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A new class of conjugate gradient method based on secant condition for unconstrained optimization with application

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Abstract

The Conjugate gradient algorithms are among the efficient and widely considered numerical algorithms for solving large-scale minimization problems. This is due to their low memory requirement and global convergence properties. This paper constructs a new class of conjugate gradient method based on the famous secant condition and Perry's conjugacy condition for unconstrained optimization and image restoration problems. This study is motivated by recent quasi-Newton methods available in literature. The method's main goal is to use information from the secant condition to dynamically modify the conjugacy direction. It is anticipated that this flexibility will raise the optimization process's general efficiency. The suggested algorithm's convergence qualities are established through theoretical analyzes, guaranteeing its dependability and efficiency. An interesting feature of the proposed method is that the search direction possesses the descent property irrespective of the line search strategies. The global convergence of the proposed algorithm is established for uniformly convex function under the weak Wolfe line search. Furthermore, the paper explores practical applications of the proposed CG algorithm in diverse domains such as image restoration and large-scale optimization models. The efficiency of the proposed method is demonstrated through computational test, comparing the performance with other classical state-of-the-art conjugate gradient algorithms. The results demonstrate the algorithm's efficiency and superior convergence behavior, especially in cases with large-scale and complex optimization landscapes. In summary, this work presents a novel viewpoint on conjugate gradient method and provides a viable path forward for optimization strategies. Practical applications illustrate the versatility of the proposed approach and show its potential impact on effectively tackling real-world optimization difficulties.

Keywords. Conjugate gradient formula, Global convergence properties, Image Restoration, Secant Condition, Unconstrained Optimization.
2010 Mathematics Subject Classification. 90C30, 90C30, 65K05.

1. Introduction

Minimization problems are among the widely studied optimization problems that involve finding the smallest value of a function subject to some constraints. These types of optimization problems could either be constrained or unconstrained and share a common iterative procedure. However, there is a crucial distinction because both the step size and the search direction must take constraints into account [3]. An alternative method for identifying either one may result in an alternative optimization process. This study is more interested in numerical method for solving the following unconstrained optimization problem:

$$\min \{ f(x) : x \in \mathcal{R}^n \} . \quad (1.1)$$

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where $f:\mathcal{R}^n \rightarrow \mathcal{R}$ and $f \in C^2$. The unconstrained optimization problem of form Eq. (1) has several real-life applications including image restoration, robotic motion control, training of neural network, regression analysis, and many more [9, 20]. Some of the earliest methods for solving Eq. (1.1) are the trust region algorithms, Newton method, Quasi-Newton method, and their variants. An interesting feature of these methods is their locally fast convergence property. However, these methods have some major drawbacks which includes finding the minimum of a function using second-order information (the Hessian matrix) or the approximate Hession matrix during the iteration process. This procedure is always expensive, especially if large-scale problems are considered [9, 27].

To avoid the major drawbacks of the Newton-type method, researchers developed and presented the conjugate gradient method. The CG method has been considered as one of the widely and most efficient numerical methods for solving problems of the form Eq. (1.1). The CG algorithm compute its iterative points using:

$$x_{k+1} = x_k + \alpha_k d_k. \tag{1.2}$$

where x_{k+1} is the current iterative point, x_k is the previous iterate, α_k is the step-size computed using a suitable line search strategy. Also, d_k is the search direction computed via:

$$d_{k+1} = -g_{k+1} + \beta_k d_k. \tag{1.3}$$

where β_k is a scalar that differentiate different CG method. Some of the earliest CG methods β_k are presented by Fletcher-Reeves (FR) [8]. Dai-Yuan (DY) [4], and Conjugate descent (CD) [7] with their formulas defined as:

$$\beta_k^{FR} = \frac{\|g_k\|^2}{\|g_{k-1}\|^2}, \quad \beta_k^{DY} = \frac{\|g_k\|^2}{d_{k-1}^T y_{k-1}}, \quad \beta_k^{CD} = \frac{\|g_k\|^2}{-d_{k-1}^T g_{k-1}}. \tag{1.4}$$

where $y_{k-1} = g_k - g_{k-1}$ and $\|\bullet\|$ denotes the Euclidean norm. It is a known fact that the FR, CD, and DY approaches have superior theoretical convergence. However, numerical performance is influenced by ill conditioning. The Dai-Yuan method with the standard Wolfe line search, consistently produces descent directions and is globally convergent under the Lipschitz assumption [4]. Also, the FR method has the descent property under certain standard inexact line search conditions [2, 18], and the method is globally convergent under some mild assumptions [1]. For more recent studies on the convergence of CG methods [5, 19, 23].

Motivated by the above discussion, this study will develop a new variant of conjugate gradient method for unconstrained optimization with application to image restoration in section 2. Section 3 will discuss the convergence analysis of the proposed method. Preliminary results on unconstrained optimization and image restoration problem will be presented in Section 4 and Section 5 will be presented the conclusion.

