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Existence and uniqueness criteria for the solution of impulsive-implicit FDE with non-local operator

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Abstract

This article deals with the existence results and uniqueness criteria for impulsive-implicit FDE of order $0 < \beta < 1$, where authors obtained the results using Atangana-Baleanu derivative operator. The existence results and uniqueness criteria are obtained by using Krasnoselskii fixed point theorem and Banach Contraction principle. Further authors have obtained Hyer-Ulam stability. To show applicability of obtained results, a few examples are provided in details.

Keywords. Atangana Baleanu derivative operator, Krasnoselskii fixed point theorem, Banach contraction principle, and Hyer-Ulam stability.
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Uncorrected Proof

1. Introduction

The study of fractional differential equations has increased exponentially in the last several decades because of its vast range of applications in disciplines like chemistry, physics, biology, nuclear dynamics, etc. see in [23]. Due to their tremendous application, many researchers are using different derivative operators such as Riemann Liouvile, Hilfer, Caputo, Hadamard, Caputo-Fabrizio, Atangana-Baleanu, Riezs derivative operators [1, 15, 20, 26]. Fractional-order differential equations can now be solved using a variety of analytical and numerical methods. For example, Jassim et al. [11] suggested a hybrid method for solving fractional partial differential equations based on the homotopy analysis Sumudu transform method and illustrated its efficacy with a number of examples. Additionally, Mohammed et al. [18] looked into a higher-dimensional time-fractional Fokas equation and used several analytical techniques to find exact solutions. Now a days many researchers are applying Caputo-Fabrizio deivative operator for modeling various problems in engineering sciences because of their nonlocal property see in [2, 21, 33]. Furthermore, these type of derivatives are utilized in an exponential decay kernel to a novel HIV/AIDS epidemic model which includes an anti-retrovirus treatment compartment, optimal control and synchronization, dynamical system with chaotic, non-chaotic and hyper-chaotic behaviors, nonstandard finite difference scheme and non-identical synchronization of a novel fractional chaotic system one can see in [5–7, 19, 25]. In addition, the research has been done on existence, uniqueness and stability results. We refer to some of the articles [9, 17, 24, 29].

Additionally, the mathematical modeling of various real-world problem is studied by many researchers see in [22, 27, 31]. Also it is made easier by the inherent structure of impulsive differential equations. It is possible to measure the influence of the instantaneous model mutation and give a theoretical foundation for the practical application by taking the impulse effect into account in the continuous differential equation. Consequently, academics are also quite interested in solving impulsive fractional differential equation problems. Readers can review the theories of impulsive FDE by visiting [4, 12–14, 16, 28, 30].

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The following impulsive fractional differential equation was examined by Yang and Zhang in [32]

$$\begin{aligned} {}^{C}D^{\alpha}_{0+}\eta(p) &= G(p,\eta(p)), p \in J = (0,1), p \neq p_k, \\ \Delta\eta(p) &= \zeta(p_k) = I_k(\eta(p_k)), k = 1,2,3,\cdots,m, \\ \eta(0) &= g(\eta). \end{aligned} \tag{1.1}$$

The existence of solution for the class of fractional differential equations involving the Caputo-Fabrizio FD studied by Eiman et al. in [10].

$$\begin{aligned} {}^{CF}_{0}D^{\theta}_{l}z(l) &= F(l, z(l), {}^{CF}_{0}D^{\theta}_{l}z(l)), l \in [0,T], \\ z(0) &= z_0, z_0 \in \mathbb{R}. \end{aligned} \tag{1.2}$$

Inspired by the previously completed work, we used the Atangana Baleanu derivative operator to achieve results on the existence and uniqueness for the below mention Impulsive-Implicit FDE.

$$\begin{aligned} {}^{AB}_{0}D^{\beta}_{h}m(h) &= G(h, m(h), {}^{AB}_{0}D^{\beta}_{h}m(h)), h \in [0,a], h \neq h_k, \\ \Delta m(h_k) &= I_k(m(h_k)), k = 1,2,3,\cdots,n, \\ m(0) &= g(m). \end{aligned} \tag{1.3}$$

where $G : [0,a] \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}$, ${}^{AB}_{0}D^{\beta}_{h}$ is Atangana Baleanu derivative operator of order $0 < \beta < 1$, a continuous function $g : \mathbb{R} \to \mathbb{R}, I_k : \mathbb{R} \to \mathbb{R}$.
$\Delta m(h_k) = m(h_k^+) - m(h_k^-)$, $0 = h_0 < h_1 < \cdots < h_n < h_{n+1} = a$, right side of Eq. (1.3) vanish at $h = 0$.
"In this section, we provide several fundamental definitions and key theorems necessary for the development and analysis of the results derived from this study."

Definition 1.1 (Atangana Baleanu Fractional Derivative Operator)**.** [3] If $f(x) \in C^2[a,b]$ and $a < x < b$ then Atangana Baleanu Fractional Derivative is denoted by ${}^{AB}D^{\alpha}f(x)$ and is defined as follows

$${}^{AB}D^{\alpha}f(x) = \frac{M(\alpha)}{1-\alpha}\int_0^x E_{\alpha}\left(\frac{-\alpha(x-t)}{1-\alpha}\right) f'(t)dt.$$

Definition 1.2. Atangana Baleanu Fractional Integral Operator[3] If $f(x) \in C^2[a,b]$ and $a < x < b$ then Atangana Baleanu Fractional Integral is denoted by ${}^{AB}I^{\alpha}f(x)$ and is defined as follows

$${}^{AB}I^{\alpha}f(x) = \frac{1-\alpha}{M(\alpha)}f(x) + \frac{\alpha}{M(\alpha)\Gamma\alpha}\int_0^x f(s)(x-s)^{\alpha-1}ds.$$

Theorem 1.3. *[30] Let Y be non empty, convex and closed subset of X. Consider two operators T, S such that*

1. *$T(y_1) + S(y_2) \in Y, \forall y_1, y_2 \in Y$.*
2. *T is contraction operator.*
3. *S is continuous and compact. then there exists at least one solution $y \in X$ such that $T(y) + S(y) = y$.*

Definition 1.4 (H-U Stable)**.** The said problem Eq. (1.3) is H-U stable if any $\epsilon > 0$ for the given inequality

$$|{}^{AB}_{0}D^{\beta}_{t}m(h) - G(h, m(h), {}^{AB}_{0}D^{\beta}_{h}m(h))| \leq \epsilon, \forall\ h \in [0,a].$$

then, $\exists$ unique solution $\overline{m}(h)$ with a constant Z such that

$$|m(h) - \overline{m}(h)| \leq Z\epsilon, \forall\ h \in [0,a].$$

Remark 1.5. Suppose there exists a function $\Psi(h)$ which depend on $m \in Y$ with $\Psi(0) = 0$, $\Psi(a) = 0$ such that

1. $|\Psi(h)| \leq \epsilon,\ \forall\ h \in [0,a]$.
2. ${}^{AB}_{0}D^{\beta}m(h) = G(h, m(h), {}^{AB}_{0}D^{\beta}m(h)) + \Psi(h),\ 0 < \beta < 1,\ \forall h \in [0,a]$.

