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Certain new integral results associated with incomplete I-function

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Abstract

This article explores new integral solutions for generalized Mittag-Leffler function, I-function, general class of polynomials and incomplete I-function. To show the significance and applicability of our findings, we examine certain circumstances involving significant special functions. The findings obtained are very general in nature and may be used to drive a wide variety of interesting fractional integration operators and their applications.

Keywords. Incomplete I-function, General class of polynomials, Generalized Mittag-Leffler function, I-function.
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1. Introduction

The incomplete I-functions [1–3, 8] are exceptionally assists in deriving novel and previously outcomes due to its widespread use in Scientific discipline and Technology . The Mittag-Leffler function contribute significantly to solution of fractional differential equation, modeling anomalous diffusion processes, viscoelasticity and memory-dependent system. The I-function [9, 10] being a very general class of special functions, unifies and extends many well-Known functions especially the hypergeometric function, Fox H- function [7]. Due to its general nature, it has been widely used in soling complex integral transforms and differential equations arising in mathematical physics and engineering problems.

1.1. **I-Function.** I-function provides a highly general framework that unifies a wide variety of special function, consisting the Fox H- function [11], Meijer G- function and several hypergeometric type functions. The I-function was developed by Saxena and it's generalisation of Fox's H-function [7]. The illustration and analytical expression of the I-function [9, 10] are as follows.

$$I^{m'_i,n'_i}_{p'_i,q'_i,r'}[z] = I^{m'_i,n'_i}_{p'_i,q'_i,r'}\left\{ z \left| \begin{matrix} (t_j,\tau_j)_{1,n'_i}\,; (t_{ji},\tau_{ji})_{n'_i+1,p'_i} \\ (b_j,\beta_j)_{1,m'_i}\,; (b_{ji},\beta_{ji})_{m'_i+1,q'_i} \end{matrix} \right. \right\} = \sum_{h=1}^{m'}\sum_{k=0}^{\infty} \frac{(-1)^k \phi'(\eta_{h,k})}{\beta_h k!} z^{\eta_{h,k}}, \tag{1.1}$$

where

$$\phi'(\eta_{h,k}) = \frac{\prod_{j=1}^{m'_i}\Gamma(b_j-\beta_j\eta_{h,k})\prod_{j=1}^{n'_i}\Gamma(1-t_j+\tau_j\eta_{h,k})}{\sum_{i=1}^{r'}\left\{\prod_{j=m'_i+1}^{q'_i}\Gamma(1-b_{ji}+\beta_{ji}\eta_{h,k})\prod_{j=n'_i+1}^{p'_i}\Gamma(t_{ji}-\tau_{ji}\eta_{h,k})\right\}},$$

and

$$\eta_{h,k} = \frac{b_h+k}{\beta_h}.$$

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1.2. **Incomplete I-Function.** The incomplete I- functions established through Bansal and Kumar [1] are defined as

$$ {}^{(\Gamma)}I^{e,f}_{g_l,h_l,s}(z) = {}^{(\Gamma)}I^{e,f}_{g_l,h_l,s}\left[z \middle| \begin{array}{l} (V_1, A_1, x), (V_j, A_j)_{2,f}\,(V_{jl}, A_{jl})_{f+1,g_l} \\ (W_j, G_j)_{1,e}\,(W_{jl}, G_{jl})_{e+1,h_l} \end{array} \right] = \frac{1}{2\pi i}\int_{\Omega} \mathrm{K}(\xi, x) z^{-\xi} d\xi\,, \tag{1.2} $$

where $K(\xi, x) = \frac{\Gamma(1-V_1-A_1\xi,x)\prod_{j=1}^{e}\Gamma(W_j+G_j\xi)\prod_{j=2}^{f}\Gamma(1-V_j-A_j\xi)}{\sum_{l=1}^{s}\left[\prod_{j=e+1}^{h_l}\Gamma(1-W_{jl}-G_{jl}\xi)\prod_{j=f+1}^{g_l}\Gamma(V_{jl}+A_{jl}\xi)\right]}$.

and

$$ {}^{(\gamma)}I^{e,f}_{g_l,h_l,s}(z) = {}^{(\gamma)}I^{e,f}_{g_l,h_l,s}\left[z \middle| \begin{array}{l} (V_1, A_1, x), (V_j, A_j)_{2,f}\,(V_{jl}, A_{jl})_{f+1,g_l} \\ (W_j, G_j)_{1,e}\,(W_{jl}, G_{jl})_{e+1,h_l} \end{array} \right] = \frac{1}{2\pi i}\int_{\Omega} \mathrm{L}(\xi, x) z^{-\xi} d\xi\,, \tag{1.3} $$

where

$$ L(\xi, x) = \frac{\gamma\left(1 - V_1 - A_1\xi, x\right)\prod_{j=1}^{e}\Gamma\left(W_j + G_j\xi\right)\prod_{j=2}^{f}\Gamma\left(1 - V_j - A_j\xi\right)}{\sum_{l=1}^{s}\left[\prod_{j=e+1}^{h_l}\Gamma\left(1 - W_{jl} - G_{jl}\xi\right)\prod_{j=f+1}^{g_l}\Gamma\left(V_{jl} + A_{jl}\xi\right)\right]}. $$

For convergence and other conditions see the work by Bansal and Kumar [1].

1.3. **Generalized Extended Mittag-Leffler Function (GEMLF).** The general class of polynomials provides an effective tool for generating broader families of integral representations. It enables the derivation of unified results containing several class polynomial systems as special cases. Choudhary et al. [4] studied the generalized extended Mittag-Leffler function (GEMLF) is described as.

$$ E^{\Psi;\gamma}_{v,\omega}(z; \alpha, \beta, \lambda) = \sum_{l=0}^{\infty} \frac{B^{\beta,\lambda}_{\alpha}(\Psi + l, \gamma - \Psi)}{B(\Psi, \gamma - \Psi)} \frac{(\gamma)_l z^l}{\Gamma(vl + \omega)}, \tag{1.4} $$

where $\mathrm{Re}(\gamma) > 0, \mathrm{Re}(\Psi) > 0, Re(\omega) > 0, \mathrm{Re}(v) > 0, \mathrm{Re}(\alpha) > 0, \min(\mathrm{Re}(\gamma - l), \mathrm{Re}(\Psi + l))\,\mathrm{Re}(B), \mathrm{Re}(\lambda)) > 0$, in Eq. (1.4) $B^{\beta,\lambda}_{\alpha}(x, y)$ represents the generalized Beta type function [1] for further specification and characteristics of GEMLF, refer to the work of Choudhary et al. [4].

1.4. **General Class of Polynomials (GCP).** The general class of polynomials (GCP) [11] is formulated as.

$$ S^M_N(z) = \sum_{r=0}^{[N/M]} \frac{(-N)_{Mr}}{r!} \zeta_{N,r} z^r (N = 0, 1, 2, \ldots). \tag{1.5} $$

In Eq. (1.5) r represents an arbitrary positive integer and $\zeta_{N,r}(N, r \geq 0)$ indicate the arbitrary constants.

2. Main Results

In the current section we explored the four significant theorems on the multiplecation of the I-function, generalized extended Mittag-Leffler (ML) function, the general class of polynomials and the incomplete I-function. The result obtain in the present section are explored in the form of many significant formulas for the generalized extended ML function, I-function, general class of polynomials and incomplete I-function in form of several simpler special functions arising in science and technology problems.

Theorem 2.1. *The following integrals is obtained in this section.*

$$ \int_0^1\int_0^1 \left(\frac{1-x}{1-xy}y\right)^{\sigma}\left(\frac{1-y}{1-xy}\right)^{\rho}\left(\frac{1-xy}{(1-x)(1-y)}\right) E^{\Psi,\gamma}_{v,\omega}\left[\kappa_1\left(\frac{1-x}{1-xy}y\right)^{\alpha_1}\left(\frac{1-y}{1-xy}\right)^{\beta_1}\right] $$
$$ I^{m_i',n_i'}_{p_i',q_i',r'}\left[\kappa_2\left(\frac{1-x}{1-xy}y\right)^{\alpha_2}\left(\frac{1-y}{1-xy}\right)^{\beta_2}\right] S^M_N\left[\kappa_3\left(\frac{1-x}{1-xy}y\right)^{\alpha_3}\left(\frac{1-y}{1-xy}\right)^{\beta_3}\right] $$
$$ {}^{(\Gamma)}I^{e,f}_{g_l,h_l,s}\left[\kappa_4\left(\frac{1-x}{1-xy}y\right)^{\alpha_4}\left(\frac{1-y}{1-xy}\right)^{\beta_4}\right] dxdy $$

$$
\begin{aligned}
&= \sum_{r=0}^{[N/M]} \sum_{h=1}^{m'} \sum_{l,k=0}^{\infty} \frac{B_{\alpha}^{\beta,\lambda}\left(\Psi+l,\gamma-\Psi\right)(\gamma)_l}{B(\Psi,\gamma-\Psi)\Gamma(vl+\omega)} \frac{(-1)^k \phi'\left(\eta_{h,k}\right)}{\beta_h k!} \frac{(-N)_{M_r}}{r!} \zeta_{N,r} \left(\kappa_1\right)^l \left(\kappa_2\right)^{\eta_{h,k}} \left(\kappa_3\right)^r \\
&\times {}^{(\Gamma)}\,\mathrm{I}_{g_{l+2},h_{l+1},s}^{e,f+2} \left[k_4 Z^{(\alpha_4+\beta_4)} \left| \begin{array}{c} \left(V_1,A_1,x\right),\left(1-\sigma-\alpha_1 l-\alpha_2\eta_{h,k}-\alpha_3 r;\alpha_4\right) \\ \left(1-\rho-\beta_1 l-\beta_2\eta_{h,k}-\beta_3 r;\beta_4\right)\left(V_j,A_j\right)_{2,f}\left(V_{jl},A_{jl}\right)_{f+1,g_l} \\ \left(W_j,G_j\right)_{1,e}\left(W_{jl},G_{jl}\right)_{e+1,h_l}\left(1-\sigma-\rho-\left(\alpha_1+\beta_1\right)l\right. \\ \left.-\left(\alpha_2+\beta_2\right)m-\left(\alpha_3+\beta_3\right)r\right);\left(\alpha_4+\beta_4\right) \end{array} \right. \right],
\end{aligned}
$$

where all the parameters are real/complex numbers satisfying the convergence conditions.

Provided that:

$$
\mathrm{Re}\left(\sigma+1+\alpha_1\mathrm{l}+\alpha_2\eta_{\mathrm{h,k}}+\alpha_3\mathrm{r}+\alpha_4\frac{W_j}{G_j}\right)>0, \mathrm{Re}\left(\rho+1+\beta_1\mathrm{l}+\beta_2\eta_{\mathrm{h,k}}+\beta_3\mathrm{r}+\beta_4\frac{W_j}{G_j}\right)>0,
$$

$$
|\mathrm{argk}_4| \leq \frac{1}{2}T\pi, T>0,
$$

where $j=1,2,\ldots,e;(\rho,\sigma,\alpha_1,\beta_1,\alpha_2,\beta_2,\alpha_3,\beta_3,\alpha_4,\beta_4)>0,$ *and* $|k_3|<1$.