2. New method and algorithm

Recently, Wei et al.,[26] constructed a modified secant condition as follows:

$$s_k^T Q(u_k) s_k = s_k^T y_k + 2(f_k - f_{k+1}) + (g_{k+1} + g_k)^T s_k. \tag{2.1}$$

In general derivatives difference y_k defined the relative as:

$$y_k = g_{k+1} - g_k = Q(x_k) s_k. \tag{2.2}$$

Multiplying Eq. (2.2) by s_k^T implies:

$$s_k^T y_k = s_k^T g_{k+1} - s_k^T g_k. \tag{2.3}$$

Putting Eq. (2.1) in Eq. (2.2), and by advantage of Perry's conjugacy condition $d_{k+1}^T y_k = -s_k^T g_{k+1}$ will produce:

$$d_{k+1}^T y_k = -s_k^T y_k + 2(f_k - f_{k+1}) + (g_{k+1} + g_k)^T s_k - s_k^T g_k. \tag{2.4}$$

Using $d_{k+1} = -g_{k+1} + \beta_k s_k$in Eq. (1.4) reduces to:

$$\beta_k s_k^T y_k = g_{k+1}^T y_k - s_k^T y_k + 2(f_k - f_{k+1}) + (g_{k+1} + g_k)^T s_k - s_k^T g_k. \tag{2.5}$$

After some simplification, Eq. (2.5) becomes:

$$\beta_k = \frac{g_{k+1}^T y_k}{s_k^T y_k} - \frac{s_k^T y_k + 2(f_k - f_{k+1}) + (g_{k+1} + g_k)^T s_k}{s_k^T y_k} - \frac{s_k^T g_k}{s_k^T y_k}. \tag{2.6}$$

Algorithm 1. Modified conjugate gradient Algorithm (**BILIA**).
Step 0: Given the initial point $x_0 \in \Re^n$, $\epsilon \in (0,1)$ as positive constants, $\beta_k > 0$. Set $k = 0$.
Step 1: If $\|g_k\| \le \epsilon$, stop.
Step 2: Update the search direction d_k via

$$d_{k+1} = -g_{k+1} + \beta_k s_k. \tag{2.7}$$

Step 3: Compute the step size α_k based on some line search strategies such that:

$$f(x_k + \alpha_k d_k) \le f(x_k) + \delta \alpha_k \mathfrak{g}_k^T d_k. \tag{2.8}$$

$$d_k^T \mathfrak{g}(x_k + \alpha_k d_k) \ge \sigma\, d_k^T \mathfrak{g}_k. \tag{2.9}$$

tep 4: Obtain new iterative points x_{k+1} using Eq.2.
Step 5: Set $k = k+1$ and return to step 2.

3. Convergence properties

This section will discuss the convergence analysis of the new formula. To achieve the convergence result, the following assumption is needed.

Assumption A.
(i). The level set$\Omega = \{x \in R^n | f(x) \le f(x_1)\}$ is bounded.
(ii). In some neighborhood Q of Ω, $f$is continuously differentiable, and its gradient is Lipschitz continuous; that is, for all, $x, y \in Q$, there exists a constant $L > 0$ such that:

$$\|g(x) - g(y)\| \le L \|x - y\|. \tag{3.1}$$

and by recalling strongly convex function definition, it implies that for some constant $\mu > 0$:

$$(\nabla f(x) - \nabla f(y))^T (x - y) \ge \mu \|x - y\|^2. \tag{3.2}$$

or equivalently:

$$y_k^T s_k \ge \mu \|s_k\|^2 \text{ and } \mu \|s_k\|^2 \le y_k^T s_k \le L \|s_k\|^2. \tag{3.3}$$

For more details see [26].

Theorem 3.1. *Suppose assumption holds. Then search directions generated by the proposed algorithm of CG are descent directions.*

Proof. For $k = 0$, $g_0^T d_0 = -\|g_0\|^2$. Let $d_k^T g_k < 0\ \forall k$, then, multiplying Eq. (2.7) by g_{k+1} will yield:

$$d_{k+1}^T g_{k+1} = -g_{k+1}^T g_{k+1} + \beta_k s_k^T g_{k+1}. \tag{3.4}$$

To ease the proof of descent condition, the study simplifies the proposed better β_k. So, it follows that:

$$d_{k+1}^T g_{k+1} = -\|g_{k+1}\|^2 + \left(\frac{g_{k+1}^T y_k}{s_k^T y_k} - \frac{s_k^T g_{k+1}}{s_k^T y_k} \right) s_k^T g_{k+1}. \tag{3.5}$$

which reduces to:

$$d_{k+1}^T g_{k+1} = -\|g_{k+1}\|^2 + \frac{g_{k+1}^T y_k s_k^T g_{k+1}}{s_k^T y_k} - \frac{(s_k^T g_{k+1})^2}{s_k^T y_k}. \tag{3.6}$$

By the inequality $w^T v \leq \frac{1}{2}(\|w\|^2+\|v\|^2$, with $v= (s_k^T g_{k+1})y_k$ and $w= (y_k^T s_k)g_{k+1}$ will indicate:

$$\frac{g_{k+1}^T y_k s_k^T g_{k+1}}{s_k^T y_k} \leq \frac{\frac{1}{2}\left[\|g_{k+1}\|^2 (y_k^T s_k)^2 + (s_k^T g_{k+1})^2 \|y_k\|^2\right]}{(s_k^T y_k)^2}. \tag{3.7}$$

As Eq. (3.7) in Eq. (3.6) will reduce to:

$$d_{k+1}^T g_{k+1} \leq -\|g_{k+1}\|^2 + \frac{\frac{1}{2}\left[\|g_{k+1}\|^2 (y_k^T s_k)^2 + (s_k^T g_{k+1})^2 \|y_k\|^2\right]}{(s_k^T y_k)^2} - \frac{(s_k^T g_{k+1})^2}{s_k^T y_k}. \tag{3.8}$$

Similarly, by seeing $y_k^T y_k \leq L s_k^T y_k$ in Eq.21 reduces to:

$$d_{k+1}^T g_{k+1} \leq -\frac{1}{2}\|g_{k+1}\|^2 + \left[\frac{1}{2}L - 1\right] \frac{(s_k^T g_{k+1})^2}{s_k^T y_k}. \tag{3.9}$$

Since $0 < L < 1$, then the search direction is descent which completes the prove. □

The following theorem states that the innovative technique for uniformly convex functions has a global convergence property.

