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A novel algorithm based on spectral confluent Appell-Changhee polynomials for solving nonlinear models arising in mathematical physics

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Abstract

The main objective of this research work is to construct an approximate solution to the fractional Cahn-Allen (CA) problem, the time fractional wave-like (WL) problem and the time fractional Korteweg-de Vries (KdV) problem using Caputo fractional derivative. The proposed method based on the spectral confluent Appell-Changhee collocation algorithm coupled with residual power series (RPS) approach. The error bound of the proposed method are discussed in detail. The simulation results obtained are illustrated in both graphs and tables. The numerical results obtained are compared with other published techniques. The results show that the presented method is efficient, accurate and robust, and thus can be applied to both complex linear and nonlinear problems.

Keywords. Confluent Changhee polynomials, Residual power series approach, Fractional derivatives, Cahn-Allen problem, Wave-like problem, Korteweg-de Vries problem, Numerical results.

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1. Introduction

Fractional partial differential equations (FPDEs) are known to be used to model many physical phenomena such as kinetic theory, nonlinear optics, heat transfer, quantum mechanics, fluid flow, plasma, electromagnetic waves, control theory and biology [8, 11, 12, 19, 22, 36, 38, 50, 51, 54**?**]. For a long time, many mathematicians have made great efforts to solve FPDEs by utilizing several effective analytical and numerical techniques including spectral collocation method [46], Adomian decomposition method [16], Haar wavelet method [3], Laplace Adomian decomposition method [45], Gegenbauer wavelet method [47], alternating direction implicit method [52], B-spline collocation method [53], natural transform method [32], fractional reduced differential transform method [48], homotopy perturbation Sumudu transform method [49], Generalized Mittag-Leffler function method [7, 9], new iterative method [59], explicit finite difference method [1] and fractional residual power method [10, 42, 43].

In this study, we will study an approximate solution of the fractional CA problem, the fractional WL problem, and the fractional KdV problem which take the following forms:

- ♦ CA Problem

$$\mathscr{D}_t^{\sigma}\ \mathbf{P}(\omega,t) - \mathscr{D}_\omega^{2}\mathbf{P}(\omega,t) + \mathbf{P}^3(\omega,t) - \mathbf{P}(\omega,t) = \mathcal{F}(\omega,t),\ 0<\sigma\leq 1,\ \omega\in[0,1],\ t\in[0,1], \tag{1.1}$$

subject to initial condition (IC):

$$\mathbf{P}(\omega,0) = \mathcal{H}(\omega), \tag{1.2}$$

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boundary conditions (BCs):

$$\begin{cases}\mathbf{P}(0,t)=\mathcal{A}_1(t),\\ \mathbf{P}(1,t)=\mathcal{A}_2(t).\end{cases} \tag{1.3}$$

- WL Problem

$$\mathscr{D}_t^{2\sigma}\ \mathbf{P}(\omega,t)=\mathscr{R}\big(\mathbf{P}(\omega,t)\big)+\mathscr{T}\big(\mathbf{P}(\omega,t)\big),\ \frac{1}{2}<\sigma\leq 1,\ \omega\in[0,1],\ t\in[0,1], \tag{1.4}$$

subject to ICs:

$$\begin{cases}\mathbf{P}(\omega,0)=\mathcal{C}_1(\omega),\\ \mathscr{D}_t^{\sigma}\mathbf{P}(\omega,0)=\mathcal{C}_2(\omega),\end{cases} \tag{1.5}$$

BCs:

$$\begin{cases}\mathbf{P}(0,t)=\mathfrak{F}_1(t),\\ \mathbf{P}(1,t)=\mathfrak{F}_2(t),\end{cases} \tag{1.6}$$

where $\mathscr{R}$ and $\mathscr{T}$ are linear and non-linear term, respectively.

- KdV Problem

$$\mathscr{D}_t^{\sigma}\ \mathbf{P}(\omega,t)=\mathscr{L}\big(\mathbf{P}(\omega,t)\big)+\mathscr{N}\big(\mathbf{P}(\omega,t)\big)=0,\ 0<\sigma\leq 1,\ \omega\in[0,1],\ t\in[0,1], \tag{1.7}$$

subject to IC:

$$\mathbf{P}(\omega,0)=\mathcal{P}(\omega), \tag{1.8}$$

BCs:

$$\begin{cases}\mathbf{P}(0,t)=\mathcal{Q}_1(t),\\ \mathbf{P}(1,t)=\mathcal{Q}_2(t),\end{cases} \tag{1.9}$$

where $\mathscr{L}$ and $\mathscr{N}$ are linear and non-linear term, respectively.

The CA problem is a mathematical model that studies the phase separation processes in binary alloys. The CA problem has various applications in scientific phenomena such as plasma physics, fluid dynamics, quantum mechanics, and mathematical biology. There are many algorithms to obtain the solution of the CA problem such as the first integral method [56], Haar wavelet method [21], homotopy analysis method (HAM) [17], $(\frac{G'}{G})$ expansion method [20], tanh method [55], Exp-function method [58], Kudryashov method [23], fractional reduced differential transform method (FRDTM) [40], reproducing kernel method (RKM) [44], explicit power series method [24], finite difference method and Fourier spectral method [37], extended and cubic B-spline (ExCBS) method [18].

WL problems are of great importance in various fields of applied sciences such as mathematical physics, nonlinear hydrodynamics, mathematical physics, engineering physics, biophysics and plasma physics. In recent decades, many researchers have solved these problems in different ways. Khalouta and Kadem [27, 28] obtained an approximate solution for WL problems with variable coefficients using the natural variable iteration method and the fractional natural decomposition method. The Shehu homotopy analysis method for WL problems with variable coefficients is introduced by Khalouta [29]. An approximate analytical solution for WL problems with variable coefficients is generated using the fractional residual power series technique (FRPST) [30]. Alaroud et.al [4] used the Laplace residual power series technique (LRPST) for solving WL problems with variable coefficients.

Korteweg de Vries (KdV) problems have been widely applied in materials science phenomena as a model for nonlinear waves. KdV problems were first introduced in 1895 by Korteweg and de Vries to study waves on shallow water surfaces. KdV problems have described many different physical phenomena such as particle acoustic waves, collision-free hydromagnetic waves, water waves, layered internal waves, plasma physics, and quantum field theory [6, 25, 26, 35, 57]. Although KdV problems have been studied for decades, the physical behavior is still interesting. Many mathematicians have studied KdV problems using several important numerical and analytical methods [5, 31, 41, 60].

The main objective of this manuscript is to obtain an approximate solution to three problems arising in different physical phenomena such as plasma physics, fluid dynamics and quantum mechanics using the spectral confluent Appell-Changhee collocation algorithm with RPS approach. The main distinction lies in employing confluent Appell-Changhee polynomials as the basis functions within the spectral collocation method. Unlike traditional polynomial families, these polynomials possess superior approximation capabilities for fractional operators and offer greater flexibility in treating complex boundary conditions. Furthermore, they exhibit fast convergence, high approximation accuracy and lower computational costs when integrated with RPS approach. To the best of our knowledge, this is the first attempt to combine confluent Appell-Changhee polynomials with RPS approach, which demonstrates both originality and practical efficiency.

The manuscript is structured as follows: In section 2, we present some preliminaries defns and notation. In section 3, we derive an explicit formula for fractional derivative of Confluent Appell-Changhee polynomials. Error bound is constructed in section 4. In section 5, we derive the methodology for the solution of proposed problems. As for sections 6 and 7, we display proposed problems and discuss the results through graphs and tables. Finally, the conclusion is introduced in section 8.

2. Principal Definitions and Theories

In this part, we recall briefly few major concepts and theories which can be utilized in this study.

2.1. Fractional Calculus (FC).

Definition 2.1. The time Riemann-Liouville (R-L) fractional integral of order σ for a function $\mathbf{P}(\omega,t)$ is defined as (see [33, 34, 39]):

$$\mathscr{I}_t^{\sigma}\mathbf{P}(\omega,t) = \begin{cases} \frac{1}{\Gamma(\sigma)}\displaystyle\int_0^t (t-\upsilon)^{\sigma-1}\,\mathbf{P}(\omega,\upsilon)d\upsilon, & t>\upsilon>0,\\ \mathbf{P}(\omega,t), & \sigma=0. \end{cases} \tag{2.1}$$

Definition 2.2. For $\sigma \in \Re^+$, the time fractional derivative for function $\mathbf{P}(\omega,t)$ in Caputo sense is described as (see [33, 34, 39]):

$$\mathscr{D}_t^{\sigma}\mathbf{P}(\omega,t) = \begin{cases} \mathscr{I}_t^{n-\sigma}\mathscr{D}_t^{n}\mathbf{P}(\omega,t), & t>0,\ n-1<\sigma<n,\\ \frac{\partial^n \mathbf{P}(\omega,t)}{\partial t^n}, & \sigma=n\in\mathbb{N}. \end{cases} \tag{2.2}$$

Lemma 2.3. *For $\sigma>0$, $\mathscr{K}\in\Re$, $t\geq 0$, $n-1<\sigma<n$, $\mathscr{D}^{\sigma}$ and $\mathscr{I}^{\sigma}$. Then the following properties are satisfied (see [33, 34, 39]):*

❶ $\mathscr{D}^{\sigma}\mathscr{K}=0$.
❷ $\mathscr{D}^{\sigma}t^{\lambda}=\frac{\Gamma(\lambda+1)}{\Gamma(\lambda-\sigma+1)}t^{\lambda-\sigma}$.
❸ $\mathscr{D}_t^{\sigma}\mathscr{I}_t^{\sigma}\mathbf{P}(\omega,t)=\mathbf{P}(\omega,t)$.