Lemma 1.6. *The unique solution of the given initial value problem*

$$\begin{aligned} {}^{AB}_{0}D^{\beta}_{h}m(h) &= F(h), 0 < \beta < 1, \\ m(0) &= m_0 \in \mathbb{R}, \end{aligned}$$

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is given by

$$m(h) = m_0 + (1-\beta)[F(h) - F(0)] + \beta \int_0^h (h-s)^{\beta-1} F(s)ds.$$

We are using the following notations throughout our work:

- Let $C([0,a],\mathbb{R})$ be space of all continuous function on $[0,a]$ such that $\|m\|_C = sup_{h\in[0,a]}|m(h)|$.
- Let $Y = PC([0,a],\mathbb{R})$ be a Banach space. Define $Y = \{m : T \to \mathbb{R}|\ m \in C((h_k, h_{k+1}], \mathbb{R}), k = 1, 2, \cdots, n\}$ and $\exists\ m(h_k^+), m(h_k^-), k = 1, 2, 3, \cdots, n.$ with the norm $\|m\|_{PC} = max_{h\in[0,a]}|m(h)|$.

2. Theorem environments

This section is dedicated to the principal analytical findings of the current study regarding the examined fractional differential equation. We start by finding an equivalent integral formulation of the problem, which is very important for the analysis that follows. This transformation allows fixed point methods to be used in the right function space. In this framework, the Banach contraction principle is used to find conditions that are strong enough for uniqueness. Then, Krasnoselskii's fixed point theorem is used to find conditions that are strong enough for existence. The subsequent lemma represents the preliminary step in this direction, offering an equivalent formulation that is essential for deriving the solution of the specified fractional differential equation.

Lemma 2.1. *Let $0 < \beta < 1$ and a continuous function $\eta : [0,a] \to \mathbb{R}$. A function $m \in Y$ is called solution of the given impulsive problem Eq. (2.1)*

$$\begin{aligned} {}^{AB}_{0}D^{\beta}_{h} m(h) &= \eta(h), \beta \in (0,1),\ h \in [0,a],\ h \neq h_k, \quad (2.1)\\ \Delta m(h_k) &= I_k(m(h_k)),\ k = 1,2,3,\cdots,n,\\ m(0) &= g(m). \quad (2.2)\end{aligned}$$

iff it satisfies

$$m(h) = \begin{cases} g(m) + (1-\beta)[\eta(h)-\eta(0)] + \beta\int_0^h (h-s)^{\beta-1}\eta(s)ds, h \in [0,h_1], \\ g(m) + (1-\beta)[\eta(h)-\eta(0)] + I_1(m(h_1^-)) + \beta\int_0^{h_1}(h_1-s)^{\beta-1}\eta(s)ds \\ +\beta\int_0^h (h-s)^{\beta-1}\eta(s)ds, & h \in [h_1,h_2], \\ g(m) + (1-\beta)[\eta(h)-\eta(0)] + I_1(m(h_1^-)) + I_2(m(h_2^-)) \\ +\beta\int_0^{h_1}(h_1-s)^{\beta-1}\eta(s)ds + \beta\int_0^{h_2}(h_2-s)^{\beta-1}\eta(s)ds + \beta\int_0^h(h-s)^{\beta-1}\eta(s)ds, & h \in [h_2,h_3], \\ \cdots \\ g(m) + (1-\beta)[\eta(h)-\eta(0)] + \sum_{l=1}^k I_l(m(h_l^-)) \\ +\beta\sum_{l=1}^k\int_0^{h_l}(h_l-s)^{\beta-1}\eta(s)ds + \beta\int_0^h(h-s)^{\beta-1}\eta(s)ds, & h \in [h_k,h_{k+1}], k = 1,2,\cdots,n. \end{cases} \quad (2.3)$$

Proof. Consider $m(h)$ satisfies Eq. (2.1)
If $h \in [0,h_1]$ then

$${}^{AB}_{0}D^{\beta}_{h} m(h) = \eta(h), h \in (0,h_1],\ \text{with } m(0) = g(m).$$

Using Lemma 1.6, we get the following

$$m(h) = g(m) + (1-\beta)[\eta(h)-\eta(0)] + \beta\int_0^h (h-s)^{\beta-1}\eta(s)ds,$$

After applying impulsive condition $m(h_1^-)$, we can find

$$m(h_1^-) = g(m) + (1-\beta)[\eta(h_1)-\eta(0)] + \beta\int_0^{h_1}(h_1-s)^{\beta-1}\eta(s)ds, \quad (2.4)$$

Now, if $h \in [h_1,h_2]$ then

$${}^{AB}_{0}D^{\beta}_{h} m(h) = \eta(h), h \in (h_1,h_2],$$

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with

$$m(h_1^+) = m(h_1^-) + I_1(m(h_1^-)),$$

Again using Lemma 1.6, we can find

$$m(h) = m(h_1^+) + (1-\beta)[\eta(h) - \eta(h_1)] + \beta \int_0^h (h-s)^{\beta-1}\eta(s)ds,$$

$$m(h) = m(h_1^-) + I_1(m(h_1^-)) + (1-\beta)[\eta(h) - \eta(h_1)] + \beta \int_0^h (h-s)^{\beta-1}\eta(s)ds,$$