Theorem 3.2. *Suppose that $\{x_k\}$ is the sequence generated by Algorithm BILIA with β_k in Eq. (2.6), then:*

$$\lim_{k\to\infty} inf \|g_k\| = 0. \tag{3.10}$$

Proof. The Cauchy-Schwartz and Triangular inequalities have been used to obtain:

$$\begin{aligned} \|d_{k+1}\| &= \|g_{k+1}\| + \beta_k^{\text{BILIA}} \|s_k\| \\ &\leq \|g_{k+1}\| + \frac{\|g_{k+1}\| \, L \, \|s_k\|^2}{\mu \|s_k\|^2} + \frac{\|g_{k+1}\| \, \|s_k\|^2}{\mu \|s_k\|^2} \\ &\leq \left(1 + \frac{L}{\mu} + \frac{1}{\mu}\right) \|g_{k+1}\| \\ &\leq \left[\frac{\mu + L + 1}{\mu}\right] \|g_{k+1}\|. \end{aligned} \tag{3.11}$$

which implies:

$$\sum_{k\geq 1} \frac{1}{\|d_{k+1}\|^2} \geq \left(\frac{\mu}{\mu+L+1}\right) \frac{1}{\Gamma} \sum_{k\geq 1} 1 = \infty. \tag{3.12}$$

□

Thus, given that f is a uniformly convex function and Lemma 3.2 [17], then:

$$\lim_{k\to\infty} inf \|g_k\| = 0. \tag{3.13}$$

4. Results and discussion

In this section, the computational efficiency of the proposed method will be assessed using various metrics to support the excellent theoretical results obtained in the previous section. The benchmark problems considered for the computations as presented in Table 1 are from Andrei[14] with different dimensions and initial points. To evaluate the efficiency of the proposed method (BILIA), the performance was compared with the classical Fletcher and Reeves (FR) method[6] based on number of iteration (NOI) and number of function evaluation (NOF). Furthermore, the method is applied to solve image restoration problems arising in different real-life situations. All algorithms for this computation were coded on MATLAB R2023b programming software and run on a PC with intel COREi5 processor with CPU 2.3GHZ and 4GB of RAM. All experiments are performed under the weak Wolfe line search and the computation is stopped whenever $\|g(x_k)\| < 10^{-6}$. Table 1 below presents the numerical results of the proposed method with comparison of the classical FR method.

Table 1: The numerical results of the FR and BILIA methods.

Functions	**Dim**	**FR**		**BILIA**	
		NOI	**NOF**	**NOI**	**NOF**
Extended Rosenbrock	100	47	93	40	79
	1000	78	131	40	88
Extended White & Holst	100	43	88	35	76
	1000	46	92	34	65
Extended Beale	100	32	52	14	27
	1000	22	42	13	25
Penalty	100	10	27	10	27
	1000	24	191	26	180
Trigonometric	100	19	35	18	34
	1000	38	65	34	62
Extended Tridiagonal 1	100	32	64	14	28
	1000	77	129	11	22
Extended Three Expo Terms	100	15	25	19	28
	1000	127	3531	44	991
Generalized Tridiagonal 2	100	37	67	38	63
	1000	73	115	66	101
Extended Himmelblau	100	12	25	11	21
	1000	14	29	22	35
Extended Powell	100	180	313	67	126
	1000	Fail	Fail	Fail	Fail
Extended Maratos	100	89	174	69	154
	1000	107	211	Fail	Fail
Quadratic Diagonal Perturbed	100	124	231	82	152
	1000	445	711	224	413
Extended Wood	100	71	110	26	51
	1000	47	84	28	54
Extended Hiebert	100	101	217	79	174
	1000	101	214	79	171
Quadratic QF1	100	108	174	96	144
	1000	313	520	354	547
Extended Quadratic Penalty QP2	100	32	65	27	56
	1000	53	116	35	83
Tridiagonal Perturbed Quadratic	100	106	166	95	144
	1000	335	541	321	504
SINCOS	100	15	31	8	17
	1000	8	17	6	13
DIXMAANE (CUTE)	100	121	218	158	257
	1000	345	634	346	563
Partial Perturbed Quadratic	100	74	123	72	111
	1000	370	616	271	451
Almost Perturbed Quadratic	100	98	157	99	149
	1000	314	519	324	501
DENSCHNF (CUTE)	100	22	38	21	36
	1000	22	37	23	41
LIARWHD (CUTE)	100	23	45	16	31
	1000	27	55	22	49
ENGVAL1 (CUTE)	100	34	57	35	37
	1000	142	3616	35	439
DENSCHNA (CUTE)	100	20	33	10	19
	1000	19	35	9	18

Continued on next page

Functions	**Dim**	**FR**		**BILIA**	
		NOI	NOF	NOI	NOF
DENSCHNC (CUTE)	100	49	80	13	25
	1000	129	166	13	26
DENSCHNB (CUTE)	100	12	25	7	15
	1000	11	23	7	15
Extended Block-Diagonal BD2	100	122	156	12	23
	1000	130	166	12	23
Generalized quartic GQ1	100	11	24	9	20
	1000	9	22	7	18
Generalized quartic GQ2	100	112	147	40	59
	1000	110	145	36	57

