Research Paper
Computational Methods for Differential Equations
http://cmde.tabrizu.ac.ir
Vol. *, No. *, *, pp. 1-11
DOI:10.22034/cmde.2025.67577.3227

Exploring mathematical modeling of non-linear Inviscid and Viscid Burgers' equations: A comprehensive study

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Abstract

Present study investigates the application of the power series method to solve non-linear inviscid and viscid Burgers' equations, a fundamental equation for modeling various physical phenomena that include fluid dynamics and traffic flow. We derive approximate solutions under specific initial conditions, demonstrating the effectiveness of the power series approach for obtaining accurate results. The solutions obtained are expressed in series form, which can be further simplified into closed analytic forms. Our findings indicates that the accuracy of the derived solutions improves with the inclusion of additional terms in the series expansion. This research highlights the advantages of the power series method in solving a robust mathematical tool for addressing dynamical systems with non-linearity. It provides solution with accuracy, efficiency, and minimal computations.

Keywords. Power Series Method, Non-linear Burgers' Equation, Mathematical Modeling.
2010 Mathematics Subject Classification. 40G10, 35L60, 93A30.

1. Introduction

Burgers' equation (BE) is a significant non-linear PDE that finds applications in various physical phenomena. It is a special case of the well-known Navier–Stokes equation, which plays a crucial role in fluid dynamics. This equation holds great importance as a mathematical model with wide-ranging applications. Its significance extends to fluid dynamics, where it is employed to study fluid behavior and flow patterns. Additionally, BE is also used in the theory of shock waves, helping researchers understand and predict the behavior of these intense and sudden pressure waves.

As BE plays an important role in various scientific fields, researches have explored its applications and studied numerous analytic and numerical method to obtain its solution. A numerical study of BE was examined [2, 24]. Gorguis [14] in 2006, examined the difference between the decomposition method and the Cole-Hopf transformation for solving BEs. Wazwaz [25] in 2008, derived traveling wave solutions of the Burgers, Huxley, and Fisher equations, as well as their combined forms, using the tanh-coth method. The comparison of Variational Iteration Method (VIM) and Homotopy Perturbation Method (HPM) for solving BE in fluid dynamics was addressed by Noorzad et al. [22] in 2008.

Moving on to different methods, the Homotopy Perturbation Method and the Homotopy Analysis Method is applied various kind of non-linear PDEs [5, 6]. The implementation of Differential Transform Method for solving the Burgers and coupled Burgers equations was carried out in [1]. A new analytical method called Variational Homotopy Perturbation Method is presented in [9], which is a combination of the well-known VIM and HPM, for solving the one-dimensional BE. Efficient numerical techniques were discussed in [21], presenting effective approaches for solving BE. Furthermore, Çenesiz [7] in 2017 demonstrated new exact solutions of Burgers' type equations with conformable derivatives. Advancing into [11], the Elzaki was obtained the exact solution of non-linear PDEs by utilizing the New Laplace Variational Iteration Method. In [16, 27], Adomian Decomposition Method was applied for solving BE.

Received: 29 May 2025; Accepted: 25 May 2026.
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Additionally, Ali et al. [3] studied the reduction of asymptotic approximate expansion of Navier–Stokes equation and solving the inviscid BE through similarity transformation. Finally, in [4, 26], researchers employed various methods, including Variational Iteration Method, Finite Difference Scheme, and Natural Decomposition Method, to tackle the non-linear BE. Also, Chaturvedi and Graham [8] investigated the vanishing viscosity limit of the one-dimensional BE near nondegenerate shock formation. Gie et al. [13] introduced new semi-analytic shooting methods for the stationary viscid BE, by improving the classical time differencing methods. An efficient semi-analytical technique, VIM, to solve multi-dimensional BE in [15]. In addition, they discussed method to solve 2D, 3D, and systems of Burgers' equations.