Proof. We have

$$
\begin{aligned}
& E_{v,\omega}^{\Psi,\gamma}\left[\kappa_1\left(\frac{1-x}{1-xy}y\right)^{\alpha_1}\left(\frac{1-y}{1-xy}\right)^{\beta_1}\right] I_{p_i',q_i',r'}^{m_i',n_i'}\left[\kappa_2\left(\frac{1-x}{1-xy}y\right)^{\alpha_2}\left(\frac{1-y}{1-xy}\right)^{\beta_2}\right] \\
&\times S_N^M\left[\kappa_3\left(\frac{1-x}{1-xy}y\right)^{\alpha_3}\left(\frac{1-y}{1-xy}\right)^{\beta_3}\right] {}^{(\Gamma)}I_{g_l,h_l,s}^{e,f}\left[\kappa_4\left(\frac{1-x}{1-xy}y\right)^{\alpha_4}\left(\frac{1-y}{1-xy}\right)^{\beta_4}\right] \\
&= \sum_{l=0}^{\infty} \frac{B_{\alpha}^{\beta,\lambda}(\Psi+l,\gamma-\Psi)(\gamma)_l}{B(\Psi,\gamma-\Psi)\Gamma(vl+\omega)} \left\{\kappa_1\left(\frac{1-x}{1-xy}y\right)^{\alpha_1}\left(\frac{1-y}{1-xy}\right)^{\beta_1}\right\}^l \\
&\times \sum_{h=1}^{m'}\sum_{k=0}^{\infty} \frac{(-1)^k\phi'\left(\eta_{h,k}\right)}{\beta_h k!} \left\{\kappa_2\left(\frac{1-x}{1-xy}y\right)^{\alpha_2}\left(\frac{1-y}{1-xy}\right)^{\beta_2}\right\}^{\eta_{h,k}} \\
&\times \sum_{r=0}^{[N/M]} \frac{(-N)_{M_r}}{r!}\zeta_{N,r} \left\{\kappa_3\left(\frac{1-x}{1-xy}y\right)^{\alpha_3}\left(\frac{1-y}{1-xy}\right)^{\beta_3}\right\}^r \\
&\times \frac{1}{2\pi i}\int_{\Omega} K(\xi,x)\left\{\kappa_4\left(\frac{1-x}{1-xy}y\right)^{\alpha_4}\left(\frac{1-y}{1-xy}\right)^{\beta_4}\right\}^{-\xi}.
\end{aligned}
\tag{2.1}
$$

Eq. (2.1) Multiplying by $\left(\frac{1-x}{1-xy}y\right)^{\sigma}\left(\frac{1-y}{1-xy}\right)^{\rho}\left(\frac{1-xy}{(1-x)(1-y)}\right)$ and integrating w.r.t. x and y over the interval [0,1], we obtain the required integral representation. Since the series involved are uniformly convergent under the given conditions, the order of integration and summation can be exchanged. We obtain

$$
\begin{aligned}
& \int_0^1\int_0^1\left(\frac{1-x}{1-xy}y\right)^{\sigma}\left(\frac{1-y}{1-xy}\right)^{\rho}\left(\frac{1-xy}{(1-x)(1-y)}\right) \\
&\times E_{v,\omega}^{\Psi,\gamma}\left[\kappa_1\left(\frac{1-x}{1-xy}y\right)^{\alpha_1}\left(\frac{1-y}{1-xy}\right)^{\beta_1}\right] I_{p_i',q_i',r'}^{m_i',n_i'}\left[\kappa_2\left(\frac{1-x}{1-xy}y\right)^{\alpha_2}\left(\frac{1-y}{1-xy}\right)^{\beta_2}\right] S_N^M\left[\kappa_3\left(\frac{1-x}{1-xy}y\right)^{\alpha_3}\left(\frac{1-y}{1-xy}\right)^{\beta_3}\right] \\
&\times {}^{(\Gamma)}I_{g_l,h_l,s}^{e,f}\left[\kappa_4\left(\frac{1-x}{1-xy}y\right)^{\alpha_4}\left(\frac{1-y}{1-xy}\right)^{\beta_4}\right] dxdy \\
&= \int_0^1\int_0^1\left(\frac{1-x}{1-xy}y\right)^{\sigma}\left(\frac{1-y}{1-xy}\right)^{\rho}\left(\frac{1-xy}{(1-x)(1-y)}\right)
\end{aligned}
$$

$$\sum_{l=0}^{\infty}\frac{B_{\alpha}^{\beta,\lambda}(\Psi+l,\gamma-\Psi)(\gamma)_l}{B(\Psi,\gamma-\Psi)\Gamma(vl+\omega)}\left\{\kappa_1\left(\frac{1-x}{1-xy}y\right)^{\alpha_1}\left(\frac{1-y}{1-xy}\right)^{\beta_1}\right\}^{l}$$
$$\sum_{h=1}^{m'}\sum_{k=0}^{\infty}\frac{(-1)^k\phi'(\eta_{h,k})}{\beta_h k!}\left\{\kappa_2\left(\frac{1-x}{1-xy}y\right)^{\alpha_2}\left(\frac{1-y}{1-xy}\right)^{\beta_2}\right\}^{\eta_{h,k}}\sum_{r=0}^{[N/M]}\frac{(-N)_{M_r}}{r!}\zeta_{N,r}\left\{\kappa_3\left(\frac{1-x}{1-xy}y\right)^{\alpha_3}\left(\frac{1-y}{1-xy}\right)^{\beta_3}\right\}^{r}$$
$$\times\frac{1}{2\pi i}\int_{\Omega}K(\xi,x)\left\{\kappa_4\left(\frac{1-x}{1-xy}y\right)^{\alpha_4}\left(\frac{1-y}{1-xy}\right)^{\beta_4}\right\}^{-\xi}d\xi dxdy$$
$$=\sum_{r=0}^{[N/M]}\sum_{h=1}^{m'}\sum_{l,k=0}^{\infty}\frac{B_{\alpha}^{\beta,\lambda}(\Psi+l,\gamma-\Psi)(\gamma)_l}{B(\Psi,\gamma-\Psi)\Gamma(vl+\omega)}\frac{(-1)^k\phi'(\eta_{h,k})}{\beta_h k!}\frac{(-N)_{M_r}}{r!}\zeta_{N,r}(\kappa_1)^l(\kappa_2)^{\eta_{h,k}}(\kappa_3)^r\frac{1}{2\pi i}\int_{-\mathrm{i}\infty}^{+i\infty}K(\xi,x)(\kappa_4)^{-\xi}d\xi$$
$$\times\left[\int_0^1\int_0^1\left(\frac{1-x}{1-xy}y\right)^{\sigma+\alpha_1 l+\alpha_2\eta_{h,k}+\alpha_3 r-\alpha_4\xi}\left(\frac{1-y}{1-xy}\right)^{\rho+\beta_1 l+\beta_2\eta_{h,k}+\beta_3 r-\beta_4\xi}\left(\frac{1-xy}{(1-x)(1-y)}\right)dxdy\right]$$
$$=\sum_{r=0}^{[N/M]}\sum_{h=1}^{m'}\sum_{l,k=0}^{\infty}\frac{B_{\alpha}^{\beta,\lambda}(\Psi+l,\gamma-\Psi)(\gamma)_l}{B(\Psi,\gamma-\Psi)\Gamma(vl+\omega)}\frac{(-1)^k\phi'(\eta_{h,k})}{\beta_h k!}\frac{(-N)_{M_r}}{r!}\zeta_{N,r}(\kappa_1)^l(\kappa_2)^{\eta_{h,k}}(\kappa_3)^r$$
$$\times^{(\Gamma)}\mathrm{I}_{g_{l+2},h_{l+1},s}^{e,f+2}\left[k_4 Z^{(\alpha_4+\beta_4)}\left|\begin{array}{c}(V_1,A_1,x),(1-\sigma-\alpha_1 l-\alpha_2\eta_{h,k}-\alpha_3 r;\alpha_4)\\(1-\rho-\beta_1 l-\beta_2\eta_{h,k}-\beta_3 r;\beta_4)(V_j,A_j)_{2,f}(V_{jl},A_{jl})_{f+1,g_l}\\(W_j,G_j)_{1,e}^{\prime}(W_{jl},G_{jl})_{e+1,h_l}(1-\sigma-\rho-(\alpha_1+\beta_1)l\\-(\alpha_2+\beta_2)m-(\alpha_3+\beta_3)r);(\alpha_4+\beta_4)\end{array}\right.\right].$$

□

Theorem 2.2. *The following result holds.*

$$\int_0^1\int_0^1\left(\frac{1-x}{1-xy}y\right)^{\sigma}\left(\frac{1-y}{1-xy}\right)^{\rho}\left(\frac{1-xy}{(1-x)(1-y)}\right)E_{v,\omega}^{\Psi,\gamma}\left[\kappa_1\left(\frac{1-x}{1-xy}y\right)^{\alpha_1}\left(\frac{1-y}{1-xy}\right)^{\beta_1}\right]$$
$$I_{p_i',q_i',r'}^{m_i',n_i'}\left[\kappa_2\left(\frac{1-x}{1-xy}y\right)^{\alpha_2}\left(\frac{1-y}{1-xy}\right)^{\beta_2}\right]S_N^M\left[\kappa_3\left(\frac{1-x}{1-xy}y\right)^{\alpha_3}\left(\frac{1-y}{1-xy}\right)^{\beta_3}\right]$$
$$\times^{(\gamma)}I_{g_l,h_l,s}^{e,f}\left[\kappa_4\left(\frac{1-x}{1-xy}y\right)^{\alpha_4}\left(\frac{1-y}{1-xy}\right)^{\beta_4}\right]dxdy$$
$$=\sum_{r=0}^{[N/M]}\sum_{h=1}^{m'}\sum_{l,k=0}^{\infty}\frac{B_{\alpha}^{\beta,\lambda}(\Psi+l,\gamma-\Psi)(\gamma)_l}{B(\Psi,\gamma-\Psi)\Gamma(vl+\omega)}\frac{(-1)^k\phi'(\eta_{h,k})}{\beta_h k!}\frac{(-N)_{M_r}}{r!}\zeta_{N,r}(\kappa_1)^l(\kappa_2)^{\eta_{h,k}}(\kappa_3)^r$$
$$\times^{(\gamma)}\mathrm{I}_{g_{l+2},h_{l+1},s}^{e,f+2}\left[k_4 Z^{(\alpha_4+\beta_4)}\left|\begin{array}{c}(V_1,A_1,x),(1-\sigma-\alpha_1 l-\alpha_2\eta_{h,k}-\alpha_3 r;\alpha_4)\\(1-\rho-\beta_1 l-\beta_2\eta_{h,k}-\beta_3 r;\beta_4)(V_j,A_j)_{2,f}(V_{jl},A_{jl})_{f+1,g_l}\\(W_j,G_j)_{1,e}^{\prime}(W_{jl},G_{jl})_{e+1,h_l}(1-\sigma-\rho-(\alpha_1+\beta_1)l\\-(\alpha_2+\beta_2)m-(\alpha_3+\beta_3)r));(\alpha_4+\beta_4)\end{array}\right.\right].$$

where all the parameters are real/complex numbers satisfying the convergence conditions.
Provided that:
$\mathrm{Re}\left(\sigma+1+\alpha_1\mathrm{l}+\alpha_2\eta_{\mathrm{h,k}}+\alpha_3\mathrm{r}+\alpha_4\frac{W_j}{G_j}\right)>0, \mathrm{Re}\left(\rho+1+\beta_1\mathrm{l}+\beta_2\eta_{\mathrm{h,k}}+\beta_3\mathrm{r}+\beta_4\frac{W_j}{G_j}\right)>0,$
$|\arg k_2|<\frac{1}{2}T\pi, T>0, |\arg k_3|<\frac{1}{2}T,\pi,T'>0$ *where* $j=1,2,\ldots,e$; $(\rho,\sigma,\alpha_1,\beta_1,\alpha_2,\beta_2,\alpha_3,\beta_3,\alpha_4,\beta_4)>0$ *and* $|k_3|<1$.

Proof. We have

$$
\begin{aligned}
&E_{v,\omega}^{\Psi,\gamma}\left[\kappa_1\left(\frac{1-x}{1-xy}y\right)^{\alpha_1}\left(\frac{1-y}{1-xy}\right)^{\beta_1}\right] I_{p_i',q_i',r'}^{m_i',n_i'}\left[\kappa_2\left(\frac{1-x}{1-xy}y\right)^{\alpha_2}\left(\frac{1-y}{1-xy}\right)^{\beta_2}\right]\\
&S_N^M\left[\kappa_3\left(\frac{1-x}{1-xy}y\right)^{\alpha_3}\left(\frac{1-y}{1-xy}\right)^{\beta_3}\right] {}^{(\gamma)}I_{g_l,h_l,s}^{e,f}\left[\kappa_4\left(\frac{1-x}{1-xy}y\right)^{\alpha_4}\left(\frac{1-y}{1-xy}\right)^{\beta_4}\right]\\
&=\sum_{l=0}^{\infty}\frac{B_\alpha^{\beta,\lambda}(\Psi+l,\gamma-\Psi)(\gamma)_l}{B(\Psi,\gamma-\Psi)\Gamma(vl+\omega)}\left\{\kappa_1\left(\frac{1-x}{1-xy}y\right)^{\alpha_1}\left(\frac{1-y}{1-xy}\right)^{\beta_1}\right\}^l\\
&\times\sum_{h=1}^{m'}\sum_{k=0}^{\infty}\frac{(-1)^k\phi'(\eta_{h,k})}{\beta_h k!}\left\{\kappa_2\left(\frac{1-x}{1-xy}y\right)^{\alpha_2}\left(\frac{1-y}{1-xy}\right)^{\beta_2}\right\}^{\eta_{h,k}}\\
&\times\sum_{r=0}^{[N/M]}\frac{(-N)_{M_r}}{r!}\zeta_{N,r}\left\{\kappa_3\left(\frac{1-x}{1-xy}y\right)^{\alpha_3}\left(\frac{1-y}{1-xy}\right)^{\beta_3}\right\}^r\\
&\times\frac{1}{2\pi i}\int_\Omega L(\xi,x)\left\{\kappa_4\left(\frac{1-x}{1-xy}y\right)^{\alpha_4}\left(\frac{1-y}{1-xy}\right)^{\beta_4}\right\}^{-\xi}d\xi.
\end{aligned}
\tag{2.2}
$$