Using Eq. (2.4)

$$m(h) = g(m) + (1-\beta)[\eta(h_1) - \eta(0)] + \beta \int_0^{h_1} (h_1-s)^{\beta-1}\eta(s)ds,$$

$$+ I_1(m(h_1^-)) + (1-\beta)[\eta(h) - \eta(h_1)] + \beta \int_0^h (h-s)^{\beta-1}\eta(s)ds,$$

$$m(h) = g(m) + (1-\beta)[\eta(h) - \eta(0)] + \beta \int_0^{h_1} (h_1-s)^{\beta-1}\eta(s)ds,$$

$$+ I_1(m(h_1^-)) + \beta \int_0^h (h-s)^{\beta-1}\eta(s)ds, \tag{2.5}$$

Again applying impulsive condition $m(h_2^-)$ in equation Eq. (2.5), we get

$$m(h_2^-) = g(m) + (1-\beta)[\eta(h_2) - \eta(0)] + I_1(m(h_1^-)),$$

$$+ \beta \int_0^{h_1} (h_1-s)^{\beta-1}\eta(s)ds + \beta \int_0^{h_2} (h_2-s)^{\beta-1}\eta(s)ds, \tag{2.6}$$

If $h \in [h_2, h_3]$ then

$${}_{h_2}^{AB}D_h^{\beta} m(h) = \eta(h), h \in (h_2, h_3], \text{ with } m(h_2^+) = m(h_2^-) + I_2(m(h_2^-)),$$

Again using Lemma 1.6, we obtained the following

$$m(h) = m(h_2^+) + (1-\beta)[\eta(h) - \eta(h_2)] + \beta \int_0^h (h-s)^{\beta-1}\eta(s)ds,$$

$$m(h) = m(h_2^-) + I_2(m(h_2^-)) + (1-\beta)[\eta(h) - \eta(h_2)] + \beta \int_0^h (h-s)^{\beta-1}\eta(s)ds,$$

By Eq. (2.6)

$$m(h) = g(m) + (1-\beta)[\eta(h_2) - \eta(0)] + I_1(m(h_1^-))$$

$$+ \beta \int_0^{h_1} (h_1-s)^{\beta-1}\eta(s)ds + \beta \int_0^{h_2} (h_2-s)^{\beta-1}\eta(s)ds,$$

$$+ I_2(m(h_2^-)) + (1-\beta)[\eta(h) - \eta(h_2)] + \beta \int_0^h (h-s)^{\beta-1}\eta(s)ds,$$

$$m(h) = g(m) + (1-\beta)[\eta(h) - \eta(0)] + I_1(m(h_1^-)) + I_2(m(h_2^-))$$

$$+ \beta \int_0^{h_1} (h_1-s)^{\beta-1}\eta(s)ds + \beta \int_0^{h_2} (h_2-s)^{\beta-1}\eta(s)ds + \beta \int_0^h (h-s)^{\beta-1}\eta(s)ds,$$

Continuing in this way, one can find

$$m(h) = g(m) + (1-\beta)[\eta(h) - \eta(0)] + \sum_{l=1}^{k} I_l(m(h_l^-))$$

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$$+\beta\sum_{l=1}^{k}\int_0^{h_l}(h_l-s)^{\beta-1}\eta(s)ds+\beta\int_0^{h}(h-s)^{\beta-1}\eta(s)ds,\ h\in[h_k,h_{k+1}],$$

Hence $m(h)$ satisfies Eq. (2.3).

Conversely, assume $m(h)$ satisfies the Eq. (2.3), then by applying ${}_0^{AB}D_h^{\beta}$ to the Eq. (2.3), we can obtained impulsive problem Eq. (2.1). □

The following corollary gives an equivalent integral representation of the given fractional differential equation Eq. (1.3), which serves as a foundation for applying fixed point theorems.

By using above lemma, we obtained the following corollary.

Corollary 2.2. *Considering Lemma 2.1, the implusive-implicit fractional differential equation (1.3) has the following solution:*

$$m(h)=\begin{cases} g(m)+(1-\beta)[G(h,m(h),{}_0^{AB}D_h^{\beta}m(h))-G(0,y(0),{}_0^{AB}D_h^{\beta}y(0))] \\ +\beta\int_0^h(h-s)^{\beta-1}G(s,y(s),{}_0^{AB}D_s^{\beta}y(s))ds, & h\in[0,h_1], \\ g(m)+(1-\beta)[G(h,m(h),{}_0^{AB}D_h^{\beta}m(h))-G(0,m(0),{}_0^{AB}D_h^{\beta}m(0))] \\ +I_1(m(h_1^-))+\beta\int_0^{h_1}(h_1-s)^{\beta-1}G(s,m(s),{}_0^{AB}D_s^{\beta}m(s))ds \\ +\beta\int_0^h(h-s)^{\beta-1}G(s,m(s),{}_0^{AB}D_s^{\beta}m(s))ds, h\in[h_1,h_2], \\ g(m)+(1-\beta)[G(h,m(h),{}_0^{AB}D_h^{\beta}m(h))-G(0,m(0),{}_0^{AB}D_h^{\beta}m(0))] \\ +I_1(m(h_1^-))+I_2(m(h_2^-))+\beta\int_0^{h_1}(h_1-s)^{\beta-1}G(s,m(s),{}_0^{AB}D_s^{\beta}m(s))ds \\ +\beta\int_0^{h_2}(h_2-s)^{\beta-1}G(s,m(s),{}_0^{AB}D_s^{\beta}m(s))ds \\ +\beta\int_0^h(h-s)^{\beta-1}G(s,m(s),{}_0^{AB}D_s^{\beta}m(s))ds, & h\in[h_2,h_3], \\ \cdots, \\ g(m)+(1-\beta)[G(h,m(h),{}_0^{AB}D_h^{\beta}m(h))-G(0,m(0),{}_0^{AB}D_h^{\beta}m(0))] \\ +\sum_{l=1}^k I_l(m(h_l^-))+\beta\sum_{l=1}^k\int_0^{h_l}(h_l-s)^{\beta-1}G(s,m(s),{}_0^{AB}D_s^{\beta}m(s))ds \\ +\beta\int_0^h(h-s)^{\beta-1}G(s,m(s),{}_0^{AB}D_s^{\beta}m(s))ds, & h\in[h_k,h_{k+1}],\quad k=1,2,\cdots,n. \end{cases}\tag{2.7}$$

Consider $G(h,m(h),{}_0^{AB}D_h^{\beta}m(h))=G(h)$, at $h=0$, $G(0,m(0),{}_0^{AB}D_h^{\beta}m(0))=G(0)$. First we transform the given problem to fixed point problem. So let us define the fixed point operator $A:Y\to Y$ as follows:

$$Am(h)=g(m)+(1-\beta)[G(h)-G(0)]+\sum_{l=1}^{k}I_l m(h_l^-))$$
$$+\beta\sum_{l=1}^{k}\int_0^{h_l}(h_l-s)^{\beta-1}G(s)ds+\beta\int_0^{h}(h-s)^{\beta-1}G(s)ds.\tag{2.8}$$

Consider the following assumptions holds.