To further analyze the numerical performance, the study employed the performance profile tool described in [6] to demonstrate the effectiveness and robustness of each algorithm. This tool presents an important process for standardizing the comparison of the conjugate gradient methods. To measure the performance of each algorithm, the study considers the metrics defined in the study including number of iteration (NOI) and function evaluation (NOF). Suppose that n_p and n_s are sets of p problems and s solver, also, let $k_{p,s}$ to be the number of iterations or function evaluations required by solver s to solve problem p. Then, the performance ratio $r_{p,s}$ defined as

$$r_{p,s}=\frac{k_{p,s}}{\min\left\{k_{p,s}:s\in S\right\}}. \tag{4.1}$$

where S denotes the set of solvers used for comparing the performance of problem p by a solver s using the best performance by any other solver on this problem. The solver's overall performance is determined by utilizing the performance ratios distribution function φ. So, by defining:

$$\varphi\left(t\right)=\frac{1}{n_p}size\left\{p\in P:r_{p,s}\leq t\right\}. \tag{4.2}$$

then, $\varphi\left(t\right)$ is the likelihood that a performance ratio $r_{p,s}$ for solver $s\in S$ is within a factor $t\in R$ of the optimal ratio. The best solvers are those with a large probability $\varphi\left(t\right)$, provided that the set of problems P is sufficiently large.

Figures 1 and 2 illustrate the visual representation of the performance results based on NOI and NOF, particularly, Figure 1 demonstrates that the proposed BILIA method performed better than the classical FR method in terms of iterations, as it solves a higher number of the problems with less iterations. Despite the fact the FR method successfully solved more problems when compared to the proposed method, in terms of efficiency, the new algorithm produced better performance.

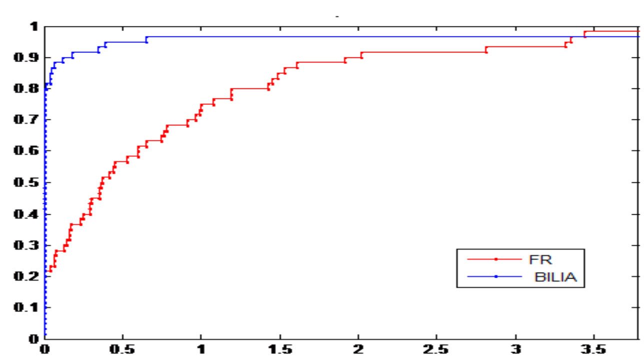

FIGURE 1. Performance based on number of iterations.

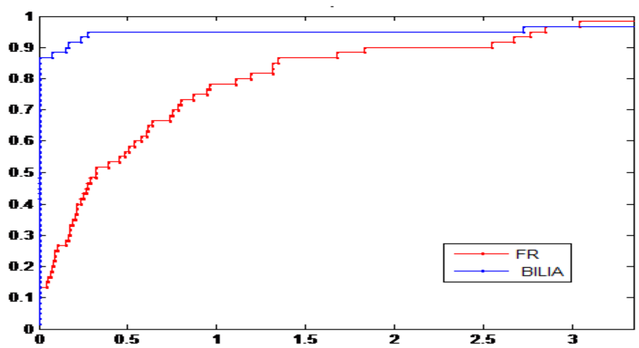

FIGURE 2. Performance based on number of function evaluations.

Similarly, in terms of NOF as displayed in Figure 2, FR algorithm as was also very competitive. However, the proposed BILIA outperformed the classical as it produced the better results in terms of the NOF. Based on Table 1, Figure 1 and 2, it can conclude that the proposed algorithm is promising.

5. Experiments on Solving Image Restoration

Image restoration and enhancement are essential for making decisions based on imaging data in different situations. Restoring the corrupted image to its original state is the primary goal of image enactment[16]. Before evaluating an image's information content, image restoration is frequently the first step. One of the most popular methods for image restoration is the Wiener deconvolution technique. This approach, in contrast to direct inverse filtering, preserves the original signal while lowering noise. Wiener deconvolution employs an ideal mean-square-error (MSE) trade-off between inverse filtering and noise smoothing. Although Wiener deconvolution works well in noisy environments, it is not as effective because of other drawbacks. For instance, there is a problem with the MSE optimality criterion when it comes to human observation. More image smoothing than the eye would prefer is the tendency. Also, Wiener deconvolution assigns an equal weight to every mistake, no matter where they are in the picture [24].

This study is interested in restoring corrupted images using the proposed conjugate gradient-based method. Let x define the real image whose index set is $A = \{1,\ 2,\ 3,\ \ldots..M\} \times \{1,\ 2,\ 3,\ \ldots..N\}$. The proposed algorithm will be applied to minimize the following image restoration model:

$$f_{\alpha}(u) = \sum_{(i,\ j)\in N} \left[|u_{i,\ j} - y_{i,\ j}| + \frac{\beta}{2}\left(2 \times S^1_{i,\ j} + S^2_{i,\ j}\right) \right]. \tag{5.1}$$

where β is the regularization parameter and

$$S^1_{i,\ j} = 2 \sum_{(m,n)\in P_{i,j}\cap N^c} \phi_{\alpha}(u_{i,\ j} - y_{m,n}), \qquad S^2_{i,\ j} = \sum_{(m,n)\in P_{i,j}\cap N} \phi_{\alpha}(u_{i,\ j} - y_{m,n}).$$