2.2. Fractional Power Series (FPS).

Definition 2.4. A power series expansion which called FPS about t_0 take the formula (see [13, 14])

$$\sum_{n=0}^{\infty}\,\Phi_n(t-t_0)^{n\sigma}=\Phi_0+\Phi_1(t-t_0)^{\sigma}+\Phi_2(t-t_0)^{2\sigma}+...,$$

where $0\leq n-1<\sigma\leq n, n\in\mathbb{N}$ and $t\geq t_0$.

Theorem 2.5. *Consider that $\mathscr{U}$ has FPS at $t=t_0$. Then it has the form:*

$$\mathscr{U}(t)=\sum_{n=0}^{\infty}\,\Phi_n(t-t_0)^{n\sigma}=\Phi_0+\Phi_1(t-t_0)^{\sigma}+\Phi_2(t-t_0)^{2\sigma}+...,$$

where $0\leq n-1<\alpha\leq n, n\in\mathbb{N}$ and $t_0\leq t\leq t_0+\Re$.

If $\mathscr{D}^{n\sigma}$ are continuous on $(t_0, t_0+\Re)$, $n=0,1,2,...,$ then

$$\Phi_n = \frac{\mathscr{D}^{n\sigma}\mathscr{U}(t_0)}{\Gamma(1+n\sigma)}.$$

Definition 2.6. A power series expansion which called multiple FPS about $t=t_0$ take the formula (see [13, 14])

$$\sum_{n=0}^{\infty}\mathscr{U}_n(t-t_0)^{n\sigma} = \mathscr{U}_0 + \mathscr{U}_1(t-t_0)^{\sigma} + \mathscr{U}_2(t-t_0)^{2\sigma} + ...,$$

Theorem 2.7. *Suppose that $\mathbf{P}(\omega,t)$ has a multiple FPS at t_0. Then it has the form:*

$$\mathbf{P}(\omega,t) = \sum_{n=0}^{\infty}\ \mathscr{U}_n(\omega)(t-t_0)^{n\sigma},$$

where $\omega \in I$, $0 \le n-1 < \sigma \le n, n \in \mathbb{N}$ and $t_0 \le t \le t_0+\Re$. If $\mathscr{D}_t^{n\sigma}\mathbf{P}(\omega,t)$ is continuous on $I\times$ $(t_0, t_0+\Re)$, $n=0,1,2,\dots$ then the coefficients will take the form

$$\mathscr{U}_n(\omega) = \frac{\mathbb{D}_t^{n\sigma}\ \mathbf{P}(\omega,t_0)}{\Gamma(1+n\sigma)}.$$

2.3. Appell-Changhee polynomials.

Definition 2.8. The Appell-Changhee polynomials of degree n take the form [2]:

$$\check{\mathcal{CH}}_n(\omega) = \sum_{j=0}^{n}\ \binom{n}{j}\check{\mathcal{CH}}_j\ \omega^{n-j}. \tag{2.3}$$

It has the following property:

$$\int_0^1 \check{\mathcal{CH}}_n(\omega)\ \check{\mathcal{CH}}_m(\omega)d\omega = \sum_{i=0}^{n}\sum_{k=0}^{n-i}\frac{(-1)^{n-i-1}\ n!\ (n-i)\binom{n-i}{k}\ \check{\mathcal{CH}}_k(1)\ \check{\mathcal{CH}}_i}{i!\ (n-i-1)!\ (2n-2i-k+1)\ \binom{2n-2i-k}{n-i}}.$$

Definition 2.9. The confluent Appell-Changhee polynomials are described as below :

$$\check{\mathcal{CH}}_n^{(\alpha,\beta)}(\omega) = \sum_{j=0}^{n}\ \binom{n}{j}\frac{(\alpha)_{n-j}}{(\beta)_{n-j}}\check{\mathcal{CH}}_j\ \omega^{n-j}, \tag{2.4}$$

where $\check{\mathcal{CH}}_j$ is the Changhee number which achieves the properties $\check{\mathcal{CH}}_0 = 1$, and

$$2\ \check{\mathcal{CH}}_n + n\ \check{\mathcal{CH}}_{n-1} = 0,\ \forall\ n \ge 1.$$

The function $\mathbf{P}(\omega,t)$ can be expressed in terms of confluent Appell-Changhee polynomials as:

$$\mathbf{P}(\omega,t) = \sum_{j=0}^{\infty}\Phi_j(t)\ \check{\mathcal{CH}}_j^{(\alpha,\beta)}(\omega). \tag{2.5}$$

In practice, an infinite series of $(n+1)$ in terms of confluent Appell-Changhee polynomials take the form:

$$\mathbf{P}_n(\omega,t) = \sum_{j=0}^{n}\Phi_j(t)\ \check{\mathcal{CH}}_j^{(\alpha,\beta)}(\omega). \tag{2.6}$$

3. Computation Fractional Derivative for Confluent Appell-Changhee Polynomials

Here, we conclude a direct approximation for $\mathscr{D}^{\sigma}_{\omega}\mathbf{P}_n(\omega,t)$ as described in following theorem.

Theorem 3.1. *The σ^{th} fractional order derivative of confluent Appell-Changhee polynomials approximation $\mathbf{P}_n(\omega,t)$ supplied in Eq. (2.6) is given as:*

$$\mathscr{D}^{\sigma}_{\omega}\mathbf{P}_n(\omega,t) = \sum_{j=\lceil\sigma\rceil}^{n}\sum_{k=0}^{j-\lceil\sigma\rceil} \Phi_j(t)\ \mathbb{X}^{(\sigma)}_{j,k}\ \omega^{j-k-\sigma}, \tag{3.1}$$

where $\mathbb{X}^{(\sigma)}_{j,k}$ is defined as:

$$\mathbb{X}^{(\sigma)}_{j,k} = \binom{j}{k}\frac{(\alpha)_{j-k}\ \Gamma(j-k+1)}{(\beta)_{j-k}\ \Gamma(j-k-\sigma+1)}\ \check{\mathcal{CH}}_k.$$

Proof. By appling Caputo fractional partial derivative on approximate function $\mathbf{P}_n(\omega,t)$ defined in (2.6), we have

$$\mathscr{D}^{\sigma}_{\omega}\mathbf{P}_n(\omega,t) = \sum_{j=0}^{n}\Phi_j(t)\mathscr{D}^{\sigma}_{\omega}\Big(\check{\mathcal{CH}}^{(\alpha,\beta)}_j(\omega)\Big). \tag{3.2}$$

According to Lemma 2.3, we have

$$\mathscr{D}^{\sigma}_{\omega}\Big(\check{\mathcal{CH}}^{(\alpha,\beta)}_j(\omega)\Big) = 0, \quad j=0,1,2,...\lceil\sigma\rceil-1, \quad \sigma>0. \tag{3.3}$$

Also, $\forall\ j=\lceil\sigma\rceil,\lceil\sigma\rceil+1,...,n$, we get

$$\begin{aligned}\mathscr{D}^{\sigma}_{\omega}\Big(\check{\mathcal{CH}}^{(\alpha,\beta)}_j(\omega)\Big) &= \sum_{k=0}^{j}\binom{j}{k}\frac{(\alpha)_{j-k}\ \check{\mathcal{CH}}_k}{(\beta)_{j-k}}\ \mathscr{D}^{\sigma}_{\omega}\Big(\omega^{j-k}\Big)\\ &= \sum_{k=0}^{j-\lceil\zeta\rceil}\binom{j}{k}\frac{(\alpha)_{j-k}\ \Gamma(j-k+1)\ \check{\mathcal{CH}}_k}{(\beta)_{j-k}\ \Gamma(j-k-\sigma+1)}\ \omega^{j-k-\sigma}.\end{aligned} \tag{3.4}$$

By substituting from Eq. (3.4) into Eq. (3.2), we obtain

$$\mathscr{D}^{\sigma}_{\omega}\mathbf{P}_n(\omega,t) = \sum_{j=\lceil\sigma\rceil}^{n}\sum_{k=0}^{j-\lceil\sigma\rceil} \Phi_j(t)\ \mathbb{X}^{(\sigma)}_{j,k}\ \omega^{j-k-\sigma}.$$

□

4. Error Bound

In this section, we derive an error bound for proposed approach as given in following theorem.