Eq. (2.1) Multiplying by $\left(\frac{1-x}{1-xy}y\right)^\sigma\left(\frac{1-y}{1-xy}\right)^\rho\left(\frac{1-xy}{(1-x)(1-y)}\right)$ and integrating w.r.t. x and y over the interval [0,1], we obtain the required integral representation. Since the series involved are uniformly convergent under the given conditions, the order of integration and summation can be interchanged. We get

$$
\begin{aligned}
&=\int_0^1\int_0^1\left(\frac{1-x}{1-xy}y\right)^\sigma\left(\frac{1-y}{1-xy}\right)^\rho\left(\frac{1-xy}{(1-x)(1-y)}\right)E_{v,\omega}^{\Psi,\gamma}\left[\kappa_1\left(\frac{1-x}{1-xy}y\right)^{\alpha_1}\left(\frac{1-y}{1-xy}\right)^{\beta_1}\right]\\
&\times I_{p_i',q_i',r'}^{m_i',n_i'}\left[\kappa_2\left(\frac{1-x}{1-xy}y\right)^{\alpha_2}\left(\frac{1-y}{1-xy}\right)^{\beta_2}\right]S_N^M\left[\kappa_3\left(\frac{1-x}{1-xy}y\right)^{\alpha_3}\left(\frac{1-y}{1-xy}\right)^{\beta_3}\right]\\
&\times{}^{(\gamma)}I_{g_l,h_l,s}^{e,f}\left[\kappa_4\left(\frac{1-x}{1-xy}y\right)^{\alpha_4}\left(\frac{1-y}{1-xy}\right)^{\beta_4}\right]dxdy\\
&=\int_0^1\int_0^1\left(\frac{1-x}{1-xy}y\right)^\sigma\left(\frac{1-y}{1-xy}\right)^\rho\left(\frac{1-xy}{(1-x)(1-y)}\right)\\
&\times\sum_{l=0}^{\infty}\frac{B_\alpha^{\beta,\lambda}(\Psi+l,\gamma-\Psi)(\gamma)_l}{B(\Psi,\gamma-\Psi)\Gamma(vl+\omega)}\left\{\kappa_1\left(\frac{1-x}{1-xy}y\right)^{\alpha_1}\left(\frac{1-y}{1-xy}\right)^{\beta_1}\right\}^l\\
&\times\sum_{h=1}^{m'}\sum_{k=0}^{\infty}\frac{(-1)^k\phi'(\eta_{h,k})}{\beta_h k!}\left\{\kappa_2\left(\frac{1-x}{1-xy}y\right)^{\alpha_2}\left(\frac{1-y}{1-xy}\right)^{\beta_2}\right\}^{\eta_{h,k}}\\
&\times\sum_{r=0}^{[N/M]}\frac{(-N)_{M_r}}{r!}\zeta_{N,r}\left\{\kappa_3\left(\frac{1-x}{1-xy}y\right)^{\alpha_3}\left(\frac{1-y}{1-xy}\right)^{\beta_3}\right\}^r\\
&\times\frac{1}{2\pi i}\int_\Omega L(\xi,x)\left\{\kappa_4\left(\frac{1-x}{1-xy}y\right)^{\alpha_4}\left(\frac{1-y}{1-xy}\right)^{\beta_4}\right\}^{-\xi}d\xi dxdy\\
&=\sum_{r=0}^{[N/M]}\sum_{h=1}^{m'}\sum_{l,k=0}^{\infty}\frac{B_\alpha^{\beta,\lambda}(\Psi+l,\gamma-\Psi)(\gamma)_l}{B(\Psi,\gamma-\Psi)\Gamma(vl+\omega)}\frac{(-1)^k\phi'(\eta_{h,k})}{\beta_h k!}\frac{(-N)_{M_r}}{r!}\zeta_{N,r}(\kappa_1)^l(\kappa_2)^{\eta_{h,k}}(\kappa_3)^r\frac{1}{2\pi i}\int_{-i\infty}^{+i\infty}L(\xi,x)(\kappa_4)^{-\xi}d\xi
\end{aligned}
$$

$$\times\left[\int_0^1\int_0^1\left(\frac{1-x}{1-xy}y\right)^{\sigma+\alpha_1 l+\alpha_2\eta_{h,k}+\alpha_3 r-\alpha_{4\xi}}\left(\frac{1-y}{1-xy}\right)^{\rho+\beta_1 l+\beta_2\eta_{h,k}+\beta_3 r-\beta_4\xi}\left(\frac{1-xy}{(1-x)(1-y)}\right)dxdy\right]$$
$$=\sum_{r=0}^{[N/M]}\sum_{h=1}^{m'}\sum_{l,k=0}^{\infty}\frac{B_{\alpha}^{\beta,\lambda}(\Psi+l,\gamma-\Psi)(\gamma)_l}{B(\Psi,\gamma-\Psi)\Gamma(vl+\omega)}\frac{(-1)^k\phi'(\eta_{h,k})}{\beta_h k!}\frac{(-N)_{M_r}}{r!}\zeta_{N,r}(\kappa_1)^l(\kappa_2)^{\eta_{h,k}}(\kappa_3)^r$$
$$\times^{(\gamma)}\mathrm{I}_{g_{l+2},h_{l+1},s}^{e,f+2}\left[k_4 Z^{(\alpha_4+\beta_4)}\left|\begin{array}{c}(V_1,A_1,x),(1-\sigma-\alpha_1 l-\alpha_2\eta_{h,k}-\alpha_3 r;\alpha_4)\\(1-\rho-\beta_1 l-\beta_2\eta_{h,k}-\beta_3 r;\beta_4)(V_j,A_j)_{2,f}(V_{jl},A_{jl})_{f+1,g_l}\\(W_j,G_j)_{1,e}^{,}(W_{jl},G_{jl})_{e+1,h_l}(1-\sigma-\rho-(\alpha_1+\beta_1)l\\-(\alpha_2+\beta_2)m-(\alpha_3+\beta_3)r));(\alpha_4+\beta_4)\end{array}\right.\right].$$

□

Theorem 2.3.

$$\int_0^\infty\int_0^\infty\phi(u+v)(u)^{\sigma-1}(v)^{\rho-1}E_{v,\omega}^{\psi,\gamma}\left[\kappa_1(u)^{\alpha_1}(v)^{\beta_1}\right]I_{p_i',q_i',r'}^{m_i',n_i'}\left[\kappa_2(u)^{\alpha_2}(v)^{\beta_2}\right]S_N^M\left[\kappa_3(u)^{\alpha_3}(v)^{\beta_3}\right]$$
$$\times^{(\Gamma)}I_{g_l,h_l,s}^{e,f}\left[\kappa_4(u)^{\alpha_4}(v)^{\beta_4}\right]dudv$$
$$=\sum_{r=0}^{[N/M]}\sum_{h=1}^{m'}\sum_{l,k=0}^{\infty}\frac{B_{\alpha}^{\beta,\lambda}(\Psi+l,\gamma-\Psi)(\gamma)_l}{B(\Psi,\gamma-\Psi)\Gamma(vl+\omega)}\frac{(-1)^k\phi'(\eta_{h,k})}{\beta_h k!}\frac{(-N)_{M_r}}{r!}\zeta_{N,r}(\kappa_1)^l(\kappa_2)^{\eta_{h,k}}(\kappa_3)^r$$
$$\times\int_0^\infty\phi(z)z^{\sigma+\rho+(\alpha_1+\beta_1)l+(\alpha_2+\beta_2)\eta_{h,k}+(\alpha_3+\beta_3)r-(\alpha_4+\beta_4)\xi-1}d\xi$$
$$\times^{(\Gamma)}\mathrm{I}_{g_{l+2},h_{l+1},s}^{e,f+2}\left[\kappa_4 Z^{(\alpha_4+\beta_4)}\left|\begin{array}{c}(V_1,A_1,x),(1-\sigma-\alpha_1 l-\alpha_2\eta_{h,k}-\alpha_3 r;\alpha_4)\\(1-\rho-\beta_1 l-\beta_2\eta_{h,k}-\beta_3 r;\beta_4)(a_j,A_j)_{2,f}(V_{jl},A_{jl})_{f+1,g_l}\\(W_j,G_j)_{1,e},(W_{jl},G_{jl})_{e+1,h_l}(1-\sigma-\rho-(\alpha_1+\beta_1)l\\-(\alpha_2+\beta_2)m-(\alpha_3+\beta_3)r));(\alpha_4+\beta_4)\end{array}\right.\right].$$

where all the parameters are real/complex numbers satisfying the convergence conditions.

Provided that:

$\mathrm{Re}\left(\sigma+\alpha_1\mathrm{l}+\alpha_2\eta_{\mathrm{h,k}}+\alpha_3\mathrm{r}+\alpha_4\frac{W_j}{G_j}\right)>0;\mathrm{Re}\left(\sigma+\alpha_1\mathrm{l}+\alpha_2\eta_{\mathrm{h,k}}+\alpha_3\mathrm{r}+\alpha_4\left(\frac{a_j-1}{A_j}\right)\right)<0.$

$\mathrm{Re}\left(\rho+\beta_1\mathrm{l}+\beta_2\eta_{\mathrm{h,k}}+\beta_3\mathrm{r}+\beta_4\frac{W_j}{G_j}\right)>0;\mathrm{Re}\left(\rho+\beta_1\mathrm{l}+\beta_2\eta_{\mathrm{h,k}}+\beta_3\mathrm{r}+\beta_4\left(\frac{a_j-1}{A_j}\right)\right)<0.$

$|\mathrm{argk}_4|<\frac{1}{2}T\pi,T>0,$ *where* $j=1,2,\ldots,e;(\rho,\sigma,\alpha_1,\beta_1,\alpha_2,\beta_2,\alpha_3,\beta_3,\alpha_4,\beta_4)>0.$

Proof. We have

$$E_{v,\omega}^{\Psi,\gamma}\left[\kappa_1(u)^{\alpha_1}(v)^{\beta_1}\right]I_{p_i',q_i',r'}^{m_i',n_i'}\left[\kappa_2(u)^{\alpha_2}(v)^{\beta_2}\right]S_N^M\left[\kappa_3(u)^{\alpha_3}(v)^{\beta_3}\right]{}^{(\Gamma)}I_{g_l,h_l,s}^{e,f}\left[\kappa_4(u)^{\alpha_4}(v)^{\beta_4}\right]$$
$$=\sum_{l=0}^{\infty}\frac{B_{\alpha}^{\beta,\lambda}(\Psi+l,\gamma-\Psi)(\gamma)_l}{B(\Psi,\gamma-\Psi)\Gamma(vl+\omega)}\left\{\kappa_1(u)^{\alpha_1}(v)^{\beta_1}\right\}^l\sum_{h=1}^{m'}\sum_{k=0}^{\infty}\frac{(-1)^k\phi'(\eta_{h,k})}{\beta_h k!}\left\{\kappa_2(u)^{\alpha_2}(v)^{\beta_2}\right\}^{(\eta_{h,k})}\tag{2.3}$$
$$\times\sum_{r=0}^{[N/M]}\frac{(-N)_{M_r}}{r!}\zeta_{N,r}\left\{\kappa_3(u)^{\alpha_3}(v)^{\beta_3}\right\}^r\times\frac{1}{2\pi i}\int_{\Omega}K(\xi,x)\left\{\kappa_4(u)^{\alpha_4}(v)^{\beta_4}\right\}^{-\xi}d\xi.$$