(A_1) The function $G:[0,a]\times\mathbb{R}\times\mathbb{R}\to\mathbb{R}$ is continuous.

(A_2) $\exists\ L>0,\ 0<M<1$ such that
$|G(h,m(h),G(h))-G(h,\overline{m}(h),\overline{G}(h))|\le L|m(h)-\overline{m}(h)|+M|G(h)-\overline{G}(h)|$.

(A_3) $I_k:\mathbb{R}\to\mathbb{R}$ are continuous functions. There exists $0<C_k<1$ such that

$$\sum_{k=1}^{n}|I_k(m(h)-I_k(\overline{m}(h)))|\le\sum_{k=1}^{n}C_k|m(h)-\overline{m}(h)|,\ \ m,\overline{m}\in\mathbb{R},k=1,2,3,\cdots,n.$$

Further, we consider $\sum_{k=1}^{n}C_k=C<1$.

(A_4) Let $g:\mathbb{R}\to\mathbb{R}$ is continuous function which satisfies the condition $|g(m)-g(\overline{m})|\le K|m-\overline{m}|$, $0<K<1$.

(A_5) $\exists$ some positive constants $P,Q,R>0$ and $0<Q<1$ such that

$$|G(h,m(h),G(h))|\le P+Q|m(h)|+R|G(h)|,\text{for each } h\in[0,a],m(h),G(h)\in\mathbb{R}.$$

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(A_6) $\exists$ positive constant $K^* > 0$ such that
$|g(m)| \leq K^*|m(h)|$, for $m(h) \in \mathbb{R}$.

Theorem 2.3. *Consider the assumptions $A_1 - A_4$, there exist a unique solution to the problem Eq. (1.3) if*

$$\left[K + C + (1 - \beta + (n+1)J^{\beta})\frac{L}{1-M}\right] < 1.$$

Proof. Let $h \in [0, a],\ m(h), \overline{m}(h) \in Y$,

$$\begin{aligned}
|Am(h) - A\overline{m}(h)| = \Bigg| & g(m) + (1-\beta)[G(h) - G(0)] + \sum_{k=1}^{n} I_k(m(h_k^-)) \\
& + \beta \sum_{k=1}^{n} \int_0^{h_k} (h_k - s)^{\beta-1} G(s)ds + \beta \int_0^h (h-s)^{\beta-1} G(s)ds \\
& - g(\overline{m}) - (1-\beta)[\overline{G}(h) - G(0)] - \sum_{k=1}^{n} I_k(\overline{m}(h_k^-)) \\
& - \beta \sum_{k=1}^{n} \int_0^{h_k} (h_k - s)^{\beta-1} \overline{G}(s)ds - \beta \int_0^h (h-s)^{\beta-1} \overline{G}(s)ds \Bigg|,
\end{aligned}$$

$$\begin{aligned}
|Am(h) - A\overline{m}(h)| \leq & |g(m) - g(\overline{m})| + (1-\beta)|G(h) - \overline{G}(h)| + \sum_{k=1}^{n} |I_k(m(h_k^-)) - I_k(\overline{m}(m_k^-))| \\
& + \beta \sum_{k=1}^{n} \int_0^{h_k} (h_k - s)^{\beta-1} |G(s) - \overline{G}(s)|ds + \beta \int_0^h (h-s)^{\beta-1} |G(s) - \overline{G}(s)|ds,
\end{aligned}$$

From assumption A_2, we get

$$|G(h) - \overline{G}(h)| \leq L|m(h) - \overline{m}(h)| + M|G(h) - \overline{G}(h)|, \tag{2.9}$$

$$|G(h) - \overline{G}(h)| - M|G(h) - \overline{G}(h)| \leq L|m(h) - \overline{m}(h)|,$$

$$|G(h) - \overline{G}(h)| \leq \frac{L}{(1-M)}|m(h) - \overline{m}(h)|.$$

Substitute Eq. (2.9) in above, we get

$$\begin{aligned}
|Am(h) - A\overline{m}(h)| \leq & K|m(h) - \overline{m}(h)| + \frac{(1-\beta)L}{1-M}|m(h) - \overline{m}(h)| + C|m(h) - \overline{m}(h)| \\
& + \beta \sum_{k=1}^{n} \int_0^{h_k} (h_k - s)^{\beta-1} \frac{L}{1-M}|m(s) - \overline{m}(s)|ds + \beta \int_0^h (h-s)^{\beta-1} \frac{L}{1-M}|m(s) - \overline{m}(s)|ds,
\end{aligned}$$

Taking maximum on both side, we get

$$\begin{aligned}
\|Am(h) - A\overline{m}(h)\|_{PC} \leq & K\|m(h) - \overline{m}(h)\|_{PC} + \frac{(1-\beta)L}{1-M}\|m(h) - \overline{m}(h)\|_{PC} + C\|m(h) - \overline{m}(h)\|_{PC} \\
& + \frac{L\beta}{1-M}\left(\sum_{k=1}^{n} \int_0^{h_k} (h_k - s)^{\beta-1} ds\right) \|m(s) - \overline{m}(s)\|_{PC} \\
& + \frac{L\beta}{1-M}\left(\int_0^h (h-s)^{\beta-1} ds\right) \|m(s) - \overline{m}(s)\|_{PC},
\end{aligned}$$

$$\sum_{k=1}^{n} \int_0^{h_k} (h_k - s)^{\beta-1} ds \leq \frac{na^{\beta}}{\beta},$$