Theorem 4.1. *Assume that $\mathscr{D}^{i\sigma}\ \mathbf{P}(\omega)\in\mathcal{C}[0,1],\ \forall\ i=0,1,...,n+1,\ \ \mathbf{P}_n(\omega)$ is the best square approximation of $\mathbf{P}(\omega)$, then the error bound is definied as:*

$$\|\mathbb{E}_n\| \le \frac{\mathcal{S}_\sigma}{\Gamma\big(1+(n+1)\sigma\big)\sqrt{1+2\sigma(n+1)}},$$

where $\mathcal{S}_\sigma \ge \mathscr{D}^{(n+1)\sigma}\mathbf{P}(\omega),\ \omega\in[0,1]$.

Proof. By using Taylor formula for the function $\mathbf{P}(\omega)$ as:

$$\mathbf{P}(\omega) = \sum_{\rho=0}^{n}\frac{\omega^{\sigma\rho}}{\Gamma(1+\sigma\rho)}\mathscr{D}^{\sigma\rho}\ \mathbf{P}(0^+) + \mathbb{K}_{n+1},$$

where

$$\mathbb{K}_{n+1} = \frac{\omega^{(n+1)\sigma}}{\Gamma\big(1+(n+1)\sigma\big)} \mathscr{D}^{(n+1)\sigma}\mathbf{P}(\vartheta), \quad \vartheta \in [0,\omega]. \tag{4.1}$$

Suppose

$$\tilde{\mathbf{P}}_n(\omega) = \sum_{\rho=0}^{n} \frac{\omega^{\sigma\rho}}{\Gamma(1+\sigma\rho)} \mathscr{D}^{\sigma\rho}\, \mathbf{P}(0^+). \tag{4.2}$$

Then

$$\|\mathbf{P}(\omega) - \tilde{\mathbf{P}}_n(\omega)\| = \mathbb{K}_{n+1} \leq \frac{\omega^{(n+1)\sigma}}{\Gamma\big(1+(n+1)\sigma\big)}\mathcal{S}_\sigma.$$

If, $\mathbf{P}_n(\omega)$ is the best square approximation for $\mathbf{P}(\omega)$, then

$$\begin{aligned}
\|\mathbb{E}_n\|^2 = \|\mathbf{P}(\omega) - \mathbf{P}_n(\omega)\|^2 &\leq \|\mathbf{P}(\omega) - \tilde{\mathbf{P}}_n(\omega)\|^2 \\
&= \int_0^1 \Big(\mathbf{P}(\omega) - \tilde{\mathbf{P}}_n(\omega)\Big)^2 d\omega \\
&\leq \int_0^1 \left(\frac{\omega^{(n+1)\sigma}}{\Gamma\big(1+(n+1)\sigma\big)}\mathcal{S}_\sigma\right)^2 d\omega \\
&\leq \left(\frac{\mathcal{S}_\sigma}{\Gamma\big(1+(n+1)\sigma\big)}\right)^2 \int_0^1 \omega^{2\sigma(n+1)} d\omega \\
&= \left(\frac{\mathcal{S}_\sigma}{\Gamma\big(1+(n+1)\sigma\big)}\right)^2 \frac{1}{1+2\sigma(n+1)}.
\end{aligned} \tag{4.3}$$

By taking the square roots of both sides, we obtain

$$\|\mathbb{E}_n\| \leq \frac{\mathcal{S}_\sigma}{\Gamma\big(1+(n+1)\sigma\big)\sqrt{1+2\sigma(n+1)}}.$$

□

5. Description of Methodology

In this section, the steps required to construct an approximate solution to the proposed problems using the proposed method will be explained.

5.1. Generalized time-Fractional CA Problem.

➤ Step 1. By appling Eqs. (2.6) and (3.1) to Eq. (1.1), we get

$$\begin{aligned}
&\sum_{j=0}^{n} \mathscr{D}_t^\sigma \Phi_j(t)\ \mathcal{C}\check{\mathcal{H}}_j^{(\alpha,\beta)}(\omega) - \sum_{j=\lceil 2\rceil}^{n} \sum_{k=0}^{j-\lceil 2\rceil} \Phi_j(t)\ \mathbb{X}_{j,k}^{(\sigma)}\ \omega^{j-k-2} \\
&+ \left(\sum_{j=0}^{n} \Phi_j(t)\ \mathcal{C}\check{\mathcal{H}}_j^{(\alpha,\beta)}(\omega)\right)^3 - \left(\sum_{j=0}^{n} \Phi_j(t)\ \mathcal{C}\check{\mathcal{H}}_j^{(\alpha,\beta)}(\omega)\right) = \mathcal{F}(\omega,t).
\end{aligned} \tag{5.1}$$

➤ Step 2. By computing Eq. (5.1) at $(n+1-\lceil\sigma\rceil)$ points ω_r, $r = 0,1,2,...,n-\lceil\sigma\rceil$ as follows:

$$\begin{aligned}
&\sum_{j=0}^{n} \mathscr{D}_t^\sigma \Phi_j(t)\ \mathcal{C}\check{\mathcal{H}}_j^{(\alpha,\beta)}(\omega_r) - \sum_{j=\lceil 2\rceil}^{n} \sum_{k=0}^{j-\lceil 2\rceil} \Phi_j(t)\ \mathbb{X}_{j,k}^{(\sigma)}\ \omega_r^{j-k-2} \\
&+ \left(\sum_{j=0}^{n} \Phi_j(t)\ \mathcal{C}\check{\mathcal{H}}_j^{(\alpha,\beta)}(\omega_r)\right)^3 - \left(\sum_{j=0}^{n} \Phi_j(t)\ \mathcal{C}\check{\mathcal{H}}_j^{(\alpha,\beta)}(\omega_r)\right) = \mathcal{F}(\omega_r,t).
\end{aligned} \tag{5.2}$$

The collocation points used are the roots of confluent Appell-Changhee polynomials $\check{\mathcal{CH}}^{(\alpha,\beta)}_{n+1-\lceil\sigma\rceil}(\omega)$, where $\lceil\sigma\rceil$ is called the ceiling function.

➤ Step 3. By substituting from Eq. (2.6) into Eqs. (1.2) and (1.3), we obtain

$$\sum_{j=0}^{n} \Phi_j(0)\; \check{\mathcal{CH}}^{(\alpha,\beta)}_j(\omega_r) = \mathcal{H}(\omega_r), \tag{5.3}$$

$$\begin{cases} \displaystyle\sum_{j=0}^{n} \Phi_j(t)\; \check{\mathcal{CH}}^{(\alpha,\beta)}_j(0) = \mathcal{A}_1(t), \\ \displaystyle\sum_{j=0}^{n} \Phi_j(t)\; \check{\mathcal{CH}}^{(\alpha,\beta)}_j(1) = \mathcal{A}_2(t). \end{cases} \tag{5.4}$$

To obtain the unknown coefficients $\Phi_j(t), \forall\, j = 0,1,2,...,n.$ combing Eq.(5.2) and Eq. (5.3), we obtain system of fractional ordinary differential equations (SFODEs), which can be solved by RPS approach.
To determine the unknown coefficients of $\Phi_j(t)$, we take $n = 3,\ \alpha = 3,\ \beta = \frac{1}{2}$ in Eq.(5.2):

$$\begin{cases} \mathscr{D}^{\sigma}_t\Phi_0(t) + \mathscr{D}^{\sigma}_t\Phi_1(t) + \frac{1}{2}\mathscr{D}^{\sigma}_t\Phi_3(t) - 32\Phi_2(t) + \Big(\Phi_0(t) + \Phi_1(t) + \frac{1}{2}\Phi_3(t)\Big)^3 \\ -\Big(\Phi_0(t) + \Phi_1(t) + \frac{1}{2}\Phi_3(t)\Big) = \mathcal{F}(\frac{1}{4}, t), \end{cases} \tag{5.5}$$

$$\begin{cases} \mathscr{D}^{\sigma}_t\Phi_0(t) + \frac{1}{4}\mathscr{D}^{\sigma}_t\Phi_1(t) + \frac{1}{16}\mathscr{D}^{\sigma}_t\Phi_3(t) - 32\Phi_2(t) + 24\Phi_3(t) + \Big(\Phi_0(t) + \frac{1}{4}\Phi_1(t) + \frac{1}{16}\Phi_3(t)\Big)^3 \\ -\Big(\Phi_0(t) + \frac{1}{4}\Phi_1(t) + \frac{1}{16}\Phi_3(t)\Big) = \mathcal{F}(\frac{1}{8}, t). \end{cases} \tag{5.6}$$

By solving Eq.(5.4), then we get

$$\begin{cases} \Phi_2(t) = -\frac{17}{16}\Phi_0(t) + \frac{1}{4}\Phi_1(t) + \frac{65}{64}\mathcal{A}_1(t) + \frac{3}{64}\mathcal{A}_2(t), \\ \Phi_3(t) = \frac{5}{8}\Phi_0(t) - \frac{1}{2}\Phi_1(t) - \frac{21}{32}\mathcal{A}_1(t) + \frac{1}{32}\mathcal{A}_2(t). \end{cases} \tag{5.7}$$