The present work focuses on obtaining exact solutions for both inviscid and viscid non-linear Burgers' equations. To achieve this, the researchers employ the Power Series Method (PSM) [17–20, 23] for several compelling reasons. The PSM can easily handle the multidimensional problem although this method is not much explored earlier to solve problems. The PSM leads to reduction in the complexity and also provides the fast convergence compared to another method in its class. Further, this method also overcomes the limitation of discretization, linearization, perturbation compared to other well known techniques. The solution provided by PSM is obtained using the recursive relation for the series coefficients and that may be simplified to closed form. This demonstrates the superiority of PSM in solving non-linear PDEs.

This paper is organized as follows. The derivation of the non-linear inviscid and viscid Burgers' equations is presented in Section 2. The Power Series Method (PSM) employed to solve the BE is described in Section 3. The implementation of the proposed method to the BE is discussed in Section 4. Section 5 presents the results and discussion, while the conclusion is outlined in Section 6.

2. Derivation of non-linear inviscid and viscid Burgers' equations

2.1. Non-linear inviscid Burgers' equation.

The conservation of density and momentum equations [10] are as follow:

$$\rho_t + q_x = 0, \qquad \frac{\partial \rho}{\partial t} \equiv \rho_t; \frac{\partial q}{\partial x} \equiv q_x \tag{2.1}$$

$$q_t + (wq)_x = 0. \tag{2.2}$$

Using (2.1) and (2.2) with $q = \rho w$, we have

$$\rho_t + w\rho_x + \rho w_x = 0, \tag{2.3}$$

$$w\rho_t + \rho w_t + w^2 \rho_x + 2\rho w w_x = 0. \tag{2.4}$$

Substituting ρ_t from (2.3) into (2.4) and simplifying, we obtain the non-linear inviscid Burgers' equation

$$w_t + w w_x = 0. \tag{2.5}$$

2.2. Non-linear viscid Burgers' equation.

In order to examine the characteristics of the discontinuous solution or shock waves, we consider a functional relation $q = \mathcal{Q}(\rho)$ and permit a jump discontinuity for both ρ and q. In numerous relevant physical problems, it would provide a more accurate approximation to take q is a function of the density ρ as well as density gradient ρ_x [10].

$$q = \mathcal{Q}(\rho) - \mu \rho_x, \tag{2.6}$$

where $\mu > 0$ is a constant.

Substituting (2.6) into (2.1), we obtain the nonlinear diffusion equation

$$\rho_t + u(\rho)\rho_x = \mu \rho_{xx}, \tag{2.7}$$

where $u(\rho) = \mathcal{Q}'(\rho)$.
Multiplying (2.7) by $u'(\rho)$, we have

$$u_t + u u_x = \mu\{u_{xx} - u''(\rho)\rho_x^2\}. \tag{2.8}$$

If $\mathcal{Q}(\rho)$ is a quadratic function in ρ, then $u(\rho)$ is linear in ρ and $u''(\rho) = 0$. Consequently, (2.8) becomes

$$u_t + uu_x = \mu u_{xx}. \tag{2.9}$$

As a simple model of turbulence, u is replaced by the fluid velocity field $w(x,t)$, leading to the non-linear viscid Burgers' equation, given by

$$w_t + ww_x = \mu w_{xx}, \tag{2.10}$$

where μ is the kinematic viscosity [10].

3. Fundamentals of the Power Series Method

We initiate our study by providing a concise initial statement of the methodology, aiming to facilitate comprehension of our analysis.