Multiplying both sides of Eq. (2.3) by $\phi(u+v)(u)^{\sigma-1}(v)^{\rho-1}$ and integrating w.r.t. u and v over the interval [0, ∞], we obtain the required integral representation. Since the involved series are uniformly convergent under the given conditions, the order of integration and summation may be interchanged. We obtain

$$=\int_0^\infty\int_0^\infty\phi(u+v)(u)^{\sigma-1}(v)^{\rho-1}E_{v,\omega}^{\Psi,\gamma}\left[\kappa_1(u)^{\alpha_1}(v)^{\beta_1}\right]I_{p_i',q_i',r'}^{m_i',n_i'}\left[\kappa_2(u)^{\alpha_2}(v)^{\beta_2}\right]S_N^M\left[\kappa_3(u)^{\alpha_3}(v)^{\beta_3}\right]$$
$$\times^{(\Gamma)}I_{g_l,h_l,s}^{e,f}\left[\kappa_4(u)^{\alpha_4}(v)^{\beta_4}\right]dudv$$

$$
\begin{aligned}
&= \int_0^\infty \int_0^\infty \phi(u+v)(u)^{\sigma-1}(v)^{\rho-1} \sum_{l=0}^{\infty} \frac{B_\alpha^{\beta,\lambda}(\Psi+l,\gamma-\Psi)(\gamma)_l}{B(\Psi,\gamma-\Psi)\Gamma(vl+\omega)} \left\{\kappa_1(u)^{\alpha_1}(v)^{\beta_1}\right\}^l \\
&\times \sum_{h=1}^{m'} \sum_{k=0}^{\infty} \frac{(-1)^k \phi'(\eta_{h,k})}{\beta_h k!} \left\{\kappa_2(u)^{\alpha_2}(v)^{\beta_2}\right\}^{\eta_{h,k}} \sum_{r=0}^{[N/M]} \frac{(-N)_{M_r}}{r!}\zeta_{N,r} \left\{\kappa_3(u)^{\alpha_3}(v)^{\beta_3}\right\}^r \\
&\times \frac{1}{2\pi i} \int_\Omega K(\xi,x) \left\{\kappa_4(u)^{\alpha_4}(v)^{\beta_4}\right\}^{-\xi} d\xi du dv \\
&= \sum_{r=0}^{[N/M]} \sum_{h=1}^{m'} \sum_{l,k=0}^{\infty} \frac{B_\alpha^{\beta,\lambda}(\Psi+l,\gamma-\Psi)(\gamma)_l}{B(\Psi,\gamma-\Psi)\Gamma(vl+\omega)} \frac{(-1)^k \phi'(\eta_{h,k})}{\beta_h k!} \frac{(-N)_{M_r}}{r!}\zeta_{N,r} (\kappa_1)^l (\kappa_2)^{\eta_{h,k}} (\kappa_3)^r \\
&\times \frac{1}{2\pi i} \int_{-\mathrm{i}\infty}^{+i\infty} K(\xi,x) (\kappa_4)^{-\xi} d\xi \\
&\times \left[\int_0^1 \int_0^1 \phi(u+v)(u)^{\sigma+\alpha_1 l+\alpha_2\eta_{h,k}+\alpha_3 r-\alpha_4\xi-1}(v)^{\rho+\beta_1 l+\beta_2\eta_{h,k}+\beta_3 r+\beta_4\xi-1} du dv\right] \\
&= \sum_{r=0}^{[N/M]} \sum_{h=1}^{m'} \sum_{l,k=0}^{\infty} \frac{B_\alpha^{\beta,\lambda}(\Psi+l,\gamma-\Psi)(\gamma)_l}{B(\Psi,\gamma-\Psi)\Gamma(vl+\omega)} \frac{(-1)^k \phi'(\eta_{h,k})}{\beta_h k!} \frac{(-N)_{M_r}}{r!}\zeta_{N,r} (\kappa_1)^l (\kappa_2)^{\eta_{h,k}} (\kappa_3)^r \\
&\times \int_0^\infty \phi(z) z^{\sigma+\rho+(\alpha_1+\beta_1)l+(\alpha_2+\beta_2)\eta_{h,k}+(\alpha_3+\beta_3)r-(\alpha_4+\beta_4)\xi-1} d\xi \\
&\times {}^{(\Gamma)}\mathrm{I}_{g_{l+2},h_{l+1},s}^{e,f+2} \left[k_4 Z^{(\alpha_4+\beta_4)} \left| \begin{array}{c} (V_1,A_1,x), (1-\sigma-\alpha_1 l-\alpha_2\eta_{h,k}-\alpha_3 r;\alpha_4) \\ (1-\rho-\beta_1 l-\beta_2\eta_{h,k}-\beta_3 r;\beta_4)\,(V_j,A_j)_{2,f}\,(V_{jl},A_{jl})_{f+1,g_l} \\ (W_j,G_j)_{1,e}^{'}\,(W_{jl},G_{jl})_{e+1,h_l}\,(1-\sigma-\rho-(\alpha_1+\beta_1)\,l \\ -(\alpha_2+\beta_2)\,m-(\alpha_3+\beta_3)\,r);(\alpha_4+\beta_4) \end{array} \right. \right].
\end{aligned}
$$

□

Theorem 2.4.

$$
\begin{aligned}
&\int_0^\infty \int_0^\infty \phi(u+v)(u)^{\sigma-1}(v)^{\rho-1} E_{v,\omega}^{\psi,\gamma}\left[\kappa_1(u)^{\alpha_1}(v)^{\beta_1}\right] I_{p_i',q_i',r'}^{m_i',n_i'}\left[\kappa_2(u)^{\alpha_2}(v)^{\beta_2}\right] S_N^M\left[\kappa_3(u)^{\alpha_3}(v)^{\beta_3}\right] \\
&\times^{(\gamma)} I_{g_l,h_l,r}^{e,f}\left[\kappa_4(u)^{\alpha_4}(v)^{\beta_4}\right] du dv \\
&= \sum_{r=0}^{[N/M]} \sum_{h=1}^{m'} \sum_{l,k=0}^{\infty} \frac{B_\alpha^{\beta,\lambda}(\Psi+l,\gamma-\Psi)(\gamma)_l}{B(\Psi,\gamma-\Psi)\Gamma(vl+\omega)} \frac{(-1)^k \phi'(\eta_{h,k})}{\beta_h k!} \frac{(-N)_{M_r}}{r!}\zeta_{N,r} (\kappa_1)^l (\kappa_2)^{\eta_{h,k}} (\kappa_3)^r \\
&\times \int_0^\infty \phi(z) z^{\sigma+\rho+(\alpha_1+\beta_1)l+(\alpha_2+\beta_2)\eta_{h,k}+(\alpha_3+\beta_3)r-(\alpha_4+\beta_4)\xi-1} d\xi \\
&{}^{(\gamma)}\mathrm{I}_{g_{l+2},h_{l+1},s}^{e,f+2} \left[\kappa_4 Z^{(\alpha_4+\beta_4)} \left| \begin{array}{c} (V_1,A_1,x), (1-\sigma-\alpha_1 l-\alpha_2\eta_{h,k}-\alpha_3 r;\alpha_4) \\ (1-\rho-\beta_1 l-\beta_2\eta_{h,k}-\beta_3 r;\beta_4)\,(a_j,A_j)_{2,f}\,(V_{jl},A_{jl})_{f+1,g_l} \\ (W_j,G_j)_{1,e}, (W_{jl},G_{jl})_{e+1,h_l}\,(1-\sigma-\rho-(\alpha_1+\beta_1)\,l \\ -(\alpha_2+\beta_2)\,m-(\alpha_3+\beta_3)\,r)\,(\alpha_4+\beta_4) \end{array} \right. \right].
\end{aligned}
$$

where all the parameters are real/complex numbers satisfying the convergence conditions.
Provided that:

$\mathrm{Re}\left(\sigma+\alpha_1 \mathrm{l}+\alpha_2\eta_{\mathrm{h,k}}+\alpha_3\mathrm{r}+\alpha_4\frac{W_j}{G_j}\right)>0; \mathrm{Re}\left(\sigma+\alpha_1\mathrm{l}+\alpha_2\eta_{\mathrm{h,k}}+\alpha_3\mathrm{r}+\alpha_4\left(\frac{a_j-1}{A_j}\right)\right)<0.$

$\mathrm{Re}\left(\rho+\beta_1\mathrm{l}+\beta_2\eta_{\mathrm{h,k}}+\beta_3\mathrm{r}+\beta_4\frac{W_j}{G_j}\right)>0; \mathrm{Re}\left(\rho+\beta_1\mathrm{l}+\beta_2\eta_{\mathrm{h,k}}+\beta_3\mathrm{r}+\beta_4\left(\frac{a_j-1}{A_j}\right)\right)<0.$

$|\mathrm{argk}_4|<\frac{1}{2}T\pi, T>0,$ *where* $j=1,2,\ldots,e; (\rho,\sigma,\alpha_1,\beta_1,\alpha_2,\beta_2,\alpha_3,\beta_3,\alpha_4,\beta_4)>0.$

Proof. We have

$$E_{v,\omega}^{\Psi,\gamma}\left[\kappa_1(u)^{\alpha_1}(v)^{\beta_1}\right] I_{p_i',q_i',r'}^{m_i',n_i'}\left[\kappa_2(u)^{\alpha_2}(v)^{\beta_2}\right] S_N^M\left[\kappa_3(u)^{\alpha_3}(v)^{\beta_3}\right] {}^{(\gamma)}I_{g_l,h_l,s}^{e,f}\left[\kappa_4(u)^{\alpha_4}(v)^{\beta_4}\right]$$

$$= \sum_{l=0}^{\infty} \frac{B_\alpha^{\beta,\lambda}(\Psi+l,\gamma-\Psi)(\gamma)_l}{B(\Psi,\gamma-\Psi)\Gamma(vl+\omega)}\left\{\kappa_1(u)^{\alpha_1}(v)^{\beta_1}\right\}^l \sum_{h=1}^{m'}\sum_{k=0}^{\infty}\frac{(-1)^k\phi'\left(\eta_{h,k}\right)}{\beta_h k!}\left\{\kappa_2(u)^{\alpha_2}(v)^{\beta_2}\right\}^{\eta_{h,k}} \tag{2.4}$$

$$\times \sum_{r=0}^{[N/M]}\frac{(-N)_{M_r}}{r!}\zeta_{N,r}\left\{\kappa_3(u)^{\alpha_3}(v)^{\beta_3}\right\}^r \times \frac{1}{2\pi i}\int_\Omega L(\xi,x)\left\{\kappa_4(u)^{\alpha_4}(v)^{\beta_4}\right\}^{-\xi}d\xi.$$

Multiplying both sides of Eq. (2.4) by $\phi(u+v)(u)^{\sigma-1}(v)^{\rho-1}$, integrating w.r.t. u and v Over the interval $[0,\infty]$, we obtain the required integral representaion. Since the involved series are uniformly convergent under the given conditions, the order of summation and integration may be interchanged. We obtain

$$= \int_0^\infty\int_0^\infty \phi(u+v)(u)^{\sigma-1}(v)^{\rho-1}E_{v,\omega}^{\Psi,\gamma}\left[\kappa_1(u)^{\alpha_1}(v)^{\beta_1}\right] I_{p_i',q_i',r'}^{m_i',n_i'}\left[\kappa_2(u)^{\alpha_2}(v)^{\beta_2}\right] S_N^M\left[\kappa_3(u)^{\alpha_3}(v)^{\beta_3}\right]$$

$$\times {}^{(\gamma)}I_{g_l,h_l,s}^{e,f}\left[\kappa_4(u)^{\alpha_4}(v)^{\beta_4}\right]dudv$$

$$= \int_0^\infty\int_0^\infty \phi(u+v)(u)^{\sigma-1}(v)^{\rho-1}\sum_{l=0}^{\infty}\frac{B_\alpha^{\beta,\lambda}(\Psi+l,\gamma-\Psi)(\gamma)_l}{B(\Psi,\gamma-\Psi)\Gamma(vl+\omega)}\left\{\kappa_1(u)^{\alpha_1}(v)^{\beta_1}\right\}^l$$

$$\times \sum_{h=1}^{m'}\sum_{k=0}^{\infty}\frac{(-1)^k\phi'\left(\eta_{h,k}\right)}{\beta_h k!}\left\{\kappa_2(u)^{\alpha_2}(v)^{\beta_2}\right\}^{\eta_{h,e}}\sum_{r=0}^{[N/M]}\frac{(-N)_{M_r}}{r!}\zeta_{N,r}\left\{\kappa_3(u)^{\alpha_3}(v)^{\beta_3}\right\}^r$$

$$\times \frac{1}{2\pi i}\int_\Omega L(\xi,x)\left\{\kappa_4(u)^{\alpha_4}(v)^{\beta_4}\right\}^{-\xi}d\xi dudv$$

$$= \sum_{r=0}^{[N/M]}\sum_{h=1}^{m'}\sum_{l,k=0}^{\infty}\frac{B_\alpha^{\beta,\lambda}(\Psi+l,\gamma-\Psi)(\gamma)_l}{B(\Psi,\gamma-\Psi)\Gamma(vl+\omega)}\frac{(-1)^k\phi'\left(\eta_{h,k}\right)}{\beta_h k!}\frac{(-N)_{M_r}}{r!}\zeta_{N,r}\left(\kappa_1\right)^l\left(\kappa_2\right)^{\eta_{h,k}}\left(\kappa_3\right)^r\ \frac{1}{2\pi i}\int_{-\mathrm{i}\infty}^{+i\infty}L(\xi,x)\left(\kappa_4\right)^{-\xi}d\xi$$

$$\times \left[\int_0^\infty\int_0^\infty \phi(u+v)(u)^{\sigma+\alpha_1 l+\alpha_2\eta_{h,k}+\alpha_3 r-\alpha_4\xi-1}(v)^{\rho+\beta_1 l+\beta_2\eta_{h,k}+\beta_3 r+\beta_4\xi-1}dudv\right]$$

$$= \sum_{r=0}^{[N/M]}\sum_{h=1}^{m'}\sum_{l,k=0}^{\infty}\frac{B_\alpha^{\beta,\lambda}(\Psi+l,\gamma-\Psi)(\gamma)_l}{B(\Psi,\gamma-\Psi)\Gamma(vl+\omega)}\frac{(-1)^k\phi'\left(\eta_{h,k}\right)}{\beta_h k!}\frac{(-N)_{M_r}}{r!}\zeta_{N,r}\left(\kappa_1\right)^l\left(\kappa_2\right)^{\eta_{h,k}}\left(\kappa_3\right)^r$$

$$\times \int_0^\infty \phi(z)z^{\sigma+\rho+(\alpha_1+\beta_1)l+(\alpha_2+\beta_2)\eta_{h,k}+(\alpha_3+\beta_3)r-(\alpha_4+\beta_4)\xi-1}d\xi$$

$$\times {}^{(\gamma)}\mathrm{I}_{g_{l+2},h_{l+1},s}^{e,f+2}\left[k_4 Z^{(\alpha_4+\beta_4)}\left|\begin{array}{c}\left(V_1,A_1,x\right),\left(1-\sigma-\alpha_1 l-\alpha_2\eta_{h,k}-\alpha_3 r;\alpha_4\right)\\ \left(1-\rho-\beta_1 l-\beta_2\eta_{h,k}-\beta_3 r;\beta_4\right)\left(V_j,A_j\right)_{2,f}\left(V_{jl},A_{jl}\right)_{f+1,g_l}\\ \left(W_j,G_j\right)_{1,e}\left(W_{jl},G_{jl}\right)_{e+1,h_l}\left(1-\sigma-\rho-\left(\alpha_1+\beta_1\right)l\right.\\ \left.-\left(\alpha_2+\beta_2\right)m-\left(\alpha_3+\beta_3\right)r\right);\left(\alpha_4+\beta_4\right)\end{array}\right.\right].$$