By substituting from Eq. (5.7) into Eq. (5.5) and Eq. (5.6) , then

$$\begin{cases} \frac{21}{16}\mathscr{D}^{\sigma}_t\Phi_0(t) + \frac{3}{4}\mathscr{D}^{\sigma}_t\Phi_1(t) - \frac{21}{64}\mathscr{D}^{\sigma}_t\mathcal{A}_1(t) + \frac{1}{64}\mathscr{D}^{\sigma}_t\mathcal{A}_2(t) + 34\Phi_0(t) - 8\Phi_1(t) \\ -\frac{65}{2}\mathcal{A}_1(t) - \frac{3}{2}\mathcal{A}_2(t) + \Big(\frac{21}{16}\Phi_0(t) + \frac{3}{4}\Phi_1(t) - \frac{21}{64}\mathcal{A}_1(t) + \frac{1}{64}A_2(t)\Big)^3 = \mathcal{F}(\frac{1}{4}, t), \end{cases} \tag{5.8}$$

$$\begin{cases} \frac{133}{128}\mathscr{D}^{\sigma}_t\Phi_0(t) + \frac{7}{32}\mathscr{D}^{\sigma}_t\Phi_1(t) - \frac{21}{512}\mathscr{D}^{\sigma}_t\mathcal{A}_1(t) + \frac{1}{512}\mathscr{D}^{\sigma}_t\mathcal{A}_2(t) + 49\Phi_0(t) - 20\Phi_1(t) \\ -\frac{192}{4}\mathcal{A}_1(t) - \frac{3}{4}\mathcal{A}_2(t) + \Big(\frac{133}{128}\Phi_0(t) + \frac{7}{32}\Phi_1(t) - \frac{21}{512}\mathcal{A}_1(t) + \frac{1}{512}\mathcal{A}_2(t)\Big)^3 = \mathcal{F}(\frac{1}{8}, t). \end{cases} \tag{5.9}$$

The solution of Eq. (5.8) and Eq. (5.9) in FPS at $t = 0$ is given by RPS approach as follows:

$$\begin{cases} \Phi_0(t) = \mathscr{F}_0 + \displaystyle\sum_{r=1}^{\infty} \mathscr{F}_r \frac{t^{r\sigma}}{\Gamma(1+r\sigma)}, \\ \Phi_1(t) = \mathscr{G}_0 + \displaystyle\sum_{r=1}^{\infty} \mathscr{G}_r \frac{t^{r\sigma}}{\Gamma(1+r\sigma)}. \end{cases} \tag{5.10}$$

Next, let $\Phi_{0r}(t)$ and $\Phi_{1r}(t)$denote the m^{th} truncated series of $\Phi_0(t)$ and $\Phi_1(t)$ which take the form:

$$\begin{cases} \Phi_{0r}(t) = \mathscr{F}_0 + \sum_{r=1}^{m} \mathscr{F}_r \frac{t^{r\sigma}}{\Gamma(1+r\sigma)}, \\ \Phi_{1r}(t) = \mathscr{G}_0 + \sum_{r=1}^{m} \mathscr{G}_r \frac{t^{r\sigma}}{\Gamma(1+r\sigma)}, \end{cases} \tag{5.11}$$

where $\mathscr{F}_0$ and $\mathscr{G}_0$ can be obtained by solving Eq. (5.3) and Eq. (5.4).
Then we write the residual function of Eq. (5.8) and Eq. (5.9) as below:

$$\begin{cases} \mathscr{RE}_1 = \frac{21}{16}\mathscr{D}_t^{\sigma}\Phi_{0r}(t) + \frac{3}{4}\mathscr{D}_t^{\sigma}\Phi_{1r}(t) - \frac{21}{64}\mathscr{D}_t^{\sigma}\mathcal{A}_1(t) + \frac{1}{64}\mathscr{D}_t^{\sigma}\mathcal{A}_2(t) + 34\Phi_{0r}(t) - 8\Phi_{1r}(t) \\ -\frac{65}{2}\mathcal{A}_1(t) - \frac{3}{2}\mathcal{A}_2(t) + \left(\frac{21}{16}\Phi_{0r}(t) + \frac{3}{4}\Phi_{1r}(t) - \frac{21}{64}\mathcal{A}_1(t) + \frac{1}{64}\mathcal{A}_2(t)\right)^3 - \mathcal{F}(\frac{1}{4}, t) = 0, \end{cases} \tag{5.12}$$

$$\begin{cases} \mathscr{RE}_2 = \frac{133}{128}\mathscr{D}_t^{\sigma}\Phi_{0r}(t) + \frac{7}{32}\mathscr{D}_t^{\sigma}\Phi_{1r}(t) - \frac{21}{512}\mathscr{D}_t^{\sigma}\mathcal{A}_1(t) + \frac{1}{512}\mathscr{D}_t^{\sigma}\mathcal{A}_2(t) + 49\Phi_{0r}(t) - 20\Phi_{1r}(t) \\ -\frac{193}{4}\mathcal{A}_1(t) - \frac{3}{4}\mathcal{A}_2(t) + \left(\frac{133}{128}\Phi_{0r}(t) + \frac{7}{32}\Phi_{1r}(t) - \frac{21}{512}\mathcal{A}_1(t) + \frac{1}{512}\mathcal{A}_2(t)\right)^3 - \mathcal{F}(\frac{1}{8}, t) = 0, \end{cases} \tag{5.13}$$

and

$$\begin{cases} \mathscr{D}_t^{(m-1)\sigma}\mathscr{RE}_1(t_0) = 0, \\ \mathscr{D}_t^{(m-1)\sigma}\mathscr{RE}_2(t_0) = 0, \quad \forall m = 1, 2, 3, ... \end{cases} \tag{5.14}$$

5.2. **Generalized time-Fractional WL Problem.**

➤ Step 1. By appling Eq. (2.6) and Eq. (3.1) into Eq. (1.4), we have

$$\sum_{j=0}^{n} \mathscr{D}_t^{2\sigma}\Phi_j(t)\ \mathcal{C}\check{\mathcal{H}}_j^{(\alpha,\beta)}(\omega) = \mathscr{R}\left(\sum_{j=0}^{n} \Phi_j(t)\ \mathcal{C}\check{\mathcal{H}}_j^{(\alpha,\beta)}(\omega)\right) + \mathscr{T}\left(\sum_{j=0}^{n} \Phi_j(t)\ \mathcal{C}\check{\mathcal{H}}_j^{(\alpha,\beta)}(\omega)\right). \tag{5.15}$$

➤ Step 2. By computing Eq. (5.15) at $(n+1-\lceil\sigma\rceil)$ points ω_r, $r = 0, 1, 2, ..., n - \lceil\sigma\rceil$ as follows:

$$\sum_{j=0}^{n} \mathscr{D}_t^{2\sigma}\Phi_j(t)\ \mathcal{C}\check{\mathcal{H}}_j^{(\alpha,\beta)}(\omega_r) = \mathscr{R}\left(\sum_{j=0}^{n} \Phi_j(t)\ \mathcal{C}\check{\mathcal{H}}_j^{(\alpha,\beta)}(\omega_r)\right) + \mathscr{T}\left(\sum_{j=0}^{n} \Phi_j(t)\ \mathcal{C}\check{\mathcal{H}}_j^{(\alpha,\beta)}(\omega_r)\right). \tag{5.16}$$

➤ Step 3. By substituting from Eq. (2.6) into Eq. (1.5) and Eq. (1.6), we have

$$\begin{cases} \sum_{j=0}^{n} \Phi_j(0)\ \mathcal{C}\check{\mathcal{H}}_j^{(\alpha,\beta)}(\omega_r) = \mathcal{C}_1(\omega_r), \\ \sum_{j=0}^{n} \mathscr{D}_t^{\sigma}\Phi_j(0)\ \mathcal{C}\check{\mathcal{H}}_j^{(\alpha,\beta)}(\omega_r) = \mathcal{C}_2(\omega_r), \end{cases} \tag{5.17}$$

$$\begin{cases} \sum_{j=0}^{n} \Phi_j(t)\ \mathcal{C}\check{\mathcal{H}}_j^{(\alpha,\beta)}(0) = \mathfrak{F}_1(t), \\ \sum_{j=0}^{n} \Phi_j(t)\ \mathcal{C}\check{\mathcal{H}}_j^{(\alpha,\beta)}(1) = \mathfrak{F}_2(t). \end{cases} \tag{5.18}$$

So, we obtain system of FODEs in $\Phi_j(t)$, $\forall\ j = 0, 1, 2, ..., n$. The SFODEs solved by utilizing RPS approach.