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$$\|Am(h) - A\overline{m}(h)\|_{PC} \leq \left[K + \frac{L(1-\beta)}{1-M} + C + \frac{nLa^{\beta}}{1-M} + \frac{La^{\beta}}{1-M}\right] \|m(h) - \overline{m}(h)\|_{PC},$$
$$\leq \left[K + C + \frac{L}{1-M}((1-\beta) + (n+1)a^{\beta})\right] \|m(h) - \overline{m}(h)\|_{PC},$$
$$\|Am(h) - A\overline{m}(h)\|_{PC} \leq \|m(h) - \overline{m}(h)\|_{PC}.$$

The operator A is contraction. Hence there exist a unique solution to the problem Eq. (1.3). □

Theorem 2.4. *Under the assumption $A_5 - A_6$ are satisfied, then the Impulsive-IFDE Eq. (1.3) has atleast one solution if*

$$0 < \left(K + \frac{L(1-\beta)}{1-M} + C\right) < 1.$$

Proof. From Eq. (2.8), define two operators say T_1, T_2 as follows

$$T_1 m(h) = g(m) + (1-\beta)[G(h) - G(0)] + \sum_{l=1}^{k} I_l(m(h_l^-)), \tag{2.10}$$

$$T_2 m(h) = \beta \sum_{l=1}^{k} \int_0^{h_l} (h_l - s)^{\beta-1} G(s) ds + \beta \int_0^{h} (h-s)^{\beta-1} G(s) ds. \tag{2.11}$$

Define $H = \{m(h) \in Y \mid \|m\|_{PC} \leq q, q > 0 \in \mathbb{R}\}$. First, we prove that T_1 is contraction mapping. For this purpose, let $m(h), \overline{m}(h) \in Y$,

$$| T_1 m(h) - T_1 \overline{m}(h) | = \Bigg| g(m) + (1-\beta)[G(h) - G(0)] + \sum_{l=1}^{k} I_l(m(h_l^-)),$$
$$- g(\overline{m}) - (1-\beta)[\overline{G}(h) - G(0)] - \sum_{l=1}^{k} I_l(\overline{m}(h_l^-))\Bigg|,$$
$$\leq |g(m) - g(\overline{m})| + (1-\beta)|G(h) - \overline{G}(h)| + \sum_{l=1}^{k} \Big| I_l(m(h)) - I_l(\overline{m}(h))\Big|.$$

Taking maximum on both side of above and using Eq. (2.9), assumptions $A_3 - A_4$ we get

$$\|T_1 m(h) - T_1 \overline{m}(h)\| \leq \left(K + \frac{(1-\beta)L}{1-M} + C\right) \|m(h) - \overline{m}(h)\|_{PC},$$
$$\left(K + \frac{(1-\beta)L}{1-M} + C\right) < 1,$$
$$\|T_1 m(h) - T_1 \overline{m}(h)\| \leq \|m(h) - \overline{m}(h)\|_{PC}.$$

Hence T_1 is contraction.
Going ahead we will be going to prove, the operator T_2 is compact and continuous.
Consider $m(h) \in Y$

$$|T_2 m(h)| = \left|\beta \sum_{l=1}^{k} \int_0^{h_l} (h_l - s)^{\beta-1} G(s) ds + \beta \int_0^{h} (h-s)^{\beta-1} G(s) ds\right|, \tag{2.12}$$

$$\leq \beta \sum_{l=1}^{k} \int_0^{h_l} (h_l - s)^{\beta-1} |G(s)| ds + \beta \int_0^{h} (h-s)^{\beta-1} |G(s)| ds, \tag{2.13}$$

Uncorrected Proof

Where, by using assumption A_5 one can obtained

$$|G(h)| \leq \frac{P}{1-R} + \frac{Q}{1-R}|m(h)|. \tag{2.14}$$

Taking maximum of Eq. (2.13) and using Eq. (2.14), we get

$$\begin{aligned}
\|T_2 m(h)\|_{PC} &\leq \beta \sum_{l=1}^{k} \int_0^{h_l} (h_l - s)^{\beta-1} \left(\frac{P}{1-R} + \frac{Q}{1-R}\|m(h)\|_{PC} \right) ds \\
&\quad + \beta \int_0^{h} (h-s)^{\beta-1} \left(\frac{P}{1-R} + \frac{Q}{1-R}\|m(h)\|_{PC} \right) ds, \\
\|T_2 m(h)\|_{PC} &\leq (n+1)a^{\beta} \left(\frac{P}{1-R} + \frac{Q}{1-R}\|m(h)\|_{PC} \right),
\end{aligned}$$

Because $\|m(h)\|_{PC} \leq q$,

$$\|T_2 m(h)\|_{PC} \leq (n+1)a^{\beta} \left(\frac{P}{1-R} + \frac{Q}{1-R} q \right) = Q^*.$$

Hence, the operator T_2 is bounded. Further, let $T_1 < T_2$ in $[0,a]$, we have

$$\begin{aligned}
|T_2 m(h_2) - T_2 m(h_1)| &= \Big| \beta \sum_{l=1}^{k} \int_0^{h_l} (h_l - s)^{\beta-1} G(s) ds + \beta \int_0^{h_2} (h_2 - s)^{\beta-1} G(s) ds \\
&\quad - \beta \sum_{l=1}^{k} \int_0^{h_l} (h_l - s)^{\beta-1} G(s) ds - \beta \int_0^{h_1} (h_1 - s)^{\beta-1} G(s) ds \Big|, \\
&\leq \beta \int_0^{h_2} (h_2 - s)^{\beta-1} |G(s)| ds + \beta \int_{h_1}^{0} (h_1 - s)^{\beta-1} |G(s)| ds,
\end{aligned}$$

Taking maximum and simplifying above, we get

$$\|T_2 y(h_2) - T_2 y(h_1)\|_{PC} \leq (h_2^{\beta} - h_1^{\beta}) \left(\frac{P}{1-R} + \frac{Qq}{1-R} \right).$$

As $h_1 \to h_2$, R.H.S of above tends to 0, then $\|T_2 m(h_2) - T_2 m(h_1)\|_{PC}$ also tends to zero. Hence T_2 is equicontinuous, by Arzela-Ascoli theorem T_2 is compact. Hence by Krasnoselskii fixed point theorem, given Impulsive-IFDE Eq. (1.3) has atleast one solution. □

3. Stability results

This section focuses on examining the stability of the derived solution for the fractional differential equation Eq. (1.3). We are especially interested in looking into the Hyers–Ulam stability of the problem under certain conditions. Using the right analytical methods, we can find the right conditions to make sure that the solution meets the Hyers–Ulam stability criteria. To achieve the desired outcomes, we initially introduce the following auxiliary lemma, which is essential for the forthcoming analysis.