$\square$

Theorem 2.5.

$$\int_0^1\int_0^1 f(uv)(1-u)^{\sigma-1}(1-v)^{\rho-1}v^\sigma E_{v,\omega}^{\Psi,\gamma}\left[\kappa_1(1-u)^{\alpha_1}(1-v)^{\beta_1}v^{\alpha_1}\right] I_{p_i',q_i',r'}^{m_i',n_i'}\left[\kappa_2(1-u)^{\alpha_2}(1-v)^{\beta_2}v^{\alpha_2}\right]$$

$$\times S_N^M\left[\kappa_3(1-u)^{\alpha_3}(1-v)^{\beta_3}v^{\alpha_3}\right]^{(\Gamma)} I_{g_l,h_l,s}^{e,f}\left[\kappa_4(1-u)^{\alpha_4}(1-v)^{\beta_4}v^{\alpha_4}\right]dudv$$

$$= \sum_{r=0}^{[N/M]}\sum_{h=1}^{m'}\sum_{l,k=0}^{\infty}\frac{B_\alpha^{\beta,\lambda}(\Psi+l,\gamma-\Psi)(\gamma)_l}{B(\Psi,\gamma-\Psi)\Gamma(vl+\omega)}\frac{(-1)^k\phi'\left(\eta_{h,k}\right)}{\beta_h k!}\frac{(-N)_{M_r}}{r!}\zeta_{N,r}\left(\kappa_1\right)^l\left(\kappa_2\right)^{\eta_{h,k}}\left(\kappa_3\right)^r$$

$$\times \int_0^1 f(\xi)(1-\xi)^{\sigma+\rho+(\alpha_1+\beta_1)l+(\alpha_2+\beta_2)\eta_{h,k}+(\alpha_3+\beta_3)r-(\alpha_4+\beta_4)\xi-1}d\xi$$

$$
{}^{(\Gamma)}\mathrm{I}^{e,f+2}_{g_{l+2},h_{l+1},s}\left[\kappa_4 Z^{(\alpha_4+\beta_4)}\left|\begin{array}{c}(V_1,A_1,x),(1-\sigma-\alpha_1 l-\alpha_2\eta_{h,k}-\alpha_3 r;\alpha_4;1)\\(1-\rho-\beta_1 l-\beta_2\eta_{h,k}-\beta_3 r;\beta_4)(V_j,A_j)_{2,f}(V_{jl},A_{jl})_{f+1,g_l}\\(W_j,G_j)_{1,e},(W_{jl},G_{jl})_{e+1,h_l}(1-\sigma-\rho-(\alpha_1+\beta_1)l\\-(\alpha_2+\beta_2)m-(\alpha_3+\beta_3)r);(\alpha_4+\beta_4)\end{array}\right.\right],
$$

where all the parameters are real/complex numbers satisfying the convergence conditions. Provided that:
$\mathrm{Re}\left(\sigma+1+\alpha_1\mathrm{l}+\alpha_2\eta_{\mathrm{h,k}}+\alpha_3\mathrm{r}+\alpha_4\frac{W_j}{G_j}\right)>0, \mathrm{Re}\left(\alpha+1+\alpha_1\mathrm{l}+\alpha_2\eta_{\mathrm{h,k}}+\alpha_3\mathrm{r}+\alpha_4\frac{W_j}{G_j}\right)>0.$
$\mathrm{Re}\left(\rho+\beta_1\mathrm{l}+\beta_2\eta_{\mathrm{h,k}}+\beta_3\mathrm{r}+\beta_4\frac{W_j}{G_j}\right)>0.$
$|\mathrm{argk}_2|\leq\frac{1}{2}T\pi, T>0,$ *where* $j=1,2,\ldots,e; (\rho,\sigma,\alpha_1,\beta_1,\alpha_2,\beta_2,\alpha_3,\beta_3,\alpha_4,\beta_4)>0.$

Proof. We have

$$
\begin{aligned}
&E^{\Psi,\gamma}_{v,\omega}\left[\kappa_1(1-u)^{\alpha_1}(1-v)^{\beta_1}(v)^{\alpha_1}\right] I^{m_i',n_i'}_{p_i',q_i',r'}\left[\kappa_2(1-u)^{\alpha_2}(1-v)^{\beta_2}(v)^{\alpha_2}\right]\\
&\times S^M_N\left[\kappa_3(1-u)^{\alpha_3}(1-v)^{\beta_3}(v)^{\alpha_3}\right]{}^{(\Gamma)}I^{m,n}_{p_l,q_l,s}\left[\kappa_4(1-u)^{\alpha_4}(1-v)^{\beta_4}(v)^{\alpha_4}\right]\\
&=\sum_{l=0}^{\infty}\frac{B^{\beta,\lambda}_{\alpha}(\Psi+l,\gamma-\Psi)(\gamma)_l}{B(\Psi,\gamma-\Psi)\Gamma(vl+\omega)}\left\{\kappa_1(1-u)^{\alpha_1}(1-v)^{\beta_1}(v)^{\alpha_1}\right\}^l\\
&\times\sum_{h=1}^{m'}\sum_{k=0}^{\infty}\frac{(-1)^k\phi'(\eta_{h,k})}{\beta_h k!}\left\{\kappa_2(1-u)^{\alpha_2}(1-v)^{\beta_2}(v)^{\alpha_2}\right\}^{\eta_{h,k}}\\
&\times\sum_{r=0}^{[N/M]}\frac{(-N)_{M_r}}{r!}\zeta_{N,r}\left\{\kappa_3(1-u)^{\alpha_3}(1-v)^{\beta_3}(v)^{\alpha_3}\right\}^r\\
&\times\frac{1}{2\pi i}\int_{\Omega}K(\xi,x)\left\{\kappa_4(1-u)^{\alpha_4}(1-v)^{\beta_4}(v)^{\alpha_4}\right\}^{-\xi}d\xi.
\end{aligned}\tag{2.5}
$$

Multiplying both sides of Eq. (2.5) by $f(uv)(1-u)^{\sigma-1}(1-v)^{\rho-1}(v)^{\sigma}$ and integrating w.r.t. u and v over the interval [0,1], we obtain the required integral representation. Since the involved series are uniformly convergent under the given conditions, the order of summation and integration may be interchanged. We obtain

$$
\begin{aligned}
&\int_0^1\int_0^1 f(uv)(1-u)^{\sigma-1}(1-v)^{\rho-1}(v)^{\sigma}E^{\Psi,\gamma}_{v,\omega}\left[\kappa_1(1-u)^{\alpha_1}(1-v)^{\beta_1}v^{\alpha_1}\right]I^{m_i',n_i'}_{p_i',q_i',r'}\left[\kappa_2(1-u)^{\alpha_2}(1-v)^{\beta_2}v^{\alpha_2}\right]\\
&\times S^M_N\left[\kappa_3(1-u)^{\alpha_3}(1-v)^{\beta_3}v^{\alpha_3}\right]{}^{(\Gamma)}I^{e,f}_{g_l,h_l,s}\left[\kappa_4(1-u)^{\alpha_4}(1-v)^{\beta_4}v^{\alpha_4}\right]dudv\\
&=\int_0^1\int_0^1 f(uv)(1-u)^{\sigma-1}(1-v)^{\rho-1}(v)^{\sigma}\sum_{l=0}^{\infty}\frac{B^{\beta,\lambda}_{\alpha}(\Psi+l,\gamma-\Psi)(\gamma)_l}{B(\Psi,\gamma-\Psi)\Gamma(vl+\omega)}\left\{\kappa_1(1-u)^{\alpha_1}(1-v)^{\beta_1}(v)^{\alpha_1}\right\}^l\\
&\times\sum_{h=1}^{m'}\sum_{k=0}^{\infty}\frac{(-1)^k\phi'(\eta_{h,k})}{\beta_h k!}\left\{\kappa_2(1-u)^{\alpha_2}(1-v)^{\beta_2}(v)^{\alpha_2}\right\}^{\eta_{h,k}}\\
&\sum_{r=0}^{[N/M]}\frac{(-N)_{M_r}}{r!}\zeta_{N,r}\left\{\kappa_3(1-u)^{\alpha_3}(1-v)^{\beta_3}(v)^{\alpha_3}\right\}^r\frac{1}{2\pi i}\int_{\Omega}K(\xi,x)\left\{\kappa_4(1-u)^{\alpha_4}(1-v)^{\beta_4}(v)^{\alpha_4}\right\}^{-\xi}d\xi dudv\\
&=\sum_{r=0}^{[N/M]}\sum_{h=1}^{m'}\sum_{l,k=0}^{\infty}\frac{B^{\beta,\lambda}_{\alpha}(\Psi+l,\gamma-\Psi)(\gamma)_l}{B(\Psi,\gamma-\Psi)\Gamma(vl+\omega)}\frac{(-1)^k\phi'(\eta_{h,k})}{\beta_h k!}\frac{(-N)_{M_r}}{r!}\zeta_{N,r}(\kappa)^l(\kappa_2)^{\eta_{h,k}}(\kappa_3)^r\\
&\times\frac{1}{2\pi i}\int_{-\mathrm{i}\infty}^{+i\infty}K(\xi,x)(\kappa_4)^{-\xi}d\xi
\end{aligned}
$$

$$\left[\int_0^1\int_0^1 f(uv)(1-u)^{\sigma+\alpha_1 l+\alpha_2\eta_{h,k}+\alpha_3 r-\alpha_4\xi-1}(1-v)^{\rho+\beta_1 l+\beta_2\eta_{h,k}+\beta_3 r+\beta_4\xi-1}v^{\sigma+\alpha_1 l+\alpha_2\eta_{h,k}+\alpha_3 r-\alpha_4\xi}dudv\right]$$
$$=\sum_{r=0}^{[N/M]}\sum_{h=1}^{m'}\sum_{l,k=0}^{\infty}\frac{B_\alpha^{\beta,\lambda}(\Psi+l,\gamma-\Psi)(\gamma)_l}{B(\Psi,\gamma-\Psi)\Gamma(vl+\omega)}\frac{(-1)^k\phi'(\eta_{h,k})}{\beta_h k!}\frac{(-N)_{M_r}}{r!}\zeta_{N,r}(\kappa)^l(\kappa_2)^{\eta_{h,k}}(\kappa_3)^r$$
$$\times\int_0^1 f(\xi)(1-\xi)^{\sigma+\rho+(\alpha_1+\beta_1)l+(\alpha_2+\beta_2)\eta_{h,k}+(\alpha_3+\beta_3)r-(\alpha_4+\beta_4)\xi-1}d\xi$$
$${}^{(\Gamma)}\mathrm{I}_{g_{l+2},h_{l+1},s}^{e,f+2}\left[\kappa_4 Z^{(\alpha_4+\beta_4)}\left|\begin{array}{c}(V_1,A_1,x),(1-\sigma-\alpha_1 l-\alpha_2\eta_{h,k}-\alpha_3 r;\alpha_4)\\(1-\rho-\beta_1 l-\beta_2\eta_{h,k}-\beta_3 r;\beta_4)(V_j,A_j)_{2,f}(V_{jl},A_{jl})_{f+1,g_l}\\(W_j,G_j)_{1,e},(W_{jl},G_{jl})_{e+1,h_l}\,(1-\sigma-\rho-(\alpha_1+\beta_1)l\\-(\alpha_2+\beta_2)m-(\alpha_3+\beta_3)r);(\alpha_4+\beta_4)\end{array}\right.\right].$$