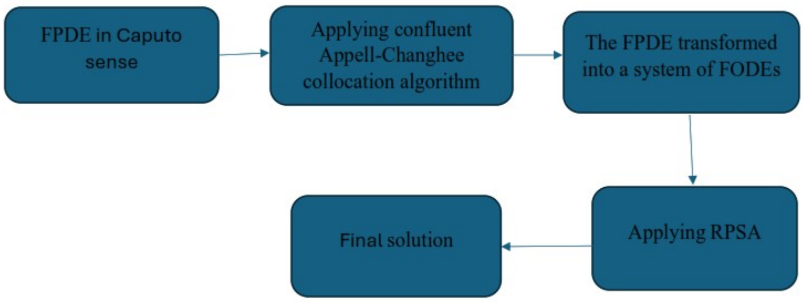

Figure 1. Flowchart describing the proposed method.

5.3. Generalized time-Fractional KdV Problem.

➤ Step 1. By appling Eq. (2.6) and Eq. (3.1) into Eq. (1.7), we have

$$\sum_{j=0}^{n} \mathscr{D}_t^{\sigma} \Phi_j(t)\, \mathcal{C}\check{\mathcal{H}}_j^{(\alpha,\beta)}(\omega) = \mathscr{L}\Big(\sum_{j=0}^{n} \Phi_j(t)\, \mathcal{C}\check{\mathcal{H}}_j^{(\alpha,\beta)}(\omega)\Big) + \mathscr{N}\Big(\sum_{j=0}^{n} \Phi_j(t)\, \mathcal{C}\check{\mathcal{H}}_j^{(\alpha,\beta)}(\omega)\Big) = 0. \tag{5.19}$$

➤ Step 2. By computing Eq. (5.19) at $(n+1-\lceil\sigma\rceil)$ points ω_r, $r = 0,1,2,...,n-\lceil\sigma\rceil$ as follows:

$$\sum_{j=0}^{n} \mathscr{D}_t^{\sigma} \Phi_j(t)\, \mathcal{C}\check{\mathcal{H}}_j^{(\alpha,\beta)}(\omega_r) = \mathscr{L}\Big(\sum_{j=0}^{n} \Phi_j(t)\, \mathcal{C}\check{\mathcal{H}}_j^{(\alpha,\beta)}(\omega_r)\Big) + \mathscr{N}\Big(\sum_{j=0}^{n} \Phi_j(t)\, \mathcal{C}\check{\mathcal{H}}_j^{(\alpha,\beta)}(\omega_r)\Big) = 0. \tag{5.20}$$

➤ Step 3. By substituting from Eq. (2.6) into Eq. (1.8) and Eq. (1.9), we get

$$\sum_{j=0}^{n} \Phi_j(0)\, \mathcal{C}\check{\mathcal{H}}_j^{(\alpha,\beta)}(\omega_r) = \mathcal{P}(\omega_r), \tag{5.21}$$

$$\begin{cases} \sum_{j=0}^{n} \Phi_j(t)\, \mathcal{C}\check{\mathcal{H}}_j^{(\alpha,\beta)}(0) = \mathcal{Q}_1(t), \\ \sum_{j=0}^{n} \Phi_j(t)\, \mathcal{C}\check{\mathcal{H}}_j^{(\alpha,\beta)}(1) = \mathcal{Q}_2(t). \end{cases} \tag{5.22}$$

Hence, we get system of FODEs in $\Phi_j(t)$, $\forall\, j = 0,1,2,...,n$. The SFODEs solved by utilizing RPS approach. All numerical outcomes in this study were done by using MATLAB R2017b. Figure 1, displays flowchart the proposed method.

6. Numerical Problems

To demonstrate how accurate, effective and reliable of suggested method in this study, we present some numerical problems.

Proposition 6.1. *Consider the non-linear time fractional CA problem [18, 44]*

$$\begin{cases} \mathscr{D}_t^{\sigma}\, \mathbf{P}(\omega,t) - \mathscr{D}_\omega^{2}\mathbf{P}(\omega,t) + \mathbf{P}^3(\omega,t) - \mathbf{P}(\omega,t) = \mathcal{F}(\omega,t),\ 0<\sigma\le 1,\ \omega\in[0,1],\ t\in[0,1], \\ \mathbf{P}(\omega,0) = 0, \\ \mathbf{P}(0,t) = 0, \quad \mathbf{P}(1,t) = 0, \end{cases} \tag{6.1}$$

where

$$\mathcal{F}(\omega,t) = \big(\omega^2-\omega\big)t\Gamma(\sigma+2) + \big(\omega^2-\omega\big)t^{3+3\sigma} - \big(\omega^2-\omega+2\big)t^{1+\sigma}.$$

An exact solution of Eq. (6.1) when $\sigma = 1$ is given by $\mathbf{P}(\omega,t) = \big(\omega^2-\omega\big)t^{1+\sigma}$.

According to exposition of the present method which shown in Section 5, we obtained an approximate solution when $n = 3$ as follows:
$\mathbf{P}_3(\omega,t) = \Phi_0(t) + \Phi_1(t)\left(6\omega - \frac{1}{2}\right) + \Phi_2(t)\left(16\omega^2 - 6\omega + \frac{1}{2}\right) + \Phi_3(t)\left(32\omega^3 - 24\omega^2 + 9\omega - \frac{3}{4}\right),$
where

$$\begin{cases} \Phi_0(t) = -\dfrac{1}{12}t^{\sigma+1}, \quad \Phi_1(t) = -\dfrac{5}{48}t^{\sigma+1} \\ \Phi_2(t) = -\dfrac{17}{16}\Phi_0(t) + \dfrac{1}{4}\Phi_1(t) \quad \textit{and} \quad \Phi_3(t) = \dfrac{5}{8}\Phi_0(t) - \dfrac{1}{2}\Phi_1(t). \end{cases}$$

Proposition 6.2. *Consider the non-linear time fractional WL problem [4]*

$$\begin{cases} \mathscr{D}_t^{2\sigma}\ \mathbf{P}(\omega,t) + \mathbf{P}(\omega,t) - \omega^2\mathscr{D}_\omega\big(\mathbf{P}_\omega\mathbf{P}_{\omega\omega}\big) + \omega^2\big(\mathbf{P}_{\omega\omega}\big)^2 = 0,\ \dfrac{1}{2} < \sigma \le 1,\ \omega \in [0,1],\ t \in [0,1], \\ \mathbf{P}(\omega,0) = 0, \quad \mathscr{D}_t^{\sigma}\mathbf{P}(\omega,0) = \omega^2, \\ \mathbf{P}(0,t) = 0, \quad \mathbf{P}(1,t) = sin(t). \end{cases} \tag{6.2}$$

An exact solution of Eq. (6.2) when $\sigma = 1$ is given by $\mathbf{P}(\omega,t) = \omega^2 sin(t)$.

According to exposition of the present method which shown in Section 5, we obtained an approximate solution when $n = 3$ as follows:
$\mathbf{P}_3(\omega,t) = \Phi_0(t) + \Phi_1(t)\left(6\omega - \frac{1}{2}\right) + \Phi_2(t)\left(16\omega^2 - 6\omega + \frac{1}{2}\right) + \Phi_3(t)\left(32\omega^3 - 24\omega^2 + 9\omega - \frac{3}{4}\right),$
where

$$\begin{cases} \Phi_0(t) = 0, \quad \Phi_1(t) = \dfrac{1}{16}\left(\dfrac{t^{\sigma}}{\Gamma(1+\sigma)} - \dfrac{t^{3\sigma}}{\Gamma(1+3\sigma)} + \dfrac{t^{5\sigma}}{\Gamma(1+5\sigma)} + \ldots\right), \\ \Phi_2(t) = -\dfrac{17}{16}\Phi_0(t) + \dfrac{1}{4}\Phi_1(t) + \dfrac{3}{64}sin(t) \quad \textit{and} \quad \Phi_3(t) = \dfrac{5}{8}\Phi_0(t) - \dfrac{1}{2}\Phi_1(t) + \dfrac{1}{32}sin(t). \end{cases}$$

Proposition 6.3. *Consider the non-linear time fractional KdV problem [31]*

$$\begin{cases} \mathscr{D}_t^{\sigma}\ \mathbf{P}(\omega,t) + \dfrac{1}{2}\mathscr{D}_\omega\big(\mathbf{P}(\omega,t)\big)^2 - \mathbf{P}_{\omega\omega\omega} = 0,\ 0 < \sigma \le 1,\ \omega \in [0,1],\ t \in [0,1], \\ \mathbf{P}(\omega,0) = 0, \\ \mathbf{P}(0,t) = \omega, \quad \mathbf{P}(1,t) = \dfrac{1}{1+t}. \end{cases} \tag{6.3}$$

An exact solution of Eq. (6.3) when $\sigma = 1$ is given by $\mathbf{P}(\omega,t) = \dfrac{\omega}{1+t}$.
According to exposition of the present method which shown in Section 5, we obtained an approximate solution when $n = 3$ as follows:

Table 1. Absolute error for present method at $\sigma = 1$ for problem (6.1).