3.1. Steps of the power series method (PSM) for solving non-linear PDEs.

Step 1. Identifying the non-linear PDE either elliptic, parabolic, or hyperbolic involving two or more independent variables and a dependent variable.
Step 2. Suppose that the dependent variable can be expressed as a double power series involving the independent variables. Assume the double power series expansion for non-linear PDE is

$$w(x,t) = \sum_{k=0}^{\infty}\sum_{l=0}^{\infty} a_{kl}x^k t^l, \tag{3.1}$$

where $w(x,t)$ is dependent variable, a_{kl} are the coefficients to be determined, and x^k and t^l represent powers of two independent variables x and t, respectively.
Step 3. Replace $w(x,t)$ and its derivatives in the non-linear PDE with their corresponding double power series expansions.
Step 4. Equate the coefficients of like powers of x and t on both sides to obtain recursive equations.
Step 5. Solve the recursive equations to determine the coefficients a_{kl}.
Step 6. Assess the convergence properties of the double power series solution. Examine the behavior of the coefficients, investigate singularities, or utilize convergence tests such as ratio tests, Cauchy-Hadamard theorem, or other convergence criteria to determine the convergence radius or validity of the solution.
Step 7. Apply initial or boundary conditions to obtain additional equations for a_{kl}.
Step 8. Once the coefficients have been determined and the convergence has been analyzed, write the double power series solution (3.1).

3.2. **Convergence Analysis.** The convergence of double power series (3.1) depends on x and t [12]. For $(x,t) = (0,0)$, then for all a_{kl}, (3.1) is convergent. If $x \in \mathbb{R}$ and $t = 0$, then (3.1) is converges iff $\sum\limits_{k=0}^{\infty} a_{k0}\, x^k$ converges and similarly for $x = 0$ and $t \in \mathbb{R}$, (3.1) is converges with $\sum\limits_{l=0}^{\infty} a_{0l}\, t^l$.

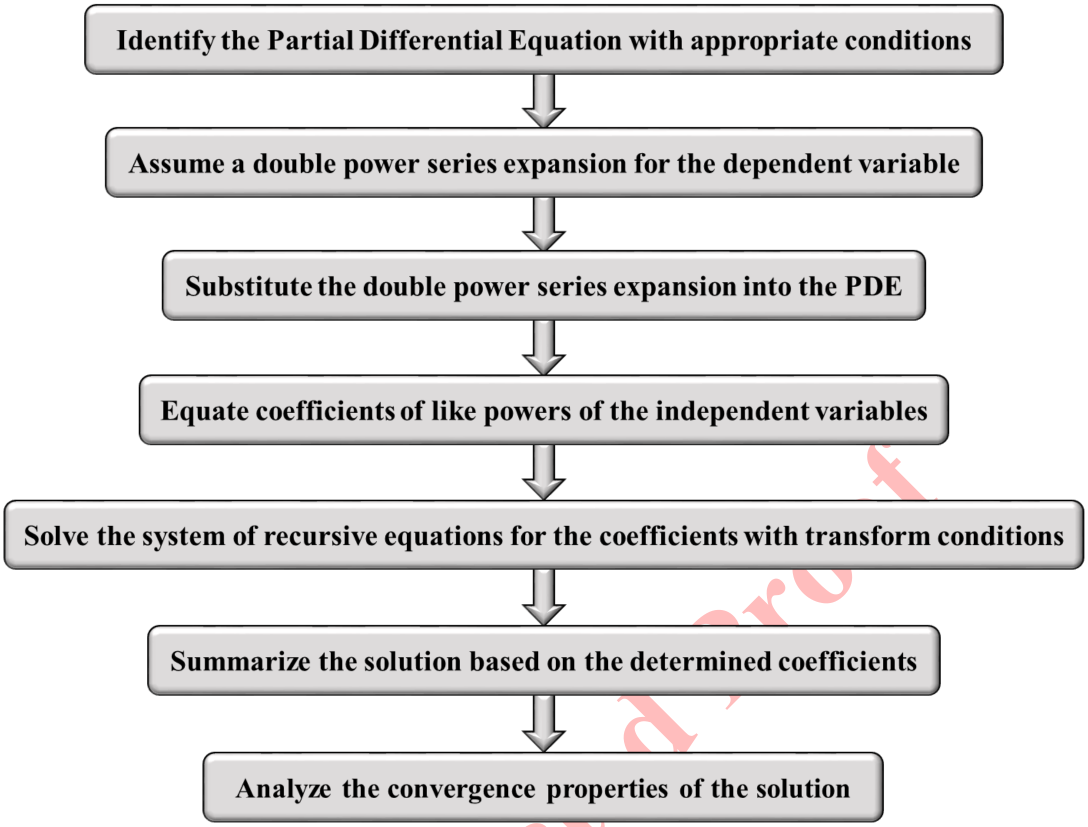

FIGURE 1. Working of power series method.