Lemma 3.1. *The solution of given problem*

$$\begin{aligned}
{}_0^{AB}D^{\beta} m(h) &= G(h, m(h), {}_0^{AB}D^{\beta} m(h)) + \Psi(h), 0 < \beta < 1, \forall h \in [0,a], h \neq h_k, \\
\Delta m(h_k) &= I_k(m(h_k)), k = 1, 2, 3, \cdots, n, \\
m(0) &= g(m).
\end{aligned}$$

is

$$m(h) = g(m) + (1-\beta)[G(h) - G(0)] + (1-\beta)[\Psi(h) - \Psi(0)] + \sum_{l=1}^{k} I_l(m(h_l^-))$$

Uncorrected Proof

$$+\beta\sum_{l=1}^{k}\int_0^{h_l}(h_l-s)^{\beta-1}G(s)ds+\beta\sum_{l=1}^{k}\int_0^{h_l}(h_l-s)^{\beta-1}\Psi(s)ds$$
$$+\beta\int_0^{h}(h-s)^{\beta-1}G(s)ds+\beta\int_0^{h}(h-s)^{\beta-1}\Psi(s)ds. \tag{3.1}$$

Where $G(h)=G(h,m(h),{}_0^{AB}D^{\beta}m(h))$ *and* $\Psi(h)=0$, *further from the solution Eq.(3.1), we get*

$$\left|m(h)-\left[g(m)+(1-\beta)[G(h)-G(0)]+\sum_{l=1}^{k}I_k(m(h_l^-))+\beta\sum_{l=1}^{k}\int_0^{h_l}(h_l-s)^{\beta-1}G(s)ds+\beta\int_0^{h}(h-s)^{\beta-1}G(s)ds\right]\right|$$
$$\leq[(1-\beta)+(n+1)a^{\beta}]\epsilon. \tag{3.2}$$

Proof. From Eq. (3.1)

$$\Bigg|m(h)-\Bigg[g(m)+(1-\beta)[G(h)-G(0)]+\sum_{l=1}^{k}I_l(m(h_m^-))$$
$$+\beta\sum_{l=1}^{k}\int_0^{h_l}(h_l-s)^{\beta-1}G(s)ds+\beta\int_0^{h}(h-s)^{\beta-1}G(s)ds\Bigg]\Bigg|,$$
$$=\left|(1-\beta)(\Psi(h)-\Psi(0))+\beta\sum_{l=1}^{k}\int_0^{h_l}(h_l-s)^{\beta-1}\Psi(s)ds+\beta\int_0^{h}(h-s)^{\beta-1}\Psi(s)ds\right|,$$
$$\leq(1-\beta)|\Psi(h)|+\beta\sum_{l=1}^{k}\int_0^{h_l}(h_l-s)^{\beta-1}|\Psi(s)|ds+\beta\int_0^{h}(h-s)^{\beta-1}|\Psi(s)|ds,$$
$$\leq(1-\beta)\epsilon+\beta\frac{a^{\beta}}{\beta}n\epsilon+\beta\frac{a^{\beta}}{\beta}\epsilon,$$
$$\leq[(1-\beta)+(n+1)a^{\beta}]\epsilon.$$

Hence proved. □

Theorem 3.2. *Using Lemma (3.1), solution of the said Impulsive-IFDE Eq. (1.3) is Hyer-Ullam's stable if there exists a constant*

$$\Omega=\frac{(1-\beta)+(n+1)a^{\beta}}{\left(1-K-C-(1-\beta)-a^{\beta}(n+1)\frac{L}{1-M}\right)}.$$

Proof. Suppose $m(h)\in Y$ be any solution of Eq. (1.3) and $\overline{m}(h)\in Y$ be unique solution of Eq. (1.3) then consider

$$|m(h)-\overline{m}(h)|=|m(h)-[g(\overline{m})+(1-\beta)[\overline{G}(h)-G(0)]+\sum_{l=1}^{k}I_l(\overline{m}(h_l^-))$$
$$+\beta\sum_{l=1}^{k}\int_0^{h_l}(h_l-s)^{\beta-1}\overline{G}(s)ds+\beta\int_0^{h}(h-s)^{\beta-1}\overline{G}(s)ds],$$

By adding and subtracting the term

$$g(m)+(1-\beta)[G(h)-G(0)]+\sum_{l=1}^{k}I_l(m(h_l^-))+\beta\sum_{l=1}^{k}\int_0^{h_l}(h_l-s)^{\beta-1}G(s)ds+\beta\int_0^{h}(h-s)^{\beta-1}G(s)ds,$$

Uncorrected Proof

In above, we get

$$\Bigg| m(h) - \Bigg[g(m) + (1-\beta)[G(h) - G(0)] + \sum_{l=1}^{k} I_l(m(h_l^-)) + \beta \sum_{l=1}^{k} \int_0^{h_l} (h_l - s)^{\beta-1} G(s) ds + \beta \int_0^{h} (h-s)^{\beta-1} G(s) ds \Bigg]$$
$$+ \Bigg[g(m) + (1-\beta)[G(h) - G(0)] + \sum_{l=1}^{k} I_l(m(h_l^-)) + \beta \sum_{l=1}^{k} \int_0^{h_l} (h_l - s)^{\beta-1} G(s) ds + \beta \int_0^{h} (h-s)^{\beta-1} G(s) ds \Bigg]$$
$$- \Bigg[g(\overline{m}) + (1-\beta)[\overline{G}(h) - G(0)] + \sum_{l=1}^{k} I_l(\overline{m}(h_l^-)) + \beta \sum_{l=1}^{k} \int_0^{h_l} (h_l - s)^{\beta-1} \overline{G}(s) ds + \beta \int_0^{h} (h-s)^{\beta-1} \overline{G}(s) ds \Bigg] \Bigg|,$$

From Eq. (3.2)

$$\leq \Big[(1-\beta) + (n+1)a^{\beta} \Big] \epsilon + |g(m) - g(\overline{m})| + (1-\beta)|G(h) - \overline{G}(h)|$$
$$+ \sum_{l=1}^{k} |I_l(m(h_l^-)) - I_l(\overline{m}(h_l^-))| + \beta \sum_{l=1}^{k} \int_0^{h_l} (h_l - s)^{\beta-1} |G(s) - \overline{G}(s)| ds$$
$$+ \beta \int_0^{h} (h-s)^{\beta-1} |G(s) - \overline{G}(s)| ds,$$