$\omega/t \rightarrow$	0.1	0.2	0.3	0.4	0.5
0.1	1.08420E-19	4.33681E-19	1.73472E-18	1.73472E-18	3.46945E-18
0.2	2.16840E-19	8.67362E-19	1.73472E-18	3.46945E-18	0.00000E+00
0.3	0.00000E+00	0.00000E+00	3.46945E-18	0.00000E+00	6.93889E-18
0.4	4.33681E-19	1.73472E-18	3.46945E-18	6.93889E-18	1.38778E-17
0.5	0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00
0.6	8.67362E-19	3.46945E-18	0.00000E+00	1.38778E-17	0.00000E+00
0.7	4.33681E-19	1.73472E-18	1.04083E-17	6.93889E-18	6.93889E-18
0.8	1.30104E-18	5.20417E-18	0.00000E+00	2.08167E-17	2.77556E-17
0.9	7.58942E-19	3.03577E-18	6.93889E-18	1.21431E-17	3.46945E-17
$\omega/t \rightarrow$	0.6	0.7	0.8	0.9	
0.1	6.93889E-18	6.93889E-18	6.93889E-18	1.38778E-17	
0.2	6.93889e-18	2.77556E-17	1.38778E-17	0.00000E+00	
0.3	1.38778E-17	1.38778E-17	0.00000E+00	2.77556E-17	
0.4	1.38778E-17	2.77556E-17	2.77556E-17	2.77556E-17	
0.5	0.00000E+00	1.38778E-17	0.00000E+00	0.00000E+00	
0.6	0.00000E+00	2.77556E-17	5.55112E-17	2.77556E-17	
0.7	4.16334E-17	5.55112E-17	2.77556E-17	2.77556E-17	
0.8	0.00000E+00	1.38778E-17	8.32667E-17	1.11022E-16	
0.9	2.77556E-17	3.46945E-17	4.85723E-17	9.71445E-17	

$$\mathbf{P}_3(\omega,t) = \Phi_0(t) + \Phi_1(t)\left(6\omega - \frac{1}{2}\right) + \Phi_2(t)\left(16\omega^2 - 6\omega + \frac{1}{2}\right) + \Phi_3(t)\left(32\omega^3 - 24\omega^2 + 9\omega - \frac{3}{4}\right),$$

where

$$\begin{cases} \Phi_0(t) = \dfrac{1}{12}\left(1 - \dfrac{t^\sigma}{\Gamma(1+\sigma)} + \dfrac{2t^{2\sigma}}{\Gamma(1+2\sigma)} - \dfrac{6t^{3\sigma}}{\Gamma(1+3\sigma)} + ...\right), \\[2ex] \Phi_1(t) = \dfrac{1}{6}\left(1 - \dfrac{t^\sigma}{\Gamma(1+\sigma)} + \dfrac{2t^{2\sigma}}{\Gamma(1+2\sigma)} - \dfrac{6t^{3\sigma}}{\Gamma(1+3\sigma)} + ...\right), \\[2ex] \Phi_2(t) = -\dfrac{17}{16}\Phi_0(t) + \dfrac{1}{4}\Phi_1(t) + \dfrac{3}{64(1+t)} \quad \textit{and} \quad \Phi_3(t) = \dfrac{5}{8}\Phi_0(t) - \dfrac{1}{2}\Phi_1(t) + \dfrac{1}{32(1+t)}. \end{cases}$$

7. Results and Discussion

Figures 2, 4, and 6 articulate the comparison between exact, approximate solution for problems 6.1, (6.2) and (6.3). Figures 3, 5, and 7 describe the performance of exact and approximate solution at $\sigma = 1$, 0.9, 0.8 and 0.7 in two dimensional graph for problems (6.1), (6.2), and (6.3). Table 1 demonstrate an absolute error of the proposed method at $\sigma = 1$ for problem (6.1). Tables 2, 3, and 4 provide the comparison of absolute error for proposed method with RKM [44] and ExCBS [18] at different values of σ for problem (6.1). Table 5 represents CPU time for proposed method and RKM [44] for problem 6.1. Tables 6 and 7 display the comparison of absolute error and approximate solution at $\sigma = 1$, 0.9, 0.8 and 0.7 when $\omega = 0.5$ with LRPST [4] for problem 6.2. Tables 8 and 9 show numerical outcomes obtained by proposed method with RPSM [31] and q-HATM [31] for problem (6.3). Table 10 show CPU time in (second) for problems (6.2) and (6.3). The above tabulars and graphics prove that the method used is accurate compared to other methods. Also, the results showed new solutions using fractional calculus which enables us to study the models better.

TABLE 2. Comparison of absolute errors at different values of σ for problem (6.1).

	3 $c\sigma = 0.9$	3 $c\sigma = 0.7$					
t	ω	RKM [44]	ExCBS [18]	present method	RKM [44]	ExCBS [18]	present method
0.1	0.1	2.58E-4	3.0507E-6	0.00000E+00	3.23E-4	7.7669E-7	4.33681E-19
	0.2	4.66E-4	5.7631E-6	4.33681E-19	5.82E-4	1.4691E-6	8.67362E-19
	0.3	6.18E-4	7.8782E-6	0.00000E+00	7.70E-4	2.0109E-6	0.00000E+00
	0.4	7.11E-4	9.2171E-6	4.33681E-19	8.85E-4	2.3548E-6	0.00000E+00
	0.5	7.42E-4	9.6748E-6	0.00000E+00	9.24E-4	2.4726E-6	0.00000E+00
	0.6	7.10E-4	9.2171E-6	0.00000E+00	8.85E-4	2.3548E-6	8.67362E-19
	0.7	6.16E-4	7.8782E-6	4.33681E-19	7.69E-4	2.0109E-6	8.67362E-19
	0.8	4.63E-4	5.7631E-6	1.73472E-18	5.79E-4	1.4691E-6	2.16840E-18
	0.9	2.55E-4	3.0507E-6	6.50521E-19	3.21E-4	7.7669E-7	1.30104E-18
0.2	0.1	7.34E-5	3.5748E-6	8.67362E-19	4.72E-5	6.8604E-7	8.67362E-19
	0.2	1.41E-4	6.7627E-6	8.67362E-19	9.50E-5	1.2984E-6	0.00000E+00
	0.3	1.98E-4	9.2577E-6	0.00000E+00	1.36E-4	1.7783E-6	0.00000E+00
	0.4	2.36E-4	1.0841E-5	1.73472E-18	1.64E-4	2.0833E-6	3.46945E-18
	0.5	2.51E-4	1.1384E-5	0.00000E+00	1.75E-4	2.1878E-6	0.00000E+00
	0.6	2.40E-4	1.0841E-5	0.00000E+00	1.68E-4	2.0833E-6	0.00000E+00
	0.7	2.04E-4	9.2577E-6	3.46945E-18	1.42E-4	1.7783E-6	5.20417E-18
	0.8	1.48E-4	6.7627E-6	2.60209E-18	1.01E-4	1.2984E-6	5.20417E-18
	0.9	7.68E-5	3.5748E-6	8.67362E-19	5.12E-5	6.8604E-7	6.07153E-18
0.3	0.1	7.11E-5	3.6734E-6	1.73472E-18	3.74E-5	6.2886E-7	1.73472E-18
	0.2	1.40E-4	6.9522 E-6	3.46945E-18	7.91E-5	1.1911E-6	3.46945E-18
	0.3	1.98E-4	9.5210E-6	0.00000E+00	1.14E-4	1.6327E-6	3.46945E-18
	0.4	2.37E-4	1.1153E-5	0.00000E+00	1.39E-4	1.9139E-6	1.04083E-17
	0.5	2.54E-4	1.1713E-5	0.00000E+00	1.51E-4	2.0105E-6	6.93889E-18
	0.6	2.45E-4	1.1153E-5	6.93889E-18	1.47E-4	1.9139E-6	6.93889E-18
	0.7	2.12E-4	9.5210E-6	1.04083E-17	1.28E-4	1.6327E-6	1.38778E-17
	0.8	1.56E-4	6.7627E-6	3.46945E-18	9.52E-5	1.1911E-6	1.04083E-17
	0.9	8.36E-5	3.5748E-6	1.73472E-18	5.08E-5	6.2886E-7	5.20417E-18

8. CONCLUSION

In this study, a powerful semi-analytical method based on confluent Appell-Changhee collocation algorithm with RPS approach was used to obtain a solution to three proposed problems. Using the properties of confluent Appell-Changhee polynomials, we transform these problems into SFODEs, which can be solved by RPS approach. All numerical results presented through tables and figures. Numerical outcomes of the method used were compared with some other published methods to determine the accuracy of the method. Furthermore, an error term of the proposed method was calculated. In this context, the numerical simulation provided by the present method is effective and has high accuracy in finding accurate results. Thus, it can be used to obtain a solution to many nonlinear fractional problems that arise in mathematical physics.

Table 3. Comparison of absolute errors at different values of σ for problem (6.1).

