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# A Spectral Stability-Based Alarm System for Detecting Process Mean and Variance Shifts

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**Abstract**
This paper presents an innovative online approach for univariate fault detection and alarm generation using the power of spectral analysis. The new approach utilizes a Spectral Stability Index (SSI) for simultaneous and sensitive monitoring of small and incipient changes in both the mean and variance of a process signal, which are common precursors to industrial alarm systems. The SSI framework accomplishes this by quantifying the deviation of the real-time power spectral density of the process signal from a reference model of normal operation. The method inherently carries an important advantage: it can detect low-level, developing faults much earlier than methods based on time-domain analysis. Its superior effectiveness is demonstrated through numerical and industrial case studies, where it significantly outperforms conventional methods, such as CUSUM, deadbands and delay timers, and more sophisticated advanced methods that have been recently introduced, such as serial and cascaded alarm systems. The results confirm the method's potential for substantially improving the responsiveness and accuracy of industrial alarm systems.

**Keywords**
Alarm systems, Incipient fault detection, Spectral stability index, Monitoring, Mean shift, Variance shift.

## 1. Introduction

Monitoring and alarm systems are intrinsic components of modern industrial practice, forming the backbone for safety, reliability, and efficiency in operations. These systems allow for the real-time monitoring of critical process parameters and permit the detection and prevention of potentially hazardous situations before they can develop into serious incidents. Given the increasing complexity and speed at which industrial processes and advanced technologies are evolving, there is an increasing need for expanded implementations of monitoring and alarm systems. Alarm systems promote operational integrity by notifying operators of deviations from specified conditions, thus enabling them to implement timely interventions that reduce the risks associated with such deviations. For the design and performance evaluation of modern alarm systems, industrially recognized standards have been developed, notably the ISA-18.2 standard [1] of the International Society of Automation and the EEMUA 191 guidelines [2] from the Engineering Equipment and Materials Users' Association. These represent critical benchmarks for alarm performance, where EEMUA 191 famously recommends that the average alarm rate for an operator should not be more than one alarm per ten minutes to avoid overload, and that the number of standing alarms should be very low. Moreover, ISA-18.2 developed a comprehensive lifecycle management framework with an emphasis on alarm rationalization so that each alarm is relevant, timely, and prioritized. The performance metrics key to this work-in particular, FAR and MAR-are directly matched with the goals of such standards: to maximize the signal-to-noise ratio of operators. The ability of the SSI method to demonstrate a superior trade-off between FAR and MAR, especially for difficult incipient faults, contributes directly to the goals of these standards and offers a pathway toward alarm systems that are easier to manage, more compliant, and effective at reducing operator burden and improving process safety.

### *1.1. Literature review*

Alarm systems have played a very important role in industrial operations; thus, extensive research efforts are conducted aiming at improving their reliability and reducing the rate of nuisance alarms. Most of the currently available alarm design methodologies are developed within the time-domain framework; they are based on simple statistical measures and incorporate fixed thresholds, which created the basis for the widespread implementation of several foundational techniques. Delay timers [3-8] avoid raising alarms due to brief fluctuations since they require the threshold violation to persist for some time, while dead bands [9, 10] introduce hysteresis zones to reduce alarm chattering due to oscillatory operating conditions. Complementary filtering approaches [11-14] try to extract the true process signal from the measurement noise by adopting various smoothing techniques. These conventional approaches have several serious fundamental limitations. The first weakness is that they are static - once calibrated, their parameters are fixed and do not change with the changing process conditions. This rigidity gives rise to an unavoidable trade-off: a system tuned for stable operation could be overly sensitive to small process variations and result in many false alarms, but when the operation moves into process transitions, the same system becomes so insensitive that it could dangerously miss real faults. Besides, all these methods focus primarily on instantaneous amplitude features but rarely take into account the temporal developments and spectral features of incipient faults. Recently, there has been a shift toward more adaptive alarm systems. For instance, [15] proposed the technique of alarm deadband adaptation through which the sensitivity would be updated in real time, whereas [16] described methodology for real-time optimization of alarm systems adapting to distributional shifts.

In recent years, with the aim of achieving greater robustness, new design methods have been introduced, which go beyond simple parameter tuning to more sophisticated structural configurations. Series alarm systems [17] have recently been proposed, in which a combination of delay timers and dead bands in series is used to create a more stringent, multi-stage triggering condition. Cascade alarm systems [18] use several alarm subsystems, operating on singular values of the signal in a sliding window, where the dead band width is dynamically designed based on the maximum amplitude deviation of the process variable [19].