□

Theorem 2.6.

$$\int_0^1\int_0^1 f(uv)(1-u)^{\sigma-1}(1-v)^{\rho-1}v^{\sigma}E_{v,\omega}^{\Psi,\gamma}\left[\kappa_1(1-u)^{\alpha_1}(1-v)^{\beta_1}v^{\alpha_1}\right]I_{p_i',q_i',r'}^{m_i',n_i'}\left[\kappa_2(1-u)^{\alpha_2}(1-v)^{\beta_2}v^{\alpha_2}\right]$$
$$\times S_N^M\left[\kappa_3(1-u)^{\alpha_3}(1-v)^{\beta_3}v^{\alpha_3}\right]{}^{(\gamma)}I_{g_l,h_l,s}^{e,f}\left[\kappa_4(1-u)^{\alpha_4}(1-v)^{\beta_4}v^{\alpha_4}\right]dudv$$
$$=\sum_{r=0}^{[N/M]}\sum_{h=1}^{m'}\sum_{l,k=0}^{\infty}\frac{B_\alpha^{\beta,\lambda}(\Psi+l,\gamma-\Psi)(\gamma)_l}{B(\Psi,\gamma-\Psi)\Gamma(vl+\omega)}\frac{(-1)^k\phi'(\eta_{h,k})}{\beta_h k!}\frac{(-N)_{M_r}}{r!}\zeta_{N,r}(\kappa)^l(\kappa_2)^{\eta_{h,k}}(\kappa_3)^r$$
$$\times\int_0^1 f(\xi)(1-\xi)^{\sigma+\rho+(\alpha_1+\beta_1)l+(\alpha_2+\beta_2)\eta_{h,k}+(\alpha_3+\beta_3)r-(\alpha_4+\beta_4)\xi-1}d\xi$$
$${}^{(\gamma)}\mathrm{I}_{g_{l+2},h_{l+1},s}^{e,f+2}\left[\kappa_4 Z^{(\alpha_4+\beta_4)}\left|\begin{array}{c}(V_1,A_1,x),(1-\sigma-\alpha_1 l-\alpha_2\eta_{h,k}-\alpha_3 r;\alpha_4)\\(1-\rho-\beta_1 l-\beta_2\eta_{h,k}-\beta_3 r;\beta_4)(V_j,A_j)_{2,f}(V_{jl},A_{jl})_{f+1,g_l}\\(W_j,G_j)_{1,e},(W_{jl},G_{jl})_{e+1,h_l}\,(1-\sigma-\rho-(\alpha_1+\beta_1)l\\-(\alpha_2+\beta_2)m-(\alpha_3+\beta_3)r);(\alpha_4+\beta_4)\end{array}\right.\right].$$

where all the parameters are real/complex numbers satisfying the convergence conditions. Provided that:
$\mathrm{Re}\left(\sigma+1+\alpha_1\mathrm{l}+\alpha_2\eta_{\mathrm{h,k}}+\alpha_3\mathrm{r}+\alpha_4\frac{W_j}{G_j}\right)>0,\mathrm{Re}\left(\alpha+1+\alpha_1\mathrm{l}+\alpha_2\eta_{\mathrm{h,k}}+\alpha_3\mathrm{r}+\alpha_4\frac{W_j}{G_j}\right)>0.$
$\mathrm{Re}\left(\rho+\beta_1\mathrm{l}+\beta_2\eta_{\mathrm{h,k}}+\beta_3\mathrm{r}+\beta_4\frac{W_j}{G_j}\right)>0.$
$|\mathrm{argk}_2|\le\frac{1}{2}T\pi, T>0,$ *where* $j=1,2,\ldots,e;(\rho,\sigma,\alpha_1,\beta_1,\alpha_2,\beta_2,\alpha_3,\beta_3,\alpha_4,\beta_4)>0.$

Proof. We have

$$E_{v,\omega}^{\Psi,\gamma}\left[\kappa_1(1-u)^{\alpha_1}(1-v)^{\beta_1}(v)^{\alpha_1}\right]I_{p_i',q_i',r'}^{m_i',n_i'}\left[\kappa_2(1-u)^{\alpha_2}(1-v)^{\beta_2}(v)^{\alpha_2}\right]$$
$$\times S_N^M\left[\kappa_3(1-u)^{\alpha_3}(1-v)^{\beta_3}(v)^{\alpha_3}\right]{}^{(\gamma)}I_{p_l,q_l,s}^{m,n}\left[\kappa_4(1-u)^{\alpha_4}(1-v)^{\beta_4}(v)^{\alpha_4}\right]$$
$$=\sum_{l=0}^{\infty}\frac{B_\alpha^{\beta,\lambda}(\Psi+l,\gamma-\Psi)(\gamma)_l}{B(\Psi,\gamma-\Psi)\Gamma(vl+\omega)}\left\{\kappa_1(1-u)^{\alpha_1}(1-v)^{\beta_1}(v)^{\alpha_1}\right\}^l$$
$$\times\sum_{h=1}^{m'}\sum_{k=0}^{\infty}\frac{(-1)^k\phi'(\eta_{h,k})}{\beta_h k!}\left\{\kappa_2(1-u)^{\alpha_2}(1-v)^{\beta_2}(v)^{\alpha_2}\right\}^{\eta_{h,k}}$$

$$\sum_{r=0}^{[N/M]} \frac{(-N)_{M_r}}{r!} \zeta_{N,r} \left\{ \kappa_3 (1-u)^{\alpha_3} (1-v)^{\beta_3} (v)^{\alpha_3} \right\}^r \frac{1}{2\pi i} \int_{\Omega} L(\xi, x) \left\{ \kappa_4 (1-u)^{\alpha_4} (1-v)^{\beta_4} (v)^{\alpha_4} \right\}^{-\xi} d\xi. \tag{2.6}$$

Eq. (2.6) Multiplying by $f(uv)(1-u)^{\sigma-1}(1-v)^{\rho-1}(v)^{\sigma}$ and integrated w.r.t. u and v over the interval [0,1], we obtain the required integral representation. Since the involved series are uniformly convergent under the given conditions, the order of summation and integration can be interchanged. We obtain

$$\int_0^1 \int_0^1 f(uv)(1-u)^{\sigma-1}(1-v)^{\rho-1}(v)^{\sigma} E_{v,\omega}^{\Psi,\gamma} \left[\kappa_1 (1-u)^{\alpha_1} (1-v)^{\beta_1} v^{\alpha_1} \right] I_{p_i',q_i',r'}^{m_i',n_i'} \left[\kappa_2 (1-u)^{\alpha_2} (1-v)^{\beta_2} v^{\alpha_2} \right]$$

$$\times S_N^M \left[\kappa_3 (1-u)^{\alpha_3} (1-v)^{\beta_3} v^{\alpha_3} \right] {}^{(\gamma)}I_{g_l,h_l,s}^{e,f} \left[\kappa_4 (1-u)^{\alpha_4} (1-v)^{\beta_4} v^{\alpha_4} \right] du dv$$

$$= \int_0^1 \int_0^1 f(uv)(1-u)^{\sigma-1}(1-v)^{\rho-1} v^{\sigma} \sum_{l=0}^{\infty} \frac{B_{\alpha}^{\beta,\lambda}(\Psi+l, \gamma-\Psi)(\gamma)_l}{B(\Psi, \gamma-\Psi)\Gamma(vl+\omega)} \left\{ \kappa_1 (1-u)^{\alpha_1} (1-v)^{\beta_1} v^{\alpha_1} \right\}^l$$

$$\times \sum_{h=1}^{m'} \sum_{k=0}^{\infty} \frac{(-1)^k \phi'(\eta_{h,k})}{\beta_h k!} \left\{ \kappa_2 (1-u)^{\alpha_2} (1-v)^{\beta_2} v^{\alpha_2} \right\}^{\eta_{h,k}} \sum_{r=0}^{[N/M]} \frac{(-N)_{M_r}}{r!} \zeta_{N,r} \left\{ \kappa_3 (1-u)^{\alpha_3} (1-v)^{\beta_3} v^{\alpha_3} \right\}^r$$

$$\times \frac{1}{2\pi i} \int_{\Omega} L(\xi, x) \left\{ \kappa_4 (1-u)^{\alpha_4} (1-v)^{\beta_4} v^{\alpha_4} \right\}^{-\xi} d\xi du dv$$

$$= \sum_{r=0}^{[N/M]} \sum_{h=1}^{m'} \sum_{l,k=0}^{\infty} \frac{B_{\alpha}^{\beta,\lambda}(\Psi+l, \gamma-\Psi)(\gamma)_l}{B(\Psi, \gamma-\Psi)\Gamma(vl+\omega)} \frac{(-1)^k \phi'(\eta_{h,k})}{\beta_h k!} \frac{(-N)_{M_r}}{r!} \zeta_{N,r} (\kappa)^l (\kappa_2)^{\eta_{h,k}} (\kappa_3)^r$$

$$\times \frac{1}{2\pi i} \int_{-i\infty}^{+i\infty} L(\xi, x) (\kappa_4)^{-\xi} d\xi$$

$$\times \left[\int_0^1 \int_0^1 f(uv)(1-u)^{\sigma+\alpha_1 l+\alpha_2 \eta_{h,k}+\alpha_3 r-\alpha_4 \xi-1} (1-v)^{\rho+\beta_1 l+\beta_2 \eta_{h,k}+\beta_3 r+\beta_4 \xi-1} v^{\sigma+\alpha_1 l+\alpha_2 \eta_{h,k}+\alpha_3 r-\alpha_4 \xi} du dv \right]$$

$$= \sum_{r=0}^{[N/M]} \sum_{h=1}^{m'} \sum_{l,k=0}^{\infty} \frac{B_{\alpha}^{\beta,\lambda}(\Psi+l, \gamma-\Psi)(\gamma)_l}{B(\Psi, \gamma-\Psi)\Gamma(vl+\omega)} \frac{(-1)^k \phi'(\eta_{h,k})}{\beta_h k!} \frac{(-N)_{M_r}}{r!} \zeta_{N,r} (\kappa)^l (\kappa_2)^{\eta_{h,k}} (\kappa_3)^r$$

$$\times \int_0^1 f(\xi)(1-\xi)^{\sigma+\rho+(\alpha_1+\beta_1)l+(\alpha_2+\beta_2)\eta_{h,k}+(\alpha_3+\beta_3)r-(\alpha_4+\beta_4)\xi-1} d\xi$$

$${}^{(\gamma)}\mathrm{I}_{g_{l+2},h_{l+1},s}^{e,f+2} \left[\kappa_4 Z^{(\alpha_4+\beta_4)} \left| \begin{array}{c} (V_1, A_1, x), (1-\sigma-\alpha_1 l-\alpha_2 \eta_{h,k}-\alpha_3 r; \alpha_4) \\ (1-\rho-\beta_1 l-\beta_2 \eta_{h,k}-\beta_3 r; \beta_4)\, (V_j, A_j)_{2,f}\, (V_{jl}, A_{jl})_{f+1,g_l} \\ (W_j, G_j)_{1,e}, (W_{jl}, G_{jl})_{e+1,h_l}\, (1-\sigma-\rho-(\alpha_1+\beta_1)\, l \\ -(\alpha_2+\beta_2)\, m-(\alpha_3+\beta_3)\, r); (\alpha_4+\beta_4) \end{array} \right. \right].$$

□

Theorem 2.7.