	3 $c\sigma = 0.9$	3 $c\sigma = 0.7$					
t	ω	RKM [44]	ExCBS [18]	present method	RKM [44]	ExCBS [18]	present method
	0.1	1.06E-6	3.6690E-6	3.46945E-18	3.31E-5	6.0350E-7	0.00000E+00
	0.2	1.42E-5	6.9461E-6	6.93889E-18	4.43E-5	1.1453E-6	0.00000E+00
	0.3	3.17E-5	9.5159E-6	6.93889E-18	4.80E-5	1.5735E-6	0.00000E+00
	0.4	4.69E-5	1.1150E-5	6.93889E-18	4.66E-5	1.8477E-6	6.93889E-18
0.4	0.5	5.70E-5	1.1710E-5	0.00000E+00	4.18E-5	1.9421E-6	6.93889E-18
	0.6	5.86E-5	1.1150E-5	6.93889E-18	3.59E-5	1.8477E-6	1.38778E-17
	0.7	5.25E-5	9.5159E-6	0.00000E+00	2.87E-5	1.5735E-6	1.38778E-17
	0.8	3.86E-5	6.9461E-6	2.42861E-17	2.09E-5	1.1453E-6	6.93889E-18
	0.9	1.99E-5	3.6690E-6	3.46945E-18	1.17E-5	6.0350E-7	3.46945E-18
	0.1	3.18E-5	3.6545E-6	0.00000E+00	4.38E-5	6.1561E-7	3.46945E-18
	0.2	3.65E-5	6.9223E-6	6.93889E-18	5.85E-5	1.1731E-6	6.93889E-18
	0.3	3.36E-5	9.4894E-6	0.00000E+00	6.41E-5	1.6189E-6	1.38778E-17
	0.4	2.63E-5	1.1124E-5	2.77556E-17	6.28E-5	1.9075E-6	1.38778E-17
0.5	0.5	1.69E-5	1.1685E-5	0.00000E+00	5.62E-5	2.0074E-6	0.00000E+00
	0.6	9.07E-6	1.1124E-5	0.00000E+00	4.70E-5	1.9075E-6	1.38778E-17
	0.7	2.63E-6	9.4894E-6	2.08167E-17	3.56E-5	1.6189E-6	1.38778E-17
	0.8	5.42E-7	6.9223E-6	6.93889E-18	2.37E-5	1.1731E-6	3.46945E-17
	0.9	1.15E-6	3.6545E-6	4.16334E-17	1.17E-5	6.1561E-7	3.81639E-17
	0.1	6.59E-5	3.6715E-6	6.93889E-18	6.78E-5	6.7886E-7	6.93889E-18
	0.2	9.17E-5	6.9623E-6	1.38778E-17	9.51E-5	1.3017E-6	1.38778E-17
	0.3	1.03E-4	9.5559E-6	0.00000E+00	1.08E-4	1.8089E-6	0.00000E+00
	0.4	1.03E-4	1.1213E-5	2.77556E-17	1.10E-4	2.1424E-6	2.77556E-17
0.6	0.5	9.49E-5	1.1782E-5	0.00000E+00	1.03E-4	2.2589E-6	2.77556E-17
	0.6	8.10E-5	1.1213E-5	1.38778E-17	9.03E-5	2.1424E-6	0.00000E+00
	0.7	6.23E-5	9.5559E-6	1.38778E-17	7.14E-5	1.8089E-6	0.00000E+00
	0.8	4.21E-5	6.9623E-6	2.08167E-17	4.96E-5	1.3017E-6	5.55112E-17
	0.9	2.10E-5	3.6715E-6	6.93889E-18	2.54E-5	6.7886E-7	3.46945E-17

Table 4. Comparison of absolute errors at different values of σ for problem (6.1).

	3 $c\sigma = 0.9$	3 $c\sigma = 0.7$					
t	ω	RKM [44]	ExCBS [18]	present method	RKM [44]	ExCBS [18]	present method
0.7	0.1	7.91E-5	3.7543E-6	6.93889E-18	7.25E-5	8.1346E-7	0.00000E+00
	0.2.	1.07E-4.	7.1331E-6	1.38778E-17.	9.65E-5	1.5715E-6	0.00000E+00
	0.3	1.20E-4	9.8119E-6	0.00000E+00	1.06E-4	2.2017E-6	1.38778E-17
	0.4	1.19E-4	1.1532E-5	2.77556E-17	1.04E-4	2.6234E-6	5.55112E-17
	0.5	1.07E-4	1.2125E-5	0.00000E+00	9.40E-5	2.7720E-6	0.00000E+00
	0.6	9.00E-5	1.1532E-5	1.38778E-17	7.87E-5	2.6234E-6	2.77556E-17
	0.7	6.75E-5	9.8191E-6	1.38778E-17	5.90E-5	2.2017E-6	1.38778E-17
	0.8	4.43E-5	7.1331E-6	8.32667E-17	3.86E-5	1.5715E-6	8.32667E-17
	0.9	2.13E-5	3.7543E-6	9.71445E-17	1.85E-5	8.1346E-7	7.63278E-17
0.8	0.1	6.96E-5	3.9420E-6	6.93889E-18	8.87E-5	1.0454E-6	6.93889E-18
	0.2	8.14E-5	7.5125E-6	1.38778E-17	1.17E-4	2.0338E-6	1.38778E-17
	0.3	7.88E-5	1.0369E-5	0.00000E+00	1.28E-4	2.8714E-6	0.00000E+00
	0.4	6.69E-5	1.2218E-5	2.77556E-17	1.27E-4	3.4402E-6	2.77556E-17
	0.5	4.85E-5	1.2859E-5	2.77556E-17	1.13E-4	3.6424E-6	0.00000E+00
	0.6	3.09E-5	1.2218E-5	2.77556E-17	9.52E-5	3.4402E-6	0.00000E+00
	0.7	1.40E-5	1.0369E-5	2.77556E-17	7.14E-5	2.8714E-6	0.00000E+00
	0.8	3.08E-6	7.5125E-6	4.16334E-17	4.68E-5	2.0338E-6	4.16334E-17
	0.9	1.71E-6	3.9420E-6	2.77556E-17	2.25E-5	1.0454E-6	3.46945E-17
0.9	0.1	4.67E-5	4.2838E-6	0.00000E+00	5.08E-5	1.4058E-6	1.38778E-17
	0.2	2.96E-5	8.1981E-6	1.38778E-17	4.09E-5	2.7504E-6	0.00000E+00
	0.3	3.02E-6	1.1368E-5	0.00000E+00	2.22E-5	3.9063E-6	0.00000E+00
	0.4	2.52E-5	1.3442E-5	2.77556E-17	5.79E-7	4.7002E-6	5.55112E-17
	0.5	5.10E-5	1.4164E-5	2.77556E-17	2.09E-5	4.9841E-6	5.55112E-17
	0.6	6.54E-5	1.3442E-5	2.77556E-17	3.54E-5	4.7002E-6	8.32667E-17
	0.7	6.93E-5	1.1368E-5	2.77556E-17	4.31E-5	3.9063E-6	2.77556E-17
	0.8	5.82E-5	8.1981E-6	1.11022E-16	3.96E-5	2.7504E-6	1.11022E-16
	0.9	3.40E-5	4.2838E-6	1.38778E-17	2.52E-5	1.4057E-6	6.93889E-17

Table 5. The CPU time in(second) at different values of σ for problem (6.1).

2 $c\sigma = 1$	3 $c\sigma = 0.9$	3 $c\sigma = 0.7$			
RKM [44]	present method	RKM [44]	present method	RKM [44]	present method
–	0.95	112.64	0.69	113.65	0.95

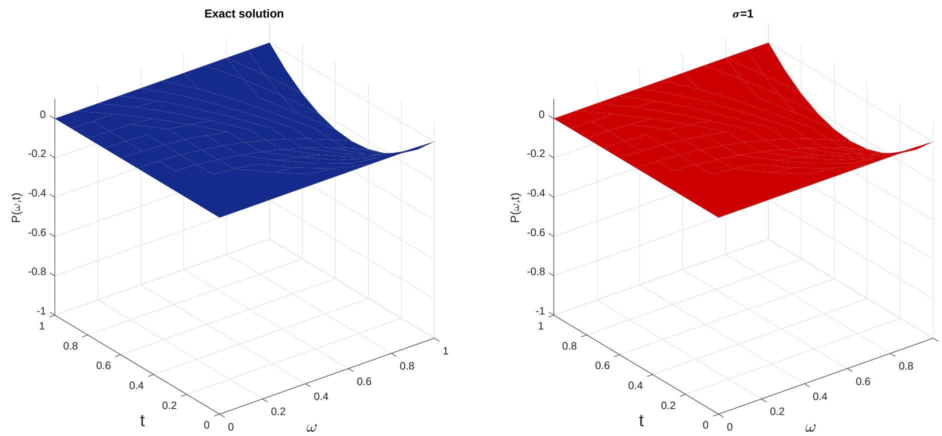

FIGURE 2. Behavior of exact and approximate solutions for problem (6.1).