Methods, such as those discussed above, represent an important step forward by introducing multi-layered decision logic and recourse to linear algebra for a more nuanced analysis of signal behaviour. The difficulties of process monitoring are also reflected in the literature dealing with risk and safety. For example, [20] introduced risk-based approaches for analysing extreme environmental loads, while [21] explored Bayesian networks for the risk-based design of process systems handling operational uncertainty. In the context of monitoring, [22] introduced an approach to enhance the accuracy of monitoring by removing certain dynamic components through Mult cointegration analysis. Similarly, [23] developed robust fault detection methodologies against challenging environments. Some references devoted to data driven approaches to fault estimation [24]. However, most of these approaches rely on heavy modelling effort or precise system identification or ample historical fault data, which in practice may be incomplete or inaccessible in many industrial applications.

It is worth mentioning that, the advances in distributed fault detection have produced elegant methodologies for networked estimation of dynamical systems. These approaches typically assume knowledge of the system dynamics (state matrix A and output matrices C) and leverage consensus protocols to enable distributed state estimation and residual generation across sensor networks. A key consideration in such methods is the rank of the system matrix (full vs. rank-deficient), which fundamentally affects observability properties, agent classification, and required network topologies. Many alarm systems adopt a fundamentally different paradigm. Alarm designing methods are usually data-driven approach that requires no model of the underlying dynamics. We operate exclusively on sensor output signals, making no assumptions about the state-space representation, system matrix rank, or observability properties of the process.

### *1.2. Contribution*

This paper presents a thorough framework for high-performance alarm system design using the SSI. The main contributions of this work are as follows:

Theoretical Proof of Early Detection for Incipient Faults: the rigorous mathematical derivation is presented that shows that the SSI method is guaranteed to provide earlier detection of incipient faults compared to the classical time-domain methods.

Generalized Framework for Variance Shift Detection: the application of the SSI beyond mean shifts is extended into the realm of detecting variance shifts with a fixed mean. The methodology derived here illustrates that the SSI responds to a uniform scaling of the power spectral density, hence offering a powerful unified spectral method for detecting changes in both the first and second moments of a signal.

Structured design methodology: The proposed approach enables a systematic design procedure where the detection threshold is first calibrated using only normal operation data to achieve a target FAR. Subsequently, the resulting MAR becomes mathematically predictable based on fault characteristics.

Altogether, these contributions make the SSI not just a signal processing technique; instead, it is an efficient and practical basis for advanced industrial alarm systems that can achieve high sensitivity to incipient faults with robust control over false alarms. The effectiveness and superiority of the proposed framework are convincingly illustrated, both by high-fidelity numerical simulations and real industrial case studies.

### *1.3. The paper organization*

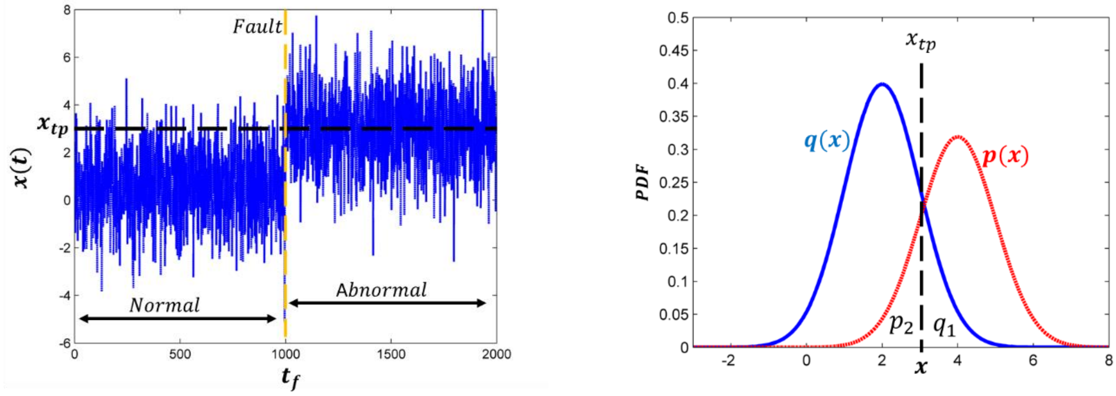
The remainder of this paper is structured as follows. Chapter 2 outlines the fundamental principles of alarm systems and the theoretical motivation for spectral methods. Chapter 3 details the design and implementation of the proposed SSI-based detection method. Chapter 4 presents the results from numerical and industrial case studies, including a comparative analysis with previous methods. Finally, Chapter 5 provides concluding remarks.