Taking maximum and using assumptions A_3 and A_4, Eq. (2.9) we get

$$\leq \Big[(1-\beta) + (n+1)a^{\beta} \Big] \epsilon + K|m - \overline{m}| + \frac{L(1-\beta)}{1-M} |m - \overline{m}|$$
$$+ C|m - \overline{m}| + \frac{\beta a^{\beta} n L}{\beta(1-M)} |m - \overline{m}| + \frac{\beta a^{\beta} L}{\beta(1-M)} |m - \overline{m}|,$$

After simplifying, we get

$$\|m - \overline{m}\|_{PC} \leq \Big[(1-\beta) + (n+1)a^{\beta} \Big] \epsilon + \Big(K + C + (1-\beta) + a^{\beta}(n+1)\frac{L}{1-M} \Big) \|m - \overline{m}\|_{PC},$$

$$\|m - \overline{m}\|_{PC} \leq \frac{\Big[(1-\beta) + (n+1)a^{\beta} \Big] \epsilon}{\Big(1 - K - C - (1-\beta) - a^{\beta}(n+1)\frac{L}{1-M} \Big)},$$

Hence $\|m - \overline{m}\|_{PC} \leq \Omega\epsilon$. Hence the solution is H-U stable. □

4. Examples

This section verifies the validity of the obtained results through the use of illustrations. We provide two examples in detail to validate our obtained results in the above sections.

Example 4.1. Consider the following Impulsive-IFDE

$$ {}_{0}^{AB}D_{h}^{\frac{1}{2}} m(h) = \frac{2 + |m(h)| + |{}_{0}^{AB}D^{\frac{1}{2}} m(h)|}{102 e^{h+3}(1 + |m(h)| + |{}_{0}^{AB}D^{\frac{1}{2}} m(h)|)}, h \in [0,1], h \neq h_k, \tag{4.1}$$
$$\Delta m\left(\frac{1}{3}\right) = \frac{|m(\frac{1}{3}^{-})|}{77 + |m(\frac{1}{3}^{-})|},$$
$$m(0) = \frac{|m(h)|}{15}.$$

Uncorrected Proof

Observe that $n = 1$, $\beta = \frac{1}{2}, h_1 = \frac{1}{3}$, ${}_0^{AB}D_h^{\frac{1}{2}} m(h) = G(h)$, $a = 1$ $g(m) = \frac{|m(h)|}{15}$
Let

$$G(h, m(h), G(h)) = \frac{2 + |m(h)| + |G(h)|}{102e^{h+3}(1 + |m(h)| + |G(h)|)},$$
$$I_1\left(m\left(\frac{1}{3}\right)\right) = \frac{|m(\frac{1}{3}^-)|}{77 + |m(\frac{1}{3}^-)|}.$$

Here G and g both are continuous function. Hence A_1 assumption hold. Next let $m(h), \overline{m}(h) \in \mathbb{R}$ and $h \in [0, 1]$. Consider

$$|G(h, m(h), G(h)) - G(h, \overline{m}(h), \overline{G}(h))| = \left| \frac{2 + |m(h)| + |G(h)|}{102e^{h+3}(1 + |m(h)| + |G(h)|)} - \frac{2 + |\overline{m}(h)| + |\overline{G}(h)|}{102e^{h+3}(1 + |\overline{m}(h)| + |\overline{G}(h)|)} \right|$$
$$\leq \frac{1}{102e^3}\Big(|m(h) - \overline{m}(h)| + |G(h) - \overline{G}(h)|\Big),$$

Hence $L = \frac{1}{102e^3} > 0$, $0 < M = \frac{1}{102e^3} < 1$. Hence assumption A_2 hold. Now, next we find

$$\left|I_1\left(m\left(\frac{1}{3}\right)\right) - I_1\left(\overline{m}\left(\frac{1}{3}\right)\right)\right| = \left|\frac{|m(\frac{1}{3}^-)|}{77 + |m(\frac{1}{3}^-)|} - \frac{|\overline{m}(\frac{1}{3}^-)|}{77 + |\overline{m}(\frac{1}{3}^-)|}\right|$$
$$\leq \frac{1}{77}\left|m\left(\frac{1}{3}^-\right) - \overline{m}\left(\frac{1}{3}^-\right)\right|$$
$$\leq \frac{1}{77}\Big|m - \overline{m}\Big|.$$

Here $0 < C = \frac{1}{77} < 1$. Therefore, assumption A_3 also holds. Clearly, assumption A_4 also holds since $0 < K = \frac{1}{15} < 1$. Next, we find P, Q, R for this consider

$$|G(h, m(h), G(h))| = \left|\frac{2 + |m(h)| + |G(h)|}{102e^{h+3}(1 + |m(h)| + |G(h)|)}\right|$$
$$\leq \frac{1}{102e^{h+3}}\left(\left|\Big(2 + |m(h)| + |G(h)|\Big)\right|\right)$$
$$\leq \frac{1}{51e^3} + \frac{1}{102e^3}|m(h)| + \frac{1}{102e^3}|G(h)|.$$

Hence $P = \frac{1}{51e^3} > 0, 0 < Q = \frac{1}{102e^3} < 1, R = \frac{1}{102e^3} > 0$. Also, assumption A_5 hold. Continuing in this way, we can find $K^* = \frac{1}{15} > 0$. Hence A_5 also holds. Hence all the assumptions holds. Consider the condition of Theorem 2.3

$$\left[K + C + (1 - \beta + (n+1)a^\beta)\frac{L}{1 - M}\right] = \left[\frac{1}{15} + \frac{1}{77} + \left(1 - \frac{1}{2} + 2\right)\frac{\frac{1}{102e^3}}{1 - \frac{1}{102e^3}}\right],$$
$$= 0.0808745 < 1.$$

Which is also true. Hence using Theorem 2.3, problem (4.1) has unique solution. Additionally, consider

$$\left(K + \frac{L(1 - \beta)}{1 - M} + C\right) = \left(\frac{1}{15} + \frac{\frac{1}{102e^3}\left(1 - \frac{1}{2}\right)}{1 - \frac{1}{102e^3}} + \frac{1}{77}\right) = 0.7989 < 1.$$