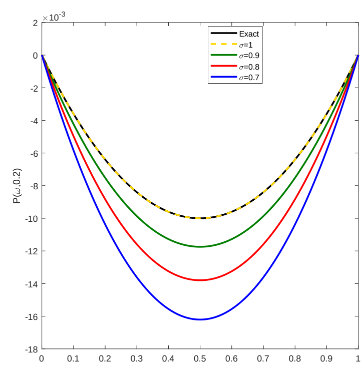

FIGURE 3. Two-dimensional plots of exact and approximate solutions at various values of σ at $t = 0.2$ for problem (6.1).

TABLE 6. Numerical results at $\omega = 0.5$. for problem (6.2).

	3 c$\sigma = 1$.		3 c Absolute error		
t	Exact	LRPST [4]	present method	LRPST [4]	present method
0.1	0.0249584	0.0249584	0.0249584	6.8695E-16	0.00000E+00
0.2	0.0496673	0.0496673	0.0496673	3.526E-13	0.00000E+00
0.3	0.0738801	0.0738801	0.0738801	1.35492E-11	0.00000E+00
0.4	0.0973546	0.0973546	0.0973546	1.80377E-10	0.00000E+00
0.5	0.119856	0.119856	0.119856	1.34252E-9	0.00000E+00
0.6	0.141161	0.141161	0.141161	6.92019E-9	0.00000E+00
0.7	0.161054	0.161054	0.161054	2.76775E-8	0.00000E+00
0.8	0.179339	0.179339	0.179339	9.19312E-8	0.00000E+00
0.9	0.195832	0.195831	0.195832	2.64952E-7	0.00000E+00

Table 7. Approximate solution at diverse of σ at $\omega = 0.5$. for problem (6.2).

	2 c$\sigma = 0.9$.	2 c$\sigma = 0.8$.	2 c$\sigma = 0.7$.			
t	LRPST [4]	present method	LRPST	present method	LRPST[4]	present method
0.1	0.0326049	0.028174	0.0422085	0.032208	0.0540002	0.037255
0.2	0.060292	0.053930	0.0723233	0.058891	0.0853825	0.064644
0.3	0.0856574	0.078435	0.0978699	0.083500	0.109684	0.089107
0.4	0.108979	0.101737	0.119923	0.106450	0.129109	0.111497
0.5	0.130281	0.123754	0.138913	0.127839	0.144648	0.132106
0.6	0.149517	0.144369	0.155073	0.147667	0.156902	0.151047
0.7	0.16662	0.163458	0.168561	0.165893	0.166296	0.168353
0.8	0.181529	0.180896	0.179507	0.182458	0.173165	0.184023
0.9	0.194193	0.196568	0.188031	0.197303	0.177787	0.198036

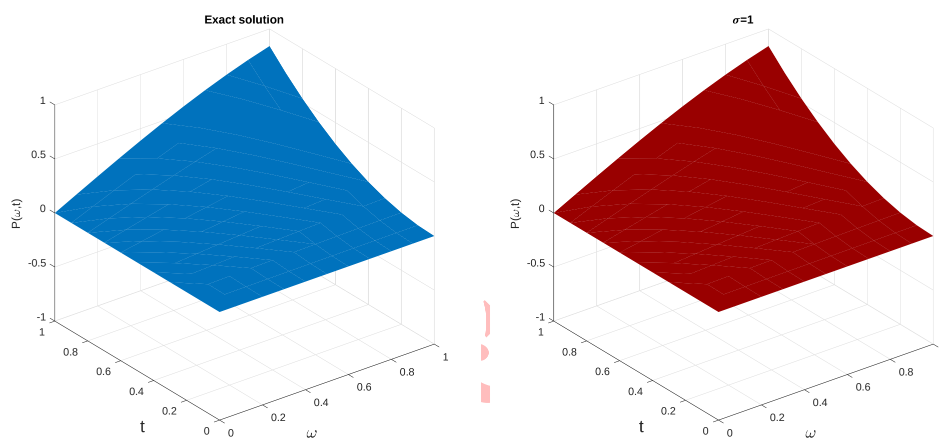

Figure 4. Behavior of exact and approximate solutions for problem (6.2).

Table 8. Comparison of absolute error for problem (6.3).

t	ω	RPSM [31]	q-HATM [31]	present method
	0.25	4E-7	2E-7	5.55112E-17
	0.5	7E-7	4E-7	5.55112E-17
0.001	0.75	1.1E-6	7E-7	1.11022E-16
	1	1.5E-6	9E-7	0.00000E+00
	0.25	9.4E-6	6.2E-6	0.00000E+00
	0.5	1.86E-5	1.24E-5	0.00000E+00
0.005	0.75	2.8E-5	1.85E-5	1.11022E-16
	1	3.74E-5	2.47E-5	0.00000E+00

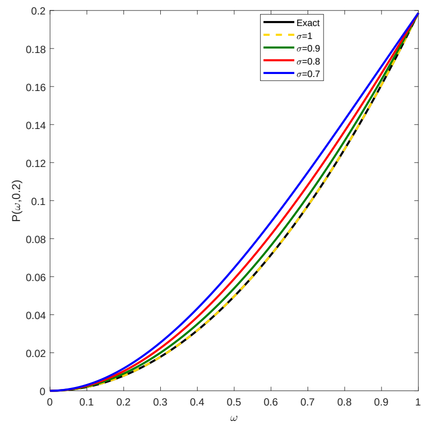

FIGURE 5. Two-dimensional plots of exact and approximate solutions at various values of σ at $t = 0.2$ for problem (6.2).

TABLE 9. Numerical outcomes at different value of t and ω for problem (6.3).

	4 c$\sigma = 1$.	5 c$\sigma = 0.9$.						
t	ω	Exact	RPSM [31]	q-HATM [31]	present method	RPSM [31]	q-HATM [31]	present method
	0.25	0.249750	0.2497498	0.2497500	0.249750	0.2494807	0.2494813.	0.249498.
	0.5	0.499500	0.4994997	0.4995000	0.499500	0.4989615	0.4989627	0.499098
0.001	0.75	0.749251	0.7492496	0.7492500	0.749251	0.7484422	0.7484440	0.748898
	1	0.999001	0.9989994	0.9990000	0.999001	0.9979230	0.9979254	0.999001
	0.25	0.248756	0.2487468	0.2487500	0.248756	0.2477814	0.2477924	0.247859
	0.5	0.497512	0.4974937	0.4975000	0.497512	0.4955629	0.4955848	0.496077
0.005	0.75	0.746269	0.7462406	0.7462501	0.746269	0.7433444	0.7433772	0.745012
	1	0.995025	0.9949874	0.9950001	0.995025	0.9911259	0.9911697	0.995025

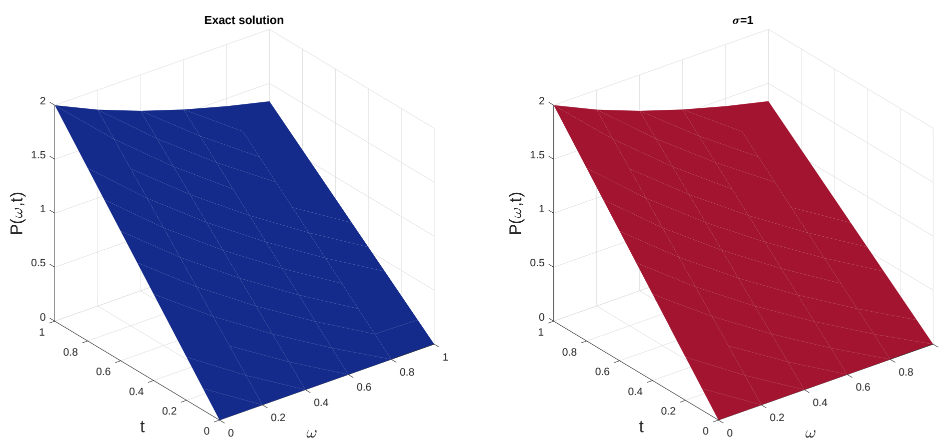

FIGURE 6. Behavior of exact and approximate solutions for problem (6.3).

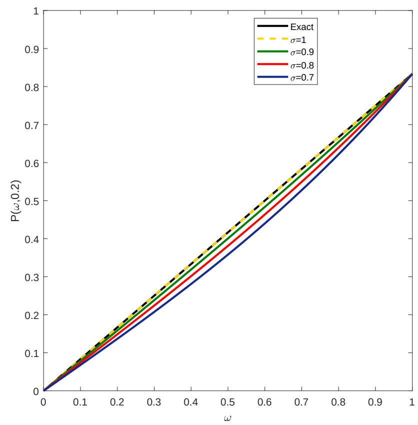

FIGURE 7. Two-dimensional plots of exact and approximate solutions at various values of σ at $t = 0.2$ for problem (6.3).

TABLE 10. CPU time in (second) at different values of σ.

	$\sigma = 1$	$\sigma = 0.9$	$\sigma = 0.8$
problem (6.2)	0.25	0.14	0.11
problem (6.3)	0.39	0.14	0.19

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