## 2. Fundamental Principles of an Alarm System

A basic alarm system can be represented by a stationary process signal $x(t)$ as the time-varying signal being monitored, a fixed threshold $x_{tp}$, and the occurrence of a fault at time $t_f$, as shown in Fig. 1.Then, the process signal is divided into normal and abnormal parts. At every moment, the system checks if $x(t)$ exceeds (or falls below, depending on the application) the threshold. If $x(t)$ breaches $x_{tp}$, the system activates the alarm (e.g., an audible siren, visual indicator, or automated response). The alarm signal $A(t)$ is defined as:

$$A(t) = \begin{cases} 0, & x(t) < x_{tp} \\ 1, & x(t) \geq x_{tp} \end{cases} \quad (1)$$

According to Fig. 1, from the normal and, abnormal historical data, the indices False Alarm Rate (FAR), and Missed Alarm Rate (MAR) can be defined as follows:




(a) (b)

Fig. 1: (a) a process variable. (b) PDFs of normal and abnormal parts.

$$FAR = q_1 = \int_{x_{tp}}^{+\infty} q(x)\, dx\,, \qquad MAR = p_2 = \int_{-\infty}^{x_{tp}} p(x)\, dx \tag{2}$$

Here, $q(x)$ and $p(x)$ are the Probability Density Functions (PDFs) of normal and abnormal parts of the process signal respectively.

An ideal alarm system would have a zero FAR and a zero MAR, which would mean that there is perfect discrimination between normal and faulty states. However, this is an unattainable ideal because of inherent uncertainties in real-world systems and measurement noise. For processes operating under stable conditions, estimates of FAR and MAR can be made by analysing the PDFs of historical data collected under known normal and fault conditions. On the other hand, many industrial processes have inherently dynamic operating conditions; this implies that the underlying process characteristics can vary with time. The estimation of static PDFs becomes problematic under such conditions since the historical data may not be representative of the current state of process operation. Therefore, any effective design of alarms for dynamic operating conditions needs methods that are capable of performing real-time adaptation and statistical inference to handle the evolving process dynamics. The central issue is to detect faults and anomalies reliably with reduced false and missed alarm rates, which becomes complicated in the presence of noise and complex interaction among system variables in dynamic environments.

## 3. Proposed Alarm System

Early detection of incipient faults is vital in an industrial alarm system for preventing catastrophic failures while minimizing false and missed alarms. Most conventional time-domain methods face difficulties in their applications in dealing with slow-developing faults because their sensitivity to fault magnitude is linear. This paper mathematically proves that the spectral stability index-based detection achieves much earlier fault detection than time-domain methods, especially for mean shifts and incipient faults. The successful application of the SSI methodology is based on two basic assumptions:

Stationary Normal Operation: Under usual, fault-free conditions, the process is assumed to be weakly stationary.

Availability of Representative Normal Data: There needs to be available a sufficiently long and representative dataset of exclusively normal operation.

### *3.1. Spectral Stability Index (SSI)*

Consider the normal operation data as

$$X_n = \{x_1, x_2, \dots, x_M\} \tag{3}$$

Where each $x_i$ is a window of length $N_w$ samples from normal operation. The reference power spectral density can be computed as

$$S_n(\omega_k) = \frac{1}{M}\sum_{i=1}^{M} |\chi_i(\omega_k)|^2 \tag{4}$$

Here, $\chi_i(\omega_k)$ is the discrete Fourier transform (DFT) of window $x_i$

$$\chi_i(\omega_k, t) = \sum_{n=0}^{N_w-1} x_i[n]\, e^{-j\frac{2\pi kn}{N_w}} \tag{5}$$