Hence by Theorem 2.4, there exist at least one solution to the given impulsive problem (4.1) on $[0, 1]$.
Lastly, we will verify the stability results for the given impulsive problem (4.1). Consider the condition of Theorem

Uncorrected Proof

3.2.

$$\Omega = \frac{(1-\beta)+(n+1)a^{\beta}}{\left(1-K-C-(1-\beta)-a^{\beta}(n+1)\frac{L}{1-M}\right)},$$
$$= \left(\frac{(1-\frac{1}{2})+2}{1-\frac{1}{15}-\frac{1}{77}-(1-\frac{1}{2})-2\frac{\frac{1}{102e^3}}{\left(1-\frac{1}{102e^3}\right)}}\right),$$
$$= 5.96132.$$

Which is constant. Hence, the given problem (4.1) is H-U stable.

Example 4.2. Consider the following Impulsive-IFDE

$$\begin{aligned} {}_0^{AB}D_h^{\frac{1}{2}}m(h) &= \frac{1}{25}+\frac{sin(m(h))+sin({}_0^{AB}D_h^{\frac{1}{2}}m(h))}{45+h^3},\ h\in[0,1], \\ \Delta m\left(\frac{1}{2}\right) &= \frac{e^{m(\frac{1}{2})}}{75}, \\ m(0) &= \frac{cos|m|}{20}. \end{aligned} \tag{4.2}$$

Observe that $n=1$, $\beta=\frac{1}{2}, h_1=\frac{1}{2}$, ${}_0^{AB}D_h^{\frac{1}{2}}m(h)=G(h)$, $a=1$, $g(m)=\frac{cos|m|}{20}$.
Let

$$G(t,m(h),G(h)) = \frac{1}{25}+\frac{sin(m(h))+sin({}_0^{AB}D_h^{\frac{1}{2}}m(h))}{45+h^3}.$$

Here, G and g both are continuous function. Hence, A_1 assumption hold. Now, let $m(h),\overline{m}(h)\in\mathbb{R}$ and $0\le h\le 1$. Consider

$$\begin{aligned} |G(h,m(h),G(h))-G(h,\overline{m}(h),\overline{G}(h))| &= \Bigg|\frac{1}{25}+\frac{sin(m(h))+sin({}_0^{AB}D_h^{\frac{1}{2}}m(h))}{45+h^3} \\ &\quad -\frac{1}{25}-\frac{sin(\overline{m}(h))+sin({}_0^{AB}D_h^{\frac{1}{2}}\overline{m}(h))}{45+h^3}\Bigg| \\ &\le \frac{1}{46}\Big(|m(h)-\overline{m}(h)|+|G(h)-\overline{G}(h)|\Big), \end{aligned}$$

Hence $L=\frac{1}{46}>0$, $0<M=\frac{1}{46}<1$. Hence assumption A_2 hold. Now, next we find

$$\begin{aligned} \left|I_1\left(m\left(\frac{1}{2}\right)\right)-I_1\left(\overline{m}\left(\frac{1}{2}\right)\right)\right| &= \left|\frac{e^{m(\frac{1}{2})}}{75}-\frac{e^{\overline{m}(\frac{1}{2})}}{75}\right| \\ &\le \frac{1}{75}|m(h)-\overline{m}(h)|. \end{aligned}$$

Here $0<C=\frac{1}{75}<1$. Therefore, assumption A_3 also holds. Clearly, assumption A_4 also holds since $0<K=\frac{1}{20}<1$. Next, we find P,Q,R for this consider

$$\begin{aligned} |G(h,m(h),G(h))| &= \left|\frac{1}{25}+\frac{sin(m(h))+sin({}_0^{AB}D_h^{\frac{1}{2}}m(h))}{45+h^3}\right| \\ &\le \frac{1}{25}+\frac{1}{46}|m(h)|+\frac{1}{46}|G(h)|. \end{aligned}$$

Uncorrected Proof

Hence $P = \frac{1}{25} > 0, 0 < Q = \frac{1}{46} < 1, R = \frac{1}{46} > 0$. Also assumption A_5 hold. Continuing in this way, we can find $K^* = \frac{1}{20} > 0$. Hence A_5 also holds. Now consider

$$\left[K + C + (1 - \beta + (n+1)a^{\beta})\frac{L}{1-M}\right] = \left[\frac{1}{20} + \frac{1}{75} + \left(1 - \frac{1}{2} + 2\right)\frac{\frac{1}{46}}{1 - \frac{1}{46}}\right],$$
$$= 0.3411 < 1.$$

Hence by Theorem 2.3, the given problem (4.2) has unique solution. Further, consider

$$\left(K + \frac{L(1-\beta)}{1-M} + C\right) = \left(\frac{1}{20} + \frac{\frac{1}{46}\left(1 - \frac{1}{2}\right)}{1 - \frac{1}{46}} + \frac{1}{75}\right)$$
$$= 0.07444 < 1.$$

Hence by Theorem 2.4, existence of solution to the problem (4.2) on $[0, 1]$ is proved.
Lastly, we will verify the stability results for the given impulsive problem (4.2). Consider the condition of Theorem 3.2.

$$\Omega = \frac{(1-\beta) + (n+1)a^{\beta}}{\left(1 - K - C - (1-\beta) - a^{\beta}(n+1)\frac{L}{1-M}\right)}$$
$$= \left(\frac{(1 - \frac{1}{2}) + 2}{1 - \frac{1}{20} - \frac{1}{75} - (1 - \frac{1}{2}) - 2\frac{\frac{1}{46}}{\left(1 - \frac{1}{46}\right)}}\right)$$
$$= 6.3739.$$

Which is constant. Hence, the given problem (4.2) is H-U stable.

5. Conclusion

In this study, using the Atangana-Baleanu derivative operator, important conditions are established for the existence and uniqueness of solutions to impulsive-implicit FDE of order $0 < \beta < 1$. The paper well illustrates these requirements by using the Banach contraction principle and the Krasnoselskii fixed point theorem. Furthermore, for the presented equations, the research validates Hyers-Ulam stability. Robustness of the acquired results is demonstrated by providing specific examples that highlight the practical relevance and applicability of these theoretical discoveries.

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