3.3. **Double power series expansions.** The double power series expansion of w_x, w_t, w_{xx} and ww_x are as follows:

$$w_x = \sum_{k=0}^{\infty}\sum_{l=0}^{\infty} (k+1)a_{(k+1)l}x^k t^l, \tag{3.2}$$

$$w_t = \sum_{k=0}^{\infty}\sum_{l=0}^{\infty} (l+1)a_{k(l+1)}x^k t^l, \tag{3.3}$$

$$w_{xx} = \sum_{k=0}^{\infty}\sum_{l=0}^{\infty} (k+1)(k+2)a_{(k+2)l}x^k t^l, \tag{3.4}$$

$$ww_x = \sum_{k=0}^{\infty}\sum_{l=0}^{\infty} \left[\sum_{r=0}^{l}\sum_{s=0}^{k} (k-s+1)a_{sr}a_{(k-s+1)(l-r)}\right] x^k t^l. \tag{3.5}$$

4. Solution of Burgers' equations

4.1. **Non-linear inviscid Burgers' equation.** Using the expansion of w_t & ww_x form (3.3) and (3.5) in non-linear inviscid Burgers' equation (2.5), we have

$$\sum_{k=0}^{\infty}\sum_{l=0}^{\infty} \left[(l+1)a_{k(l+1)} + \sum_{r=0}^{l}\sum_{s=0}^{k} (k-s+1)a_{sr}a_{(k-s+1)(l-r)}\right] x^k t^l = 0. \tag{4.1}$$

Comparing the coefficients, we get

$$(l+1)a_{k(l+1)} + \sum_{r=0}^{l}\sum_{s=0}^{k}(k-s+1)a_{sr}a_{(k-s+1)(l-r)} = 0, \ \forall\ k,l \geq 0. \tag{4.2}$$

Case 1. Let initial condition [27] is

$$w(x,0) = \beta x + \gamma, \beta \neq 0 \ \& \ \gamma \in \mathbb{R}. \tag{4.3}$$

The initial condition (4.3) should be transforms as follow:

$$a_{00} = \gamma, a_{10} = \beta \text{ and } a_{k0} = 0, k = 2,3,4,\dots\ . \tag{4.4}$$

Using (4.2) and (4.4), we obtained a_{kl} for $k,l = 1,2,3,4,5$ as shown in Table 1.

TABLE 1. Obtained values of a_{kl} for **Case 1** of subsection 4.1.

k	$l=0$	$l=1$	$l=2$	$l=3$	$l=4$	$l=5$
0	γ	$-\beta\gamma$	$\beta^2\gamma$	$-\beta^3\gamma$	$\beta^4\gamma$	$-\beta^5\gamma$
1	β	$-\beta^2$	β^3	$-\beta^4$	β^5	$-\beta^6$
2	0	0	0	0	0	0
3	0	0	0	0	0	0
4	0	0	0	0	0	0
5	0	0	0	0	0	0