For the experiment, the study considered seven sets of images including Baboon (512x512), Barbara (512x512), Brain (512x512), Camera man (512x512), Lena (512x512), Pepper (512x512), and Man (512x512), [17, 25]. The obtained results will be evaluated based on three metrics which include number of iteration (NOI) and Peak signal-to-noise ratio (PNSR). All computations are performed on MATLAB R2023b programming software and run on a PC with an intel COREi5 processor with CPU 2.3GHZ and 4GB of RAM. The results from the experiment are presented in Table 2 and Figures 3-9 as follows:

Table 2: The numerical results for image restoration of the FR and BILIA methods

IMAGES	NOISE	BILIA		FR	
		NOI	PSNR	NOI	PSNR
BABOON	0.3	31	23.29	96	23.03
	0.5	35	21.86	69	20.6
	0.7	52	20.45	5	13.38
	0.9	82	18.75	58	17.3
BARBARA	0.3	31	28.4	168	27.48
	0.5	45	26.37	142	25.24
	0.7	47	24.27	105	22.61
	0.9	70	22	160	20.85
BRAIN	0.3	35	29.67	75	24.43
	0.5	49	28.1	238	26.22
	0.7	67	25.69	175	23.98
	0.9	81	21.24	5	8.58
CAMERAMAN	0.3	33	29.61	148	27.91
	0.5	40	27.18	100	24.62
	0.7	25	21.98	5	13.17
	0.9	79	19.99	106	19.88
LENA	0.3	31	36.89	245	34.22
	0.5	41	34.09	99	26.24
	0.7	53	30.88	5	14.19
	0.9	17	13.52	130	23.43
PEPPER	0.3	30	32.95	95	28.92
	0.5	46	30.25	204	28.05
	0.7	47	26.92	110	23.86
	0.9	105	21.58	119	21.18
MAN	0.3	31	31.47	170	29.36
	0.5	41	28.88	157	26.42
	0.7	48	26.06	174	24.68
	0.9	85	21.35	132	20.76

The results presented in Table 2, Figures 3–9 illustrate the performance of the proposed BILIA method in comparison to the classical FR method. Starting with Table 1, the efficiency of the proposed formula was assessed using NOI and PSNR. The PSNR is among the widely used metric for comparing the signal's maximum potential power to the power of noise that tampers with its representation, thereby affecting its fidelity. An image's PSNR must be estimated by comparing it to the best possible clean image with the highest power. Based on the presented results, it is obvious to see that the proposed method is promising because it restored the corrupted images with the best PNSR values. Figure 2–9 presented a visual representation of the experimental results. Each column of the figure starts by displaying the original image, then, followed by the images exposed to impulse noise. The third column represents the restored images using the proposed method and the fourth column displays the images filtered using the classical FR method. Based on the results presented in the table and figure, it is obvious to say that the proposed method is efficient and promising. To enhance the theoretical foundation of the investigation, numerous studies have explored this topic from multiple perspectives [12, 13, 15, 24]. Numerous studies have explored this subject from a variety of angles [10, 11, 14, 16, 21, 22].

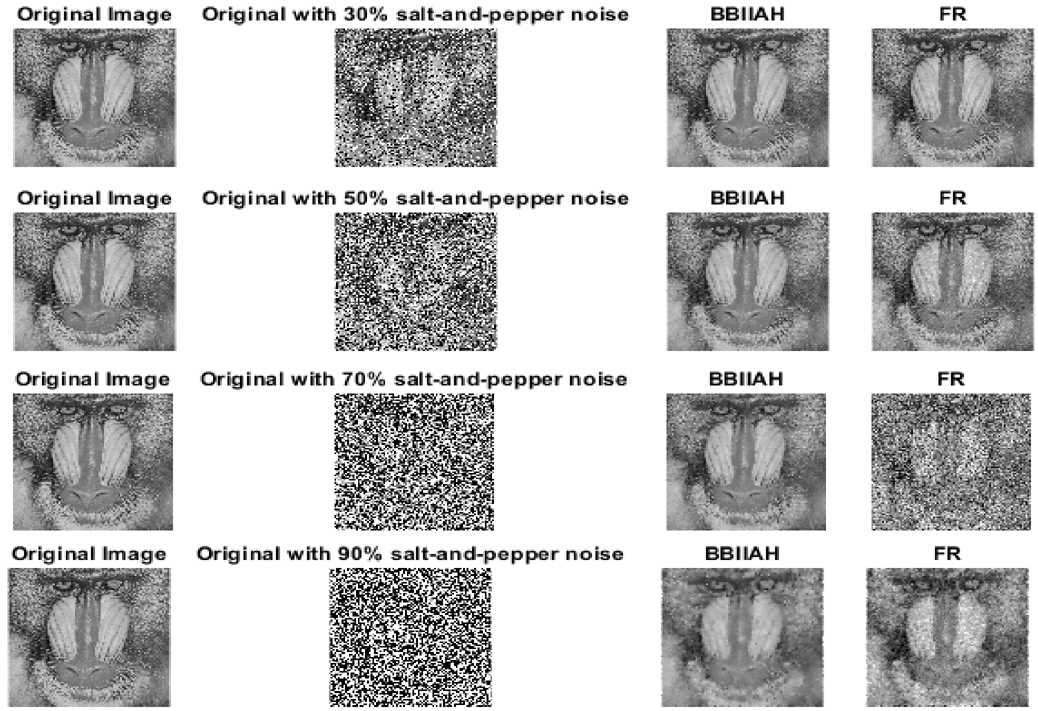

FIGURE 3. Image restoration of FR and BILIA method on Baboon image.

