tilde{K})$, and $\tilde{K}$ is SNC-closed, we obtain $S_{cl}(\tilde{K}) = \tilde{K}$. Hence $\tilde{K} = S_{int}(\tilde{K}) \cup SNC F_r(\tilde{K}) \subseteq S_{cl}(S_{int}(\tilde{K}))$. But clearly $S_{cl}(S_{int}(\tilde{K})) \subseteq S_{cl}(\tilde{K}) = \tilde{K}$. Therefore $\tilde{K} = S_{cl}(S_{int}(\tilde{K}))$, which means that $\tilde{K}$ is a SNCCD-set. $\square$

Corollary 3.5. If $\tilde{K}$ is a SNCCD-set, then $SNC F_r(S_{cl}((\tilde{K})^c)) = SNC F_r(S_{int}(\tilde{K}))$.

Proposition 3.6. If $\{\tilde{K}_j\}$ is a family of SNCCD-sets, then $S_{cl}(\cup_j \tilde{K}_j)$ is a SNCCD-set.

Proof. Since for each j there exists $\tilde{G}_j \in \tau_{\tilde{X}}$ such that $\tilde{K}_j = S_{cl}(\tilde{G}_j)$, we have $S_{cl}(\cup_j \tilde{K}_j) = S_{cl}(\cup_j (S_{cl}(\tilde{G}_j))) = S_{cl}(\cup_j \tilde{G}_j)$. Hence $\cup_j \tilde{K}_j$ is a SNCCD-set. $\square$

Proposition 3.7. If $\tilde{K} \subseteq \tilde{L}$, $\tilde{p}^{(i)} \in_i \tilde{L}$ and $\tilde{L}$ is a SNCCD-set, then $\tilde{K}$ is a SNC-set at $\tilde{p}^{(i)}$ iff $\tilde{K}$ is a SNC_b -set relatively to $\tilde{L}$ at $\tilde{p}^{(i)}$.

Proof. $\tilde{K} \subseteq \tilde{L} = S_{cl}(S_{int}(\tilde{L}))$ and $\tilde{p}^{(i)} \in_i \tilde{L} = S_{cl}(S_{int}(\tilde{L}))$, so $\tilde{K} = \tilde{K} \cap S_{cl}(S_{int}(\tilde{L}))$ is a SNC-set relative to $S_{cl}(S_{int}(\tilde{L})) = \tilde{L}$. Conversely, let $\tilde{K}$ be a SNC_b -set relative to $\tilde{L}$, so $S_{int_{\tilde{L}}}(\tilde{K}) = \emptyset$, but $S_{int}(\tilde{K}) \subseteq S_{int_{\tilde{L}}}(\tilde{K})$ and therefore $\tilde{K}$ is a SNC_b -set at $\tilde{p}^{(i)} \in_i \tilde{L}$. $\square$

Proposition 3.8. If $\tilde{K}$ is a subset of a SNCCD-set at $\tilde{p}^{(i)}$, an NC-point of $\tilde{H}$, then $\tilde{K}$ is SNC-dense relatively to $\tilde{H}$ at $\tilde{p}^{(i)}$, $i = 1, 2, 3, 4$.

Remark 3.9. (1) $\tilde{K}$ is a SNCOD-set if and only if $(\tilde{K})^c$ is a SNCCD-set.



- (2) $\tilde{K}$ is a SNCOD-set if and only if $\tilde{K}$ is a SNC-open set and $SNC F_r(\tilde{K}) \subseteq S_{cl}(S_{int}((\tilde{K})^c))$.
- (3) If $\tilde{K}$ is a SNCOD-set, then $SNC F_r(S_{cl}(\tilde{K})) = SNC F_r((S_{cl}(\tilde{K}))^c)$.
- (4) The intersection of finitely many SNCOD-sets is a SNCOD-set.
- (5) Let $SNI(\tilde{K})$ denote the NC-set of all NC-points at which $\tilde{K}$ is locally SNC-nowhere dense. Then $SNI(\tilde{K})$ is a SNCCD-set.
- (6) For any NC-set $\tilde{K}$, $\tilde{K} \cap SNI(\tilde{K}) \cap (\tilde{K} \cap SNI(\tilde{K}))^c = \emptyset$.

Proposition 3.10. *The following statements are equivalent, with $\tau_{\tilde{X}}$ an advance SNCT-space:*

- (i) Every $\tilde{G} \in \tau_{\tilde{X}}$ is SNC_{nd} .
- (ii) Every SNC_{nd} -set $\tilde{K}$ satisfies $S_{int}(\tilde{K}) = \emptyset$.
- (iii) For any $\tilde{G} \in \tau_{\tilde{X}}$, we have $Sl(\tilde{G}) \cap \tilde{G} = \emptyset$, where $Sl(\tilde{G})$ denotes the set of NC-points at which $\tilde{G}$ is locally SNC_b .

Proof. (i) $\Rightarrow$ (ii). Assume that every nonempty open set is SNC-nowhere dense. Let $\tilde{K}$ be a SNC_{nd} -set. Suppose, for contradiction, that $S_{int}(\tilde{K}) \neq \emptyset$. Then there exists a nonempty open set $\tilde{U} \in \tau_{\tilde{X}}$ such that $\tilde{U} \subseteq \tilde{K}$. Since $\tilde{U}$ is open, by assumption it is SNC_{nd} . But this contradicts the fact that an open set cannot have empty interior. Hence $S_{int}(\tilde{K}) = \emptyset$.

(ii) $\Rightarrow$ (iii). Assume that for every SNC_{nd} -set $\tilde{K}$, $S_{int}(\tilde{K}) = \emptyset$. Let $\tilde{G} \in \tau_{\tilde{X}}$. Suppose that there exists $\tilde{p}^{(i)} \in Sl(\tilde{G}) \cap \tilde{G}$. By definition of $Sl(\tilde{G})$, $\tilde{G}$ is locally SNC-boundary at $\tilde{p}^{(i)}$. Hence there exists an open set $\tilde{U}$ such that $S_{int}(\tilde{G} \cap \tilde{U}) = \emptyset$. But since $\tilde{G}$ is open and contains $\tilde{p}^{(i)}$, this contradicts assumption (ii). Therefore $Sl(\tilde{G}) \cap \tilde{G} = \emptyset$.

(iii) $\Rightarrow$ (i). Assume that for every open set $\tilde{G}$, $Sl(\tilde{G}) \cap \tilde{G} = \emptyset$. Suppose, for contradiction, that there exists a nonempty open set $\tilde{U}$ that is not SNC_{nd} . Then $S_{int}(S_{cl}(\tilde{U})) \neq \emptyset$. This implies the existence of a point $\tilde{p}^{(i)} \in \tilde{U}$ where $\tilde{U}$ is locally SNC_b . Thus $\tilde{p}^{(i)} \in Sl(\tilde{U}) \cap \tilde{U}$, which contradicts assumption (iii). Hence every open set is SNC_{nd} . $\square$

4. CONCLUSION

We introduced locally SNC-boundary sets, SNC-nowhere dense sets, SNC-closed domain sets, and SNC-open domain sets in stable Neutrosophic crisp topological spaces. The obtained results provide new insights into the local structure of Neutrosophic crisp topologies and open the door for further applications in analysis and related fields.

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Uncorrected Proof