Similarly, further values of a_{kl} can be obtained corresponding to higher values of k and l. Thus, (3.1) yields,

$$w(x,t) = \gamma - \beta\gamma t + \beta^2\gamma t^2 - \beta^3\gamma t^3 + \beta^4\gamma t^4 - \beta^5\gamma t^5 + \beta x \tag{4.5}$$

$$-\beta^2 xt + \beta^3 xt^2 - \beta^4 xt^3 + \beta^5 xt^4 - \beta^6 xt^5 + \dots, \tag{4.6}$$

which is equivalent form of the Taylor series expansion [27] of

$$w(x,t) = \frac{\beta x + \gamma}{1 + \beta t}. \tag{4.7}$$

Case 2. Let initial condition [27] is

$$w(x,0) = \beta x^2, \beta \neq 0. \tag{4.8}$$

The initial condition (4.3) should be transforms as follow:

$$a_{00} = a_{10} = 0, a_{20} = \beta \text{ and } a_{k0} = 0, k = 3,4,5\dots\ . \tag{4.9}$$

Using (4.2) and (4.9), we obtained a_{kl} for $k,l = 1,2,3,4,5$ as shown in Table 2.

TABLE 2. Obtained values of a_{kl} for **Case 2** of subsection 4.1.

k	$l=0$	$l=1$	$l=2$	$l=3$	$l=4$	$l=5$
0	0	0	0	0	0	0
1	0	0	0	0	0	0
2	β	0	0	0	0	0
3	0	$-2\beta^2$	0	0	0	0
4	0	0	$5\beta^3$	0	0	0
5	0	0	0	$-14\beta^4$	0	0

Similarly, further values of a_{kl} can be obtained corresponding to higher values of k and l.
Thus, (3.1) yields,

$$w(x,t) = \beta x^2 - 2\beta^2 x^3 t + 5\beta^3 x^4 t^2 - 14\beta^4 x^5 t^3 + \dots$$
$$= \frac{1}{2\beta t^2}(2\beta^2 x^2 t^2 - 4\beta^3 x^3 t^3 + 10\beta^4 x^4 t^4 - 28\beta^5 x^5 t^5 + \dots), \tag{4.10}$$

which is equivalent form of the Taylor series expansion [27] of

$$w(x,t) = \frac{(2\beta x t + 1) - \sqrt{1 + 4\beta x t}}{2\beta t^2}. \tag{4.11}$$

4.2. **Non-linear viscid Burgers' equation.** Consider the non-linear viscid Burgers' equation (2.10), for $\mu = 1$ [3]:

$$w_t + w w_x = w_{xx}. \tag{4.12}$$

Using (3.3) – (3.5) into (4.12), we have

$$\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\left[(l+1)a_{k(l+1)} + \sum_{r=0}^{l}\sum_{s=0}^{k}(k-s+1)a_{sr}a_{(k-s+1)(l-r)}\right] x^k t^l$$
$$= \sum_{k=0}^{\infty}\sum_{l=0}^{\infty}(k+1)(k+2)a_{(k+2)l}\, x^k t^l. \tag{4.13}$$

Comparing the coefficients, we get

$$(l+1)a_{k(l+1)} + \sum_{r=0}^{l}\sum_{s=0}^{k}(k-s+1)a_{sr}a_{(k-s+1)(l-r)} = (k+1)(k+2)a_{(k+2)l}, \ \forall\, k, l \geq 0. \tag{4.14}$$

Case 1. Let initial condition [27] is

$$w(x,0) = \beta x + \gamma, \beta \neq 0 \ \&\ \gamma \in \mathbb{R}. \tag{4.15}$$

The initial condition (4.15) should be transforms as follow:

$$a_{00} = \gamma, a_{10} = \beta \text{ and } a_{k0} = 0, k = 2, 3, 4, \dots . \tag{4.16}$$

Using (4.14) and (4.16), we obtained a_{kl} for $k, l = 1, 2, 3, 4, 5$ as shown in Table 3.