$$\int_0^1 \int_0^1 \left(\frac{1-x}{1-xy} y \right)^{\alpha+\sigma} \left(\frac{1-y}{1-xy} \right)^{\beta} \left(\frac{1}{(1-x)} \right) p_n^{(\alpha,\beta)} \left[1-2\kappa_1 \left(\frac{1-x}{1-xy} y \right)^{\alpha_1} \left(\frac{1-y}{1-xy} \right)^{\beta_1} \right]$$

$$\times E_{v,\omega}^{\Psi,\gamma} \left[\kappa_2 \left(\frac{1-x}{1-xy} y \right)^{\alpha_2} \left(\frac{1-y}{1-xy} \right)^{\beta_2} \right] I_{p_i',q_i',r'}^{m_i',n_i'} \left[\kappa_3 \left(\frac{1-x}{1-xy} y \right)^{\alpha_3} \left(\frac{1-y}{1-xy} \right)^{\beta_3} \right]$$

$$S_N^M \left[\kappa_4 \left(\frac{1-x}{1-xy} y \right)^{\alpha_4} \left(\frac{1-y}{1-xy} \right)^{\beta_4} \right] {}^{(\Gamma)}I_{g_l,h_l,s}^{e,f} \left[\kappa_5 \left(\frac{1-x}{1-xy} y \right)^{\alpha_5} \left(\frac{1-x}{1-xy} y \right)^{\beta_5} \right] dx dy$$

$$
= \sum_{r=0}^{[N/M]} \sum_{h=1}^{m'} \sum_{l,k=0}^{\infty} \frac{B_{\alpha}^{\beta,\lambda}(\Psi+l,\gamma-\Psi)(\gamma)_l}{B(\Psi,\gamma-\Psi)\Gamma(vl+\omega)} \zeta_{N,r} \frac{(-1)^k \phi'(\eta_{h,k})}{\beta_h k!} \frac{(-N)_{M_r}}{r!} \frac{(-\eta)_{k'}(\alpha+\beta+n+1)(1+\alpha)_n}{n!(1+\alpha)_{k'} k'!}
$$
$$
\times {}_2F_1\left[-n,\alpha+\beta+n+1;1+\alpha;\kappa_1\right]
$$
$$
{}^{(\Gamma)}\mathrm{I}^{e,f+2}_{g_{l+2},h_{l+1},s}\left[\kappa_5 Z^{(\alpha_5+\beta_5)} \left| \begin{array}{c} (V_1,A_1,x),(1-\alpha-\sigma-\alpha_1k'-\alpha_2 l-\alpha_3\eta_{h,k}-\alpha_4 r;\alpha_5) \\ (-\beta-\beta_1k'-\beta_2l-\beta_{3,h,k}-\beta_4r;\beta_5)\,(V_j,A_j)_{1,f}\,(V_{jl},A_{jl})_{f+1,g_l} \\ (W_j,G_j)_{1,e},(W_{jl},G_{jl})_{e+1,h_l}\,(-\alpha-\sigma-\rho-(\alpha_1+\beta_1)k' \\ -(\alpha_2+\beta_2)l-(\alpha_3+\beta_3)m)-(\alpha_4+\beta_4)r);(\alpha_5+\beta_5) \end{array}\right.\right].
$$

where all the parameters are real/complex numbers satisfying the convergence conditions.
Provided that:
$\mathrm{Re}\left(\alpha+\sigma+\alpha_1k'+\alpha_2\mathrm{l}+\alpha_3\eta_{\mathrm{h,k}}+\alpha_4\mathrm{r}+\alpha_5\frac{W_j}{G_j}\right)>0,$
$\mathrm{Re}\left(\beta+\beta_1k'+\beta_2\mathrm{l}+\beta_3\eta_{\mathrm{h,k}}+\beta_4\mathrm{r}+\beta_5\frac{W_j}{G_j}\right)>0,$
$|\mathrm{argk}_5|<\frac{1}{2}T\pi, T>0,$ *where* $j=1,2,\ldots,e;(\rho,\sigma,\alpha_1,\beta_1,\alpha_2,\beta_2,\alpha_3,\beta_3,\alpha_4,\beta_4)>0.$

Proof. We have

$$
p_n^{(\alpha,\beta)}\left[1-2\kappa_1\left(\frac{1-x}{1-xy}y\right)^{\alpha_1}\left(\frac{1-y}{1-xy}\right)^{\beta_1}\right] E^{\Psi,\gamma}_{v,\omega}\left[\kappa_2\left(\frac{1-x}{1-xy}y\right)^{\alpha_2}\left(\frac{1-y}{1-xy}\right)^{\beta_2}\right]
$$
$$
I^{m'_i,n'_i}_{p'_i,q'_i,r'}\left[\kappa_3\left(\frac{1-x}{1-xy}y\right)^{\alpha_3}\left(\frac{1-y}{1-xy}\right)^{\beta_3}\right] S_N^M\left[\kappa_4\left(\frac{1-x}{1-xy}y\right)^{\alpha_4}\left(\frac{1-y}{1-xy}\right)^{\beta_4}\right]
$$
$$
\times {}^{(\Gamma)}I^{e,f}_{g_l,h_l,s}\left[\kappa_5\left(\frac{1-x}{1-xy}y\right)^{\alpha_5}\left(\frac{1-y}{1-xy}\right)^{\beta_5}\right]
$$
$$
= \sum_{k'=0}^{\infty}\frac{(-\eta)_{k'}(\alpha+\beta+n+1)_{k'}(1+\alpha)_n}{n!(1+\alpha)_{k'}k'!}(\kappa_1)^{k'}\left(\frac{1-x}{1-xy}y\right)^{\alpha_1 k'}\left(\frac{1-y}{1-xy}\right)^{\beta_1 k'}
$$
$$
\times\sum_{l=0}^{\infty}\frac{B_{\alpha}^{\beta,\lambda}(\Psi+l,\gamma-\Psi)(\gamma)_l}{B(\Psi,\gamma-\Psi)\Gamma(vl+\omega)}\left\{\kappa_2\left(\frac{1-x}{1-xy}y\right)^{\alpha_2}\left(\frac{1-y}{1-xy}\right)^{\beta_2}\right\}^l
$$
$$
\times\sum_{h=1}^{m'}\sum_{k=0}^{\infty}\frac{(-1)^k\phi'(\eta_{h,k})}{\beta_h k!}\left\{\kappa_3\left(\frac{1-x}{1-xy}y\right)^{\alpha_3}\left(\frac{1-y}{1-xy}\right)^{\beta_3}\right\}^{\eta_{h,k}}
$$
$$
\sum_{r=0}^{[N/M]}\frac{(-N)_{M_r}}{r!}\zeta_{N,r}\left\{\kappa_4\left(\frac{1-x}{1-xy}y\right)^{\alpha_4}\left(\frac{1-y}{1-xy}\right)^{\beta_4}\right\}^r
$$
$$
\times\frac{1}{2\pi i}\int_{\Omega}K(\xi,x)\left\{\kappa_5\left(\frac{1-x}{1-xy}y\right)^{\alpha_5}\left(\frac{1-y}{1-xy}\right)^{\beta_5}\right\}^{-\xi}d\xi. \tag{2.7}
$$

Eq. (2.7) is multiplied by $\left(\frac{1-x}{1-xy}y\right)^{\alpha+\sigma}\left(\frac{1-y}{1-xy}\right)^{\beta}\left(\frac{1}{(1-x)}\right)$ and integrating w.r.t. x and y over the interval[0,1],we otained required integral representaion. Since the involved series are uniformly convergent under the given conditions,interchanging the order of integration and summation. We get

$$
\int_0^1\int_0^1\left(\frac{1-x}{1-xy}y\right)^{\alpha+\sigma}\left(\frac{1-y}{1-xy}\right)^{\beta}\left(\frac{1}{(1-x)}\right)p_n^{(\alpha,\beta)}\left[1-2\kappa_1\left(\frac{1-x}{1-xy}y\right)^{\alpha_1}\left(\frac{1-y}{1-xy}\right)^{\beta_1}\right]
$$
$$
\times E^{\Psi,\gamma}_{v,\omega}\left[\kappa_2\left(\frac{1-x}{1-xy}y\right)^{\alpha_2}\left(\frac{1-y}{1-xy}\right)^{\beta_2}\right] I^{m'_i,n'_i}_{p'_i,q'_i,r'}\left[\kappa_3\left(\frac{1-x}{1-xy}y\right)^{\alpha_3}\left(\frac{1-y}{1-xy}\right)^{\beta_3}\right]
$$

$$S_N^M\left[\kappa_4\left(\frac{1-x}{1-xy}y\right)^{\alpha_4}\left(\frac{1-y}{1-xy}\right)^{\beta_4}\right]{}^{(\Gamma)}I^{e,f}_{g_l,h_l,s}\left[\kappa_5\left(\frac{1-x}{1-xy}y\right)^{\alpha_5}\left(\frac{1-x}{1-xy}y\right)^{\beta_5}\right]dxdy$$

$$=\sum_{r=0}^{[N/M]}\sum_{h=1}^{m'}\sum_{l,k=0}^{\infty}\zeta_{N,r}\frac{(-1)^k\phi'(\eta_{h,k})}{\beta_h k!}\frac{(-N)_{M_r}}{r!}\frac{B_\alpha^{\beta,\lambda}(\Psi+l,\gamma-\Psi)(\gamma)_l}{B(\Psi,\gamma-\Psi)\Gamma(vl+\omega)}\frac{(-\eta)_{k'}(\alpha+\beta+n+1)(1+\alpha)_n}{n!(1+\alpha)_{k'}k'!}$$

$$\times(\kappa_1)^{k'}(\kappa_2)^l(\kappa_3)^{\eta_{h,k}}(\kappa_4)^r\frac{1}{2\pi i}\int_{-\mathrm{i}\infty}^{+i\infty}K(\xi,x)(a_5)^{-\xi}d\xi$$

$$\times\left[\int_0^1\int_0^1\left(\frac{1-x}{1-xy}y\right)^{\alpha+\sigma+\alpha_1k'+\alpha_2l+\alpha_3\eta_{h,k}+\alpha_4r-\alpha_5\xi}\left(\frac{1-y}{1-xy}\right)^{\beta+\beta_1k'+\beta_2l+\beta_3\eta_{h,k}+\beta_4r-\beta_5\xi}\left(\frac{1}{(1-x)}\right)dxdy\right].$$

$$=\sum_{r=0}^{[N/M]]}\sum_{h=1}^{m'}\sum_{l,k=0}^{\infty}\frac{B_\alpha^{\beta,\lambda}(\Psi+l,\gamma-\Psi)(\gamma)_l}{B(\Psi,\gamma-\Psi)\Gamma(vl+\omega)}\zeta_{N,r}\frac{(-1)^k\phi'(\eta_{h,k})}{\beta_h k!}\frac{(-N)_{M_r}}{r!}\frac{(-\eta)_{k'}(\alpha+\beta+n+1)(1+\alpha)_n}{n!(1+\alpha)_{k'}k'!}$$

$$\times{}_2F_1[-n,\alpha+\beta+n+1;1+\alpha;\kappa_1]$$

$${}^{(\Gamma)}\mathrm{I}^{e,f+2}_{g_{l+2},h_{l+1},s}\left[\kappa_5Z^{(\alpha_5+\beta_5)}\left|\begin{array}{c}(V_1,A_1,x),(1-\alpha-\sigma-\alpha_1k'-\alpha_2l-\alpha_3\eta_{h,k}-\alpha_4r;\alpha_5)\\(-\beta-\beta_1k'-\beta_2l-\beta_{3,h,k}-\beta_4r;\beta_5)(V_j,A_j)_{1,f}(V_{jl},A_{jl})_{f+1,g_l}\\(W_j,G_j)_{1,e},(W_{jl},G_{jl})_{e+1,h_l}(-\alpha-\sigma-\rho-(\alpha_1+\beta_1)k'-(\alpha_2+\beta_2)l\\-(\alpha_3+\beta_3)m)-(\alpha_4+\beta_4)r);(\alpha_5+\beta_5)\end{array}\right.\right].$$

□

Theorem 2.8.

$$\int_0^1\int_0^1\left(\frac{1-x}{1-xy}y\right)^{\alpha+\sigma}\left(\frac{1-y}{1-xy}\right)^{\beta}\left(\frac{1}{(1-x)}\right)p_n^{(\alpha,\beta)}\left[1-2\kappa_1\left(\frac{1-x}{1-xy}y\right)^{\alpha_1}\left(\frac{1-y}{1-xy}\right)^{\beta_1}\right]$$

$$\times E^{\Psi,\gamma}_{v,\omega}\left[\kappa_2\left(\frac{1-x}{1-xy}y\right)^{\alpha_2}\left(\frac{1-y}{1-xy}\right)^{\beta_2}\right]I^{m'_i,n'_i}_{p'_i,q'_i,r'}\left[\kappa_3\left(\frac{1-x}{1-xy}y\right)^{\alpha_3}\left(\frac{1-y}{1-xy}\right)^{\beta_3}\right]$$

$$S_N^M\left[\kappa_4\left(\frac{1-x}{1-xy}y\right)^{\alpha_4}\left(\frac{1-y}{1-xy}\right)^{\beta_4}\right]{}^{(\gamma)}I^{e,f}_{g_l,h_l,s}\left[\kappa_5\left(\frac{1-x}{1-xy}y\right)^{\alpha_5}\left(\frac{1-x}{1-xy}y\right)^{\beta_5}\right]dxdy$$

$$=\sum_{r=0}^{[N/M]}\sum_{h=1}^{m'}\sum_{l,k=0}^{\infty}\frac{B_\alpha^{\beta,\lambda}(\Psi+l,\gamma-\Psi)(\gamma)_l}{B(\Psi,\gamma-\Psi)\Gamma(vl+\omega)}\zeta_{N,r}\frac{(-1)^k\phi'(\eta_{h,k})}{\beta_h k!}\frac{(-N)_{M_r}}{r!}\frac{(-\eta)_{k'}(\alpha+\beta+n+1)(1+\alpha)_n}{n!(1+\alpha)_{k'}k'!}$$

$$\times{}_2F_1[-n,\alpha+\beta+n+1;1+\alpha;\kappa_1]$$

$${}^{(\gamma)}\mathrm{I}^{e,f+2}_{g_{l+2},h_{l+1},s}\left[\kappa_5Z^{(\alpha_5+\beta_5)}\left|\begin{array}{c}(V_1,A_1,x),(1-\alpha-\sigma-\alpha_1k'-\alpha_2l-\alpha_3\eta_{h,k}-\alpha_4r;\alpha_5)\\(-\beta-\beta_1k'-\beta_2l-\beta_{3,h,k}-\beta_4r;\beta_5)(V_j,A_j)_{1,f}(V_{jl},A_{jl})_{f+1,g_l}\\(W_j,G_j)_{1,e},(W_{jl},G_{jl})_{e+1,h_l}(-\alpha-\sigma-\rho-(\alpha_1+\beta_1)k'-(\alpha_2+\beta_2)l\\-(\alpha_3+\beta_3)m-)(\alpha_4+\beta_4)r);(\alpha_5+\beta_5)\end{array}\right.\right].$$

where all the parameters are real/complex numbers satisfying the convergence conditions.
Provided that:
$\mathrm{Re}\left(\alpha+\sigma+\alpha_1k'+\alpha_2\mathrm{l}+\alpha_3\eta_{\mathrm{h,k}}+\alpha_4\mathrm{r}+\alpha_5\frac{W_j}{G_j}\right)>0,$
$\mathrm{Re}\left(\beta+\beta_1k'+\beta_2\mathrm{l}+\beta_3\eta_{\mathrm{h,k}}+\beta_4\mathrm{r}+\beta_5\frac{W_j}{G_j}\right)>0,$
$|\mathrm{argk}_5|<\frac{1}{2}T\pi, T>0,$ *where* $j=1,2,\ldots,e;(\rho,\sigma,\alpha_1,\beta_1,\alpha_2,\beta_2,\alpha_3,\beta_3,\alpha_4,\beta_4)>0.$