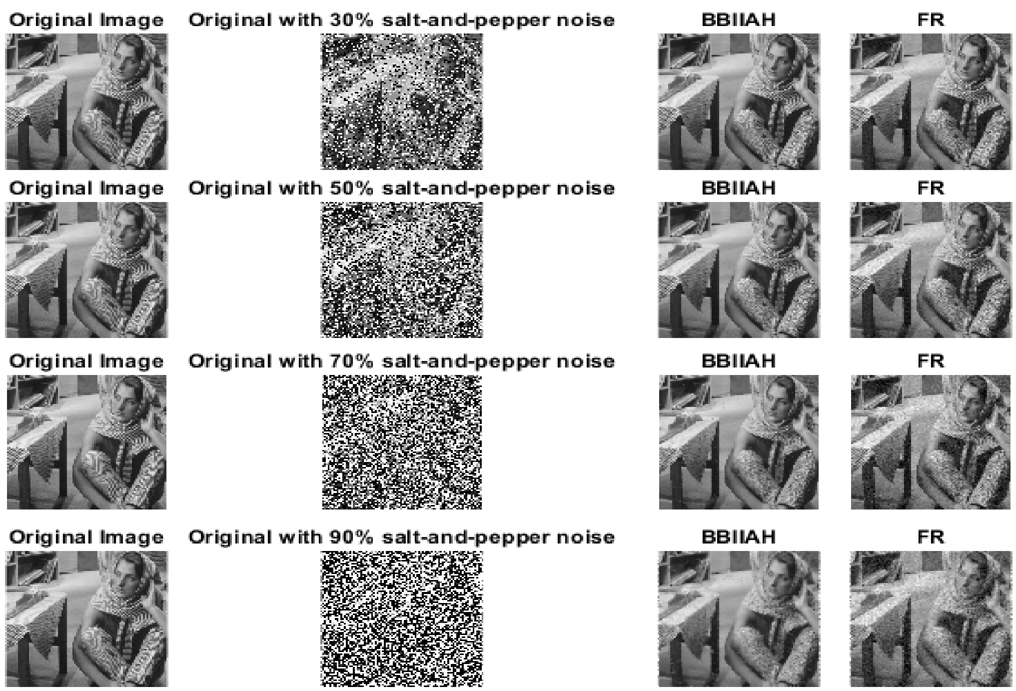

FIGURE 4. Image restoration of FR and BILIA method on Barbara image.

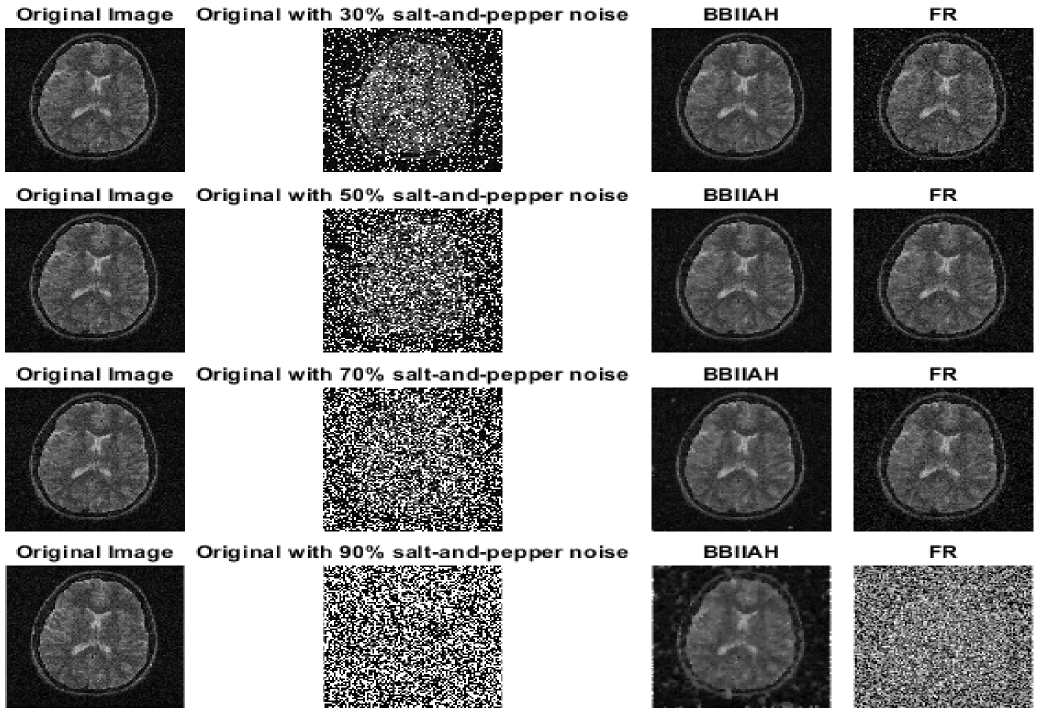

FIGURE 5. Image restoration of FR and BILIA method on Brain image.