TABLE 3. Obtained values of a_{kl} for **Case 1** of subsection 4.2.

k	$l=0$	$l=1$	$l=2$	$l=3$	$l=4$	$l=5$
0	γ	$-\beta\gamma$	$\beta^2\gamma$	$-\beta^3\gamma$	$\beta^4\gamma$	$-\beta^5\gamma$
1	β	$-\beta^2$	β^3	$-\beta^4$	β^5	$-\beta^6$
2	0	0	0	0	0	0
3	0	0	0	0	0	0
4	0	0	0	0	0	0
5	0	0	0	0	0	0

Similarly, further values of a_{kl} can be obtained corresponding to higher values of k and l.
Thus, (3.1) yields,

$$w(x,t) = \gamma - \beta\gamma t + \beta^2\gamma t^2 - \beta^3\gamma t^3 + \beta^4\gamma t^4 - \beta^5\gamma t^5 + \beta x \tag{4.17}$$
$$-\beta^2 x t + \beta^3 x t^2 - \beta^4 x t^3 + \beta^5 x t^4 - \beta^6 x t^5 + \dots, \tag{4.18}$$

which is equivalent form of the Taylor series expansion [27] of

$$w(x,t) = \frac{\beta x + \gamma}{1 + \beta t}. \tag{4.19}$$

Case 2. Let initial condition [27] is

$$w(x,0) = \gamma - 2\tanh x, \gamma \in \mathbb{R}. \tag{4.20}$$

The initial condition (4.20) should be transforms as follow:

$$a_{00} = \gamma, a_{10} = -2, a_{30} = \frac{2}{3}, a_{50} = -\frac{4}{15}, \ldots, \text{ and } a_{k0} = 0, k = 2,4,6\ldots\ . \tag{4.21}$$

Using (4.14) and (4.21), we obtained a_{kl} for $k, l = 1,2,3,4,5$ as shown in Table 4.

Table 4. Obtained values of a_{kl} for **Case 2** of subsection 4.2.

k	$l=0$	$l=1$	$l=2$	$l=3$	$l=4$	$l=5$
0	γ	2γ	0	$-\frac{2}{3}\gamma^3$	0	$\frac{4\gamma^5}{15}$
1	-2	0	$2\gamma^2$	0	$-\frac{4\gamma^4}{3}$	0
2	0	-2γ	0	$\frac{8\gamma^3}{3}$	0	0
3	$\frac{2}{3}$	0	$-\frac{8\gamma^2}{3}$	0	0	0
4	0	$\frac{4\gamma}{3}$	0	0	0	0
5	$-\frac{4}{15}$	0	0	0	0	0

Similarly, further values of a_{kl} can be obtained corresponding to higher values of k and l.
Thus, (3.1) yields,

$$\begin{aligned} w(x,t) = \gamma - 2x + 2\gamma t + \frac{2}{3}x^3 - 2\gamma x^2 t + 2\gamma^2 x t^2 - \frac{2}{3}\gamma^3 t^3 - \frac{4}{15}x^5 + \frac{4}{3}\gamma x^4 t \\ - \frac{8}{3}\gamma^2 x^3 t^2 + \frac{8}{3}\gamma^3 x^2 t^3 - \frac{4}{3}\gamma^4 x t^4 + \frac{4}{15}\gamma^5 t^5 + \ldots, \end{aligned} \tag{4.22}$$

which is equivalent form of the Taylor series expansion [27] of

$$w(x,t) = \gamma - 2\tanh(x - \gamma t). \tag{4.23}$$

5. Results and discussion

In this study, we have applied the PSM to obtain the approximate solutions for both inviscid and viscid BEs. The results indicate that the PSM provides a systematic approach to solve non-linear PDE with faster convergence. The obtained solution is expressed as power series that can be simplified to closed forms if possible. The accuracy of the solutions was assessed by comparing them with known exact solutions, demonstrating that the power series method is superior in its class. Additionally, error can be minimized by calculating more terms of the series, confirming the method's effectiveness in handling complex non-linear PDEs. A comparison between the semi-analytic solution obtained using the PSM and the exact solution at $t = 0.1$ for different values of x is presented in Tables 5–8 and illustrated graphically in Figures 2–5.