Proof.

$$
\begin{aligned}
&p_n^{(\alpha,\beta)}\left[1-2\kappa_1\left(\frac{1-x}{1-xy}y\right)^{\alpha_1}\left(\frac{1-y}{1-xy}\right)^{\beta_1}\right]E_{v,\omega}^{\Psi,\gamma}\left[\kappa_2\left(\frac{1-x}{1-xy}y\right)^{\alpha_2}\left(\frac{1-y}{1-xy}\right)^{\beta_2}\right]\\
&I_{p_i',q_i',r'}^{m_{,}',n_i'}\left[\kappa_3\left(\frac{1-x}{1-xy}y\right)^{\alpha_3}\left(\frac{1-y}{1-xy}\right)^{\beta_3}\right]S_N^M\left[\kappa_4\left(\frac{1-x}{1-xy}y\right)^{\alpha_4}\left(\frac{1-y}{1-xy}\right)^{\beta_4}\right]\\
&\times\,^{(\gamma)}I_{g_l,h_l,s}^{e,f}\left[\kappa_5\left(\frac{1-x}{1-xy}y\right)^{\alpha_5}\left(\frac{1-y}{1-xy}\right)^{\beta_5}\right]\\
&=\sum_{k'=0}^{\infty}\frac{(-\eta)_{k'}(\alpha+\beta+n+1)_{k'}(1+\alpha)_n}{n!(1+\alpha)_{k'}k'!}(\kappa_1)^{k'}\left(\frac{1-x}{1-xy}y\right)^{\alpha_1k'}\left(\frac{1-y}{1-xy}\right)^{\beta_1k'}\\
&\times\sum_{l=0}^{\infty}\frac{B_\alpha^{\beta,\lambda}(\Psi+l,\gamma-\Psi)(\gamma)_l}{B(\Psi,\gamma-\Psi)\Gamma(vl+\omega)}\left\{\kappa_2\left(\frac{1-x}{1-xy}y\right)^{\alpha_2}\left(\frac{1-y}{1-xy}\right)^{\beta_2}\right\}^l\\
&\times\sum_{h=1}^{m'}\sum_{k=0}^{\infty}\frac{(-1)^k\phi'(\eta_{h,k})}{\beta_hk!}\left\{\kappa_3\left(\frac{1-x}{1-xy}y\right)^{\alpha_3}\left(\frac{1-y}{1-xy}\right)^{\beta_3}\right\}^{\eta_{h,k}}\\
&\times\sum_{r=0}^{[N/M]}\frac{(-N)_{M_r}}{r!}\zeta_{N,r}\left\{\kappa_4\left(\frac{1-x}{1-xy}y\right)^{\alpha_4}\left(\frac{1-y}{1-xy}\right)^{\beta_4}\right\}^r\\
&\times\frac{1}{2\pi i}\int_\Omega L(\xi,x)\left\{\kappa_5\left(\frac{1-x}{1-xy}y\right)^{\alpha_5}\left(\frac{1-y}{1-xy}\right)^{\beta_5}\right\}^{-\xi}d\xi.
\end{aligned}
\tag{2.8}
$$

Eq. (2.7) is multiplied by $\left(\frac{1-x}{1-xy}y\right)^{\alpha+\sigma}\left(\frac{1-y}{1-xy}\right)^{\beta}\left(\frac{1}{(1-x)}\right)$ and integrating w.r.t. x and y over the interval [0,1], we otained required integral representaion. Since the involved series are uniformly convergent under the given conditions, interchanging the order of integration and summation. We get

$$
\begin{aligned}
&\int_0^1\int_0^1\left(\frac{1-x}{1-xy}y\right)^{\alpha+\sigma}\left(\frac{1-y}{1-xy}\right)^{\beta}\left(\frac{1}{(1-x)}\right)p_n^{(\alpha,\beta)}\left[1-2\kappa_1\left(\frac{1-x}{1-xy}y\right)^{\alpha_1}\left(\frac{1-y}{1-xy}\right)^{\beta_1}\right]\\
&\times E_{v,\omega}^{\Psi,\gamma}\left[\kappa_2\left(\frac{1-x}{1-xy}y\right)^{\alpha_2}\left(\frac{1-y}{1-xy}\right)^{\beta_2}\right]I_{p_i',q_i',r'}^{m_i',n_i'}\left[\kappa_3\left(\frac{1-x}{1-xy}y\right)^{\alpha_3}\left(\frac{1-y}{1-xy}\right)^{\beta_3}\right]\\
&S_N^M\left[\kappa_4\left(\frac{1-x}{1-xy}y\right)^{\alpha_4}\left(\frac{1-y}{1-xy}\right)^{\beta_4}\right]{}^{(\gamma)}I_{g_l,h_l,s}^{e,f}\left[\kappa_5\left(\frac{1-x}{1-xy}y\right)^{\alpha_5}\left(\frac{1-x}{1-xy}y\right)^{\beta_5}\right]dxdy\\
&=\sum_{r=0}^{[N/M]}\sum_{h=1}^{m'}\sum_{l,k=0}^{\infty}\zeta_{N,r}\frac{(-1)^k\phi'(\eta_{h,k})}{\beta_hk!}\frac{(-N)_{M_r}}{r!}\frac{B_\alpha^{\beta,\lambda}(\Psi+l,\gamma-\Psi)(\gamma)_l}{B(\Psi,\gamma-\Psi)\Gamma(vl+\omega)}\frac{(-\eta)_{k'}(\alpha+\beta+n+1)(1+\alpha)_n}{n!(1+\alpha)_{k'}k'!}\\
&\times(\kappa_1)^{k'}(\kappa_2)^l(\kappa_3)^{\eta_{h,k}}(\kappa_4)^r\frac{1}{2\pi i}\int_{-\mathrm{i}\infty}^{+i\infty}L(\xi,x)(a_5)^{-\xi}d\xi\\
&\times\left[\int_0^1\int_0^1\left(\frac{1-x}{1-xy}y\right)^{\alpha+\sigma+\alpha_1k'+\alpha_2l+\alpha_3\eta_{h,k}+\alpha_4r-\alpha_5\xi}\left(\frac{1-y}{1-xy}\right)^{\beta+\beta_1k'+\beta_2l+\beta_3\eta_{h,k}+\beta_4r-\beta_5\xi}\left(\frac{1}{(1-x)}\right)dxdy\right]\\
&=\sum_{r=0}^{[N/M]}\sum_{h=1}^{m'}\sum_{l,k=0}^{\infty}\frac{B_\alpha^{\beta,\lambda}(\Psi+l,\gamma-\Psi)(\gamma)_l}{B(\Psi,\gamma-\Psi)\Gamma(vl+\omega)}\zeta_{N,r}\frac{(-1)^k\phi'(\eta_{h,k})}{\beta_hk!}\frac{(-N)_{M_r}}{r!}\frac{(-\eta)_{k'}(\alpha+\beta+n+1)(1+\alpha)_n}{n!(1+\alpha)_{k'}k'!}\\
&\times{}_2F_1\left[-n,\alpha+\beta+n+1;1+\alpha;\kappa_1\right]
\end{aligned}
$$

$$
{}^{(\gamma)}\mathrm{I}^{e,f+2}_{g_{l+2},h_{l+1},s}\left[\begin{array}{c|c} \kappa_5 Z^{(\alpha_5+\beta_5)} & \begin{array}{c} (V_1, A_1, x), (1-\alpha-\sigma-\alpha_1 k'-\alpha_2 l-\alpha_3\eta_{h,k}-\alpha_4 r;\alpha_5) \\ (-\beta-\beta_1 k'-\beta_2 l-\beta_{3,h,k}-\beta_4 r;\beta_5)\,(V_j,A_j)_{1,n}\,(V_{jl},A_{jl})_{n+1,g_l} \\ (W_j,G_j)_{1,e},(W_{jl},G_{jl})_{e+1,h_l}\,(-\alpha-\sigma-\rho-(\alpha_1+\beta_1)\,k'-(\alpha_2+\beta_2)\,l \\ -(\alpha_3+\beta_3)\,m)-(\alpha_4+\beta_4)r);(\alpha_5+\beta_5) \end{array}\end{array}\right].
$$

□

3. Special Case

1. On specializing the values of various parameters involved in Theorems 2.1 to 2.8, then similar results can be obtained by Chaurasia and Shekhawat [5].
2. On giving some particular values of various parameters involved in Theorems 2.1 to 2.8, then similar results can be computed by Chaurasia and Singh [6].

4. Conclusions

In this paper, advanced results in integral calculus were established by using combination of the GEMLF, incomplete I-function, GCP, and I-function. At the end of our work, we emphasise that the results investigated can be utilised to create a variety of beneficial properties and outcomes of the GEMLF, incomplete I-function, GCP, and I-function. Furthermore, we may reach a number of intriguing and important conclusions by carefully selecting special values for the individual specification in the generalised extended Mittag-Leffler function, incomplete I-function, general class of polynomials, and I-function. Consequently, it is concluded that we reached in this work would immediately provide a significant number of special function insights, including various results discovered in natural science.

References

[1] M. K. Bansal and D. Kumar, *On the integral operators pertaining to a family of incomplete I-functions*, AIMS Math, *5*(2) (2020), 1247–1259.

[2] S. Bhatter, Nishant, Shyamsunder, S. D. Purohit and D. L. Suthar, *A study of incomplete I-functions relating to certain fractional integral operators*, Applied Mathematics in Science and Engineering, *31*(1) (2023), 2252996.

[3] S. Bhatter, Nishant, S. D. Purohit and D. L. Suthar, *Boros integral involving the product of special function and the incomplete I-Function*, Boletim da Sociedade Paranaense de Matemática, *43* (2025).

[4] A. Chaudhary, D. Kumar, and J. Singh , *Some fractional calculus results pertaining to Mittag-Leffler type functions*, J. Appl. Math. Statistics Informatics, *13*(1) (2017), 31–48.

[5] V. B. L. Chaurasia and A. S. Shekhawat, *Some integral properties of a general class polynomials associated with Feynman integrals*, Bull. Malays. Math. Sci. Soc. , *28*(2) (2005), 1–7.

[6] V. B. L. Chaurasia and J. Singh, *Certain integral properties pertaining to Special functions*, Math. Sci. , *19* (2010), 1–6.

[7] C. Fox, *The G and H-functions as symmetrical Fourier kernels*, Trans. Amer. Math. Soc. , *98*(3) (1961), 395–429.

[8] S. D. Purohit, D. L. Suthar, A. Al-jarrah, V. K. Vyas, and K. S. Nisar,*Certain expansion formulae involving incomplete I-Functions*, TWMS J. App. and Eng. Math. , *14*(1) (2024), 402–409.

[9] V. P. Saxena,*Formal solution of certain new pair of dual integral equations involving H-functions*,Proc. Nat. Acad. Sci. India Sect. A, *52* (1982), 366–375.

[10] V. P. Saxena,*The I-function*, Anamaya Publishers, New Delhi, 2008.

[11] H. M. Srivastava,*A contour integral involving Fox's H-function*, Indian J. Math. , *14*(1) (1972), 1–6.