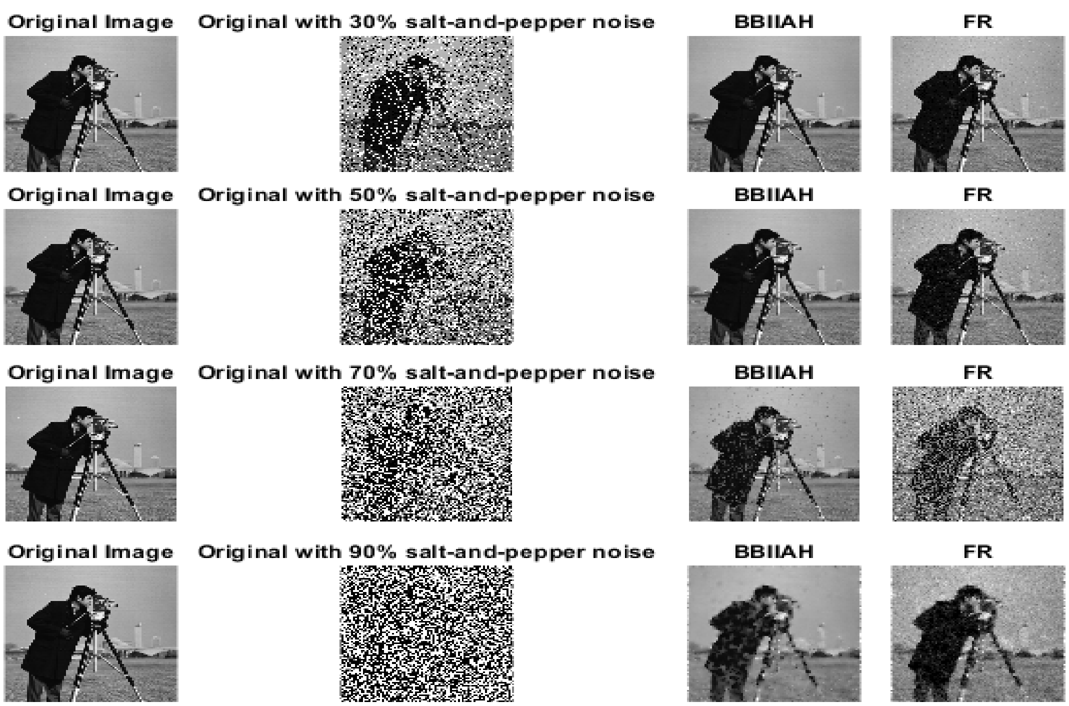

FIGURE 6. Image restoration of FR and BILIA method on Cameraman image.

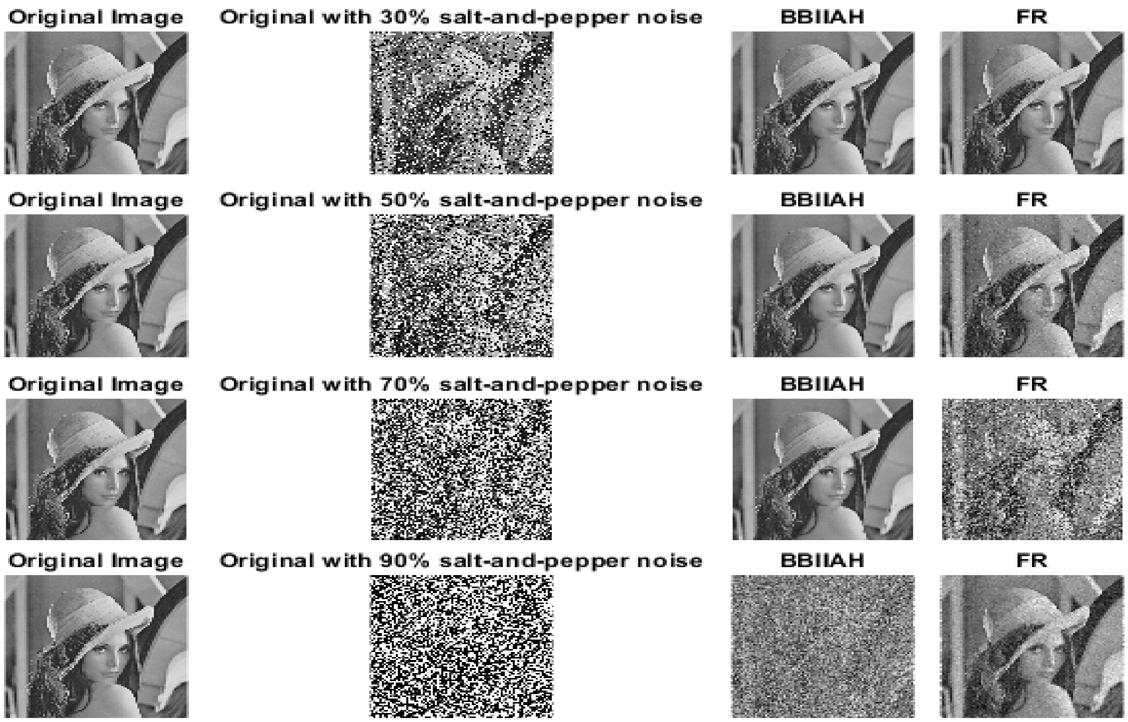

Figure 7. Image restoration of FR and BILIA method on Lena image.