The non-linear inviscid and viscid BEs describe how wave-like velocity fields evolve through the balance between non-linear motion and diffusion. In the inviscid case, the lack of viscosity lets nonlinear effects dominate, causing wave steepening and possible shock formation as fluid layers move at different speeds. When viscosity is present, diffusion smooths sharp gradients, preventing shocks and producing smooth, stable wave patterns. In essence, the inviscid form models idealized nonlinear distortion, while the viscid form captures realistic flows where diffusion tempers nonlinear effects for gradual, continuous evolution.

TABLE 5. Comparison of the PSM solution for **Case 1** of subsection 4.1 with the Exact solution at $t = 0.1$ for different values of x.

x	Exact	PSM	Error
0.1	1.0000	1.0000	0
0.2	1.0909	1.0909	0
0.3	1.1818	1.1818	0
0.4	1.2727	1.2727	0
0.5	1.3636	1.3636	0
0.6	1.4545	1.4545	0
0.7	1.5455	1.5455	0
0.8	1.6364	1.6364	0
0.9	1.7273	1.7273	0

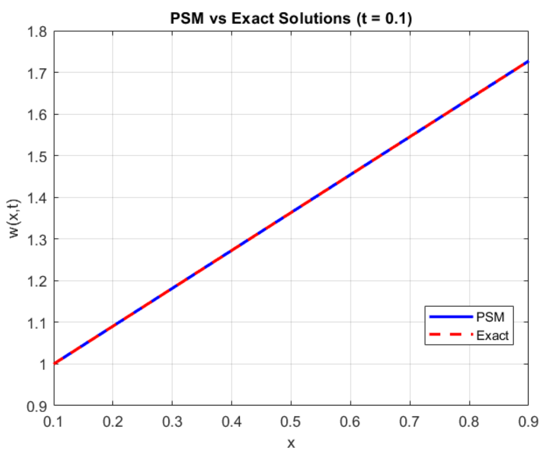

FIGURE 2. PSM and Exact solutions for **Case 1** of subsection 4.1 at $t = 0.1$ for different values of x.

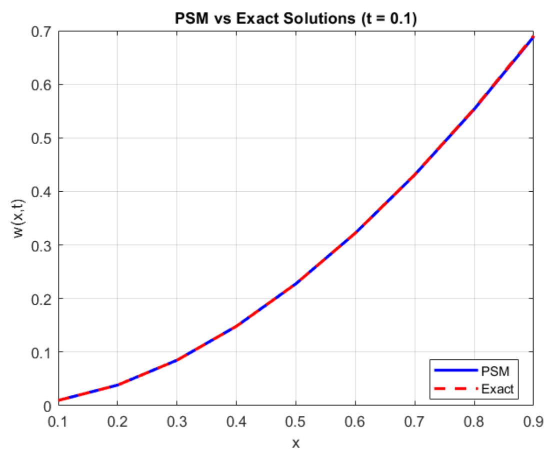

FIGURE 3. PSM and Exact solutions for **Case 2** of subsection 4.1 at $t = 0.1$ for different values of x.

6. CONCLUSION

The findings of this study provide significant insights into the physical implications of the solutions obtained for non-linear inviscid and viscid Burgers' equations. The power series method proved to be an effective tool for capturing the complex dynamics of these equations, allowing for a detailed analysis of wave behavior in fluid systems. The results demonstrate that the inviscid Burgers' equation leads to the formation of shock waves, a phenomenon that is crucial in understanding real-world fluid dynamics scenarios, such as traffic flow and shock wave propagation in gases. The solution highlights the non-linear interactions that result in shock formation, emphasizing the importance of non-linear terms in the governing equations. In contrast, the viscid Burgers' equation illustrates the stabilizing effect of viscosity on wave propagation.

The power series method may fail when the governing equation or conditions are non-analytic, causing the series expansion to break down. It can also produce unstable results near singular points or for highly nonlinear systems.

Overall, this study validates the PSM as a robust analytical tool and enhances our understanding of the physical phenomena described by BEs. The insights gained from the results underscore the significance of non-linear dynamics in fluid behavior, paving the way for further research into more complex systems and developing advanced mathematical techniques for solving non-linear PDEs.

TABLE 6. Comparison of the PSM solution for **Case 2** of subsection 4.1 with the Exact solution at $t = 0.1$ for different values of x.

x	Exact	PSM	Error
0.1	0.0098	0.0098	0
0.2	0.0385	0.0385	0
0.3	0.0850	0.0850	0
0.4	0.1484	0.1483	0.0001
0.5	0.2277	0.2277	0
0.6	0.3224	0.3222	0.0002
0.7	0.4315	0.4311	0.0004
0.8	0.5544	0.5535	0.0009
0.9	0.6905	0.6887	0.0018

TABLE 7. Comparison of the PSM solution for **Case 1** of subsection 4.2 with the Exact solution at $t = 0.1$ for different values of x.

x	Exact	PSM	Error
0.1	1.0000	1.0000	0
0.2	1.0909	1.0909	0
0.3	1.1818	1.1818	0
0.4	1.2727	1.2727	0
0.5	1.3636	1.3636	0
0.6	1.4545	1.4545	0
0.7	1.5455	1.5455	0
0.8	1.6364	1.6364	0
0.9	1.7273	1.7273	0

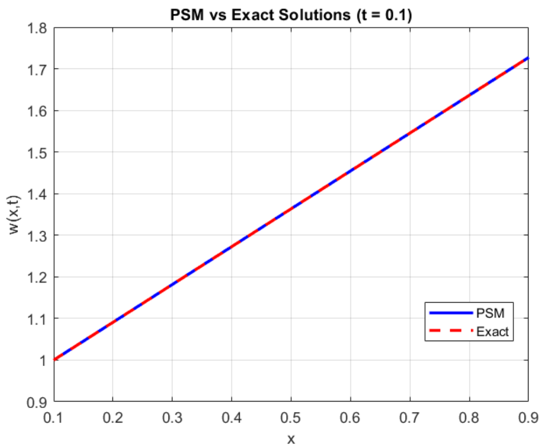

FIGURE 4. PSM vs. Exact solutions for **Case 1** of subsection 4.2 at $t = 0.1$ for different values of x.

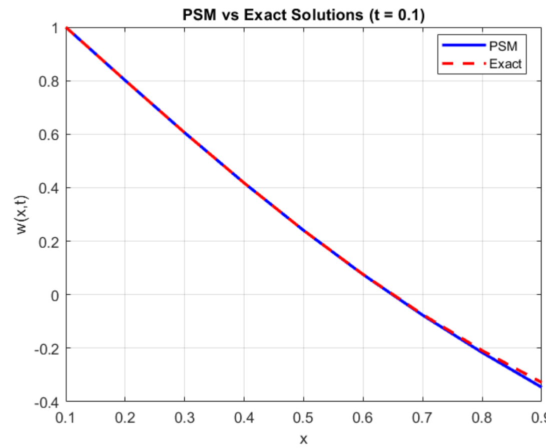

FIGURE 5. PSM vs. Exact solutions for **Case 2** of subsection 4.2 at $t = 0.1$ for different values of x.

TABLE 8. Comparison of the PSM solution for **Case 2** of subsection 4.2 with the Exact solution at $t = 0.1$ for different values of x.

x	Exact	PSM	Error
0.1	1.0000	1.0000	0
0.2	0.8007	0.8007	0
0.3	0.6052	0.6052	0
0.4	0.4174	0.4174	0
0.5	0.2401	0.2399	0.0101
0.6	0.0758	0.0750	0.0008
0.7	-0.0741	-0.0767	0.0026
0.8	-0.2087	-0.2162	0.0075
0.9	-0.3281	-0.3460	0.0179

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