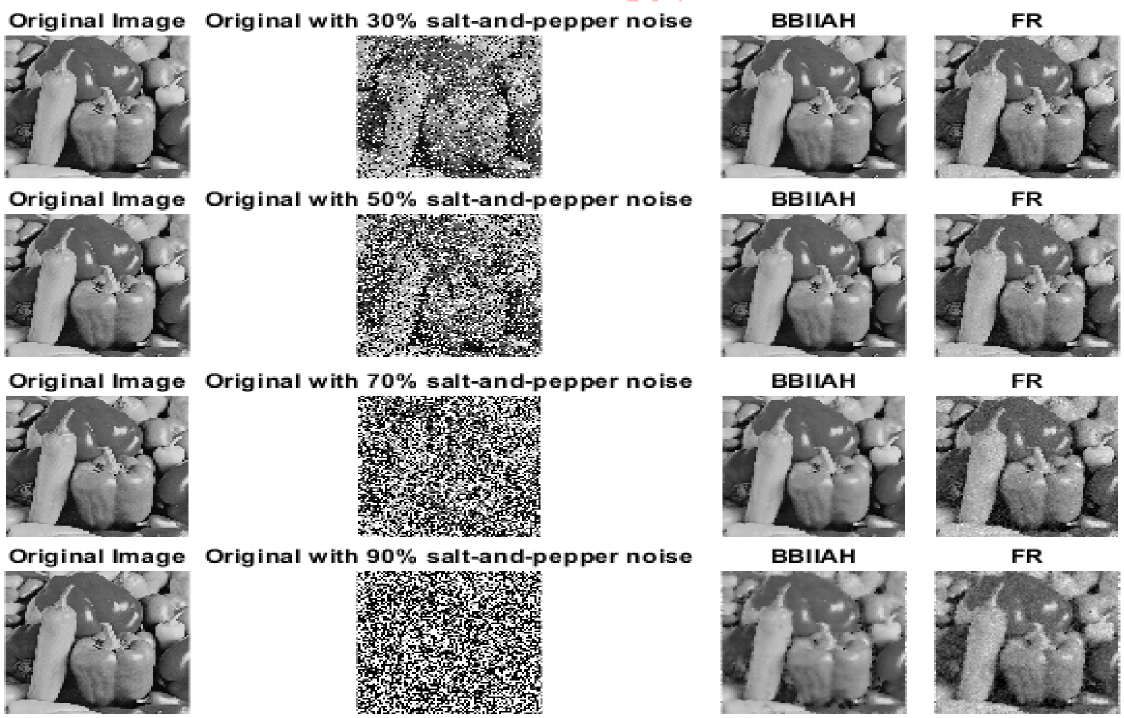

Figure 8. Image restoration of FR and BILIA method on Pepper image.

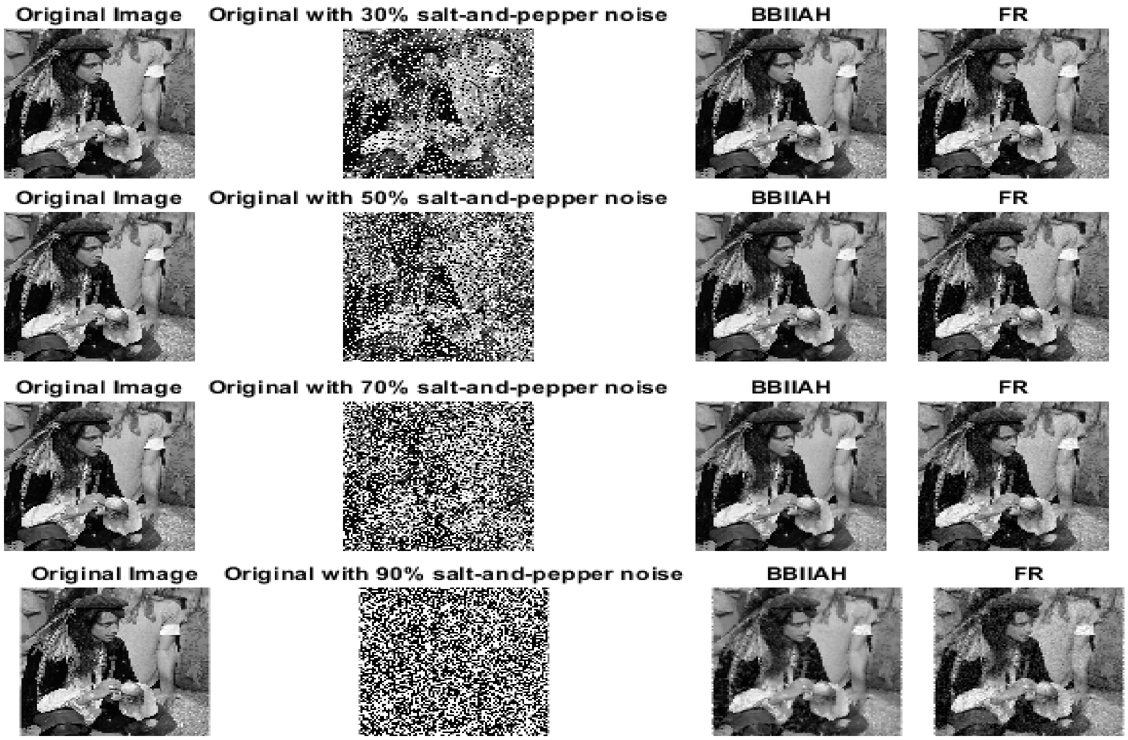

FIGURE 9. Image restoration of FR and BILIA method on Man image.

6. CONCLUSION

This paper proposed a new conjugate gradient algorithm for unconstrained optimization and image restoration problems. The new method was constructed by taking advantage of the famous Perry's conjugacy condition on a modified secant condition available in literature. The convergence result of the proposed method was established using suitable line search conditions. Preliminary results on a set of unconstrained optimization problems are presented in tables and figures. The performance evaluation of these results based on number of iteration and function evaluation show that the proposed method is promising. The method was further extended to image restoration problems and the results further demonstrated the efficiency of the new method. The authors believe that the new algorithm with other line search procedure might be much better than the original method and this can be considered an important topic that can be further explored.

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