The SSI measures the deviation of the current signal's frequency content from the normal operating condition:

$$SSI(t) = \sum_{k=k_{min}}^{k_{max}} |S_x(\omega_k, t) - S_n(\omega_k)|\Delta\omega \tag{6}$$

and

$$S_x(\omega_k, t) = |\chi_i(\omega_k, t)|^2 \tag{7}$$

### *3.2. Mean deviation*

The DC component ( $0\,Hz$ ) in the Fourier transform directly captures the signal mean

$$\chi(0) = \sum_{n=0}^{N-1} x[n] = N\,\mu_x \tag{8}$$

Where $\mu_x$ is the signal mean. Therefore

$$S_x(0) = |\chi(0)|^2 = N^2\mu_x^2 \tag{9}$$

It means a mean shift directly amplifies the DC component quadratically. Consider a signal with mean shift

$$X_{faulty}(t) = X_{normal}(t) + \Delta\mu \tag{10}$$

The power spectral density becomes

$$S_{faulty}(\omega) = S_{normal}(\omega) + |\Delta\mu|^2\delta(\omega) + 2\Delta\mu\, Re\{X_{normal}(\omega)\} \tag{11}$$

The SSI detects this through

$$SSI\,(t) = \int \left|S_{faulty}(\omega) - S_{normal}(\omega)\right|\, d\omega \geq |\Delta\mu|^2 \quad (12)$$

Now, consider a signal with slowly developing mean shift

$$x(t) = \mu_0 + \alpha t \quad (13)$$

Where $\alpha > 0$ is the drift rate. Here, small values for $\alpha$ indicate incipient faults and greater values can be considered as abrupt faults. In a sliding window with length $N$ the time domain for mean estimation can be computed as

$$\hat{\mu}(N) = \frac{1}{N}\sum_{t=1}^{N} x(t) \quad (14)$$

The expected sample mean is

$$\mathrm{E}[\hat{\mu}(N)] = \mu_0 + \alpha\frac{N+1}{2} \quad (15)$$

Then, by considering $t_d$ as detection mean shift time and $T$ as the threshold for signal. It can be concluded that

$$\mu_0 + \alpha\frac{t_d+1}{2} = T \quad (16)$$

And

$$t_d = \frac{2(T-\mu_0)}{\alpha} - 1 \approx \frac{2(T-\mu_0)}{\alpha} \quad (17)$$

Now consider the problem mean shift in frequency domain. The zero-frequency component of the Fourier transform over window length $N$

$$\chi(0) = \sum_{t=1}^{N} x(t) = N\,\mu_0 + \alpha\sum_{t=1}^{N} t \quad (18)$$

The power at DC is

$$S_x(0) = |\chi(0)|^2 = \left(N\,\mu_0 + \alpha\frac{N(N+1)}{2}\right)^2 \quad (19)$$

And the expected power is

$$E[S_x(0)] \approx \left|N\,\mu_0 + \alpha\frac{N^2}{2}\right|^2 = N^2\,{\mu_0}^2 + \alpha\mu_0 N^3 + \frac{1}{4}\alpha^2 N^4 \quad (20)$$

By considering $T_{ssi}$ as threshold, the SSI detection condition is

$$|S_x(0) - S_{normal}(0)| > T_{ssi} \quad (21)$$

Where $S_{normal}(0) = N^2\,{\mu_0}^2$, then

$$|S_x(0) - S_{normal}(0)| = \left|\alpha\mu_0 N^3 + \frac{1}{4}\alpha^2 N^4\right| > T_{ssi} \quad (22)$$

Consider $t_{ssi}$ as the time mean detection in frequency domain. Then for small $\alpha$ it can be concluded that

$$\alpha\mu_0 {t_{ssi}}^3 \approx T_{ssi} \;\rightarrow\; t_{ssi} = \sqrt[3]{\frac{T_{ssi}}{\alpha\mu_0}} \quad (23)$$

Therefore, considering equations (17) and (23) yields

$$\frac{t_d}{t_{ssi}} = \frac{\frac{2(T-\mu_0)}{\alpha}}{\sqrt[3]{\frac{T_{ssi}}{\alpha\mu_0}}} = \left(\frac{2(T-\mu_0)}{\sqrt[3]{\frac{T_{ssi}}{\mu_0}}}\right)\alpha^{\frac{-2}{3}} \quad (24)$$

Since $\alpha^{-2/3} \rightarrow \infty$ as $\alpha \rightarrow 0$, it can be concluded that

$$t_{ssi} \ll t_d \quad for\ small\ \alpha \quad (25)$$

This mathematically proves that SSI provides earlier detection than time-domain methods for incipient faults, with the advantage becoming more pronounced for slower-developing faults (smaller $\alpha$).

### *3.3. Variance deviation*

Consider a process variable with constant mean $\mu_0$ but changing variance $\varepsilon(t)$ as

$$x(t) = \mu + \varepsilon(t), \qquad \varepsilon(t) \sim \mathcal{N}(0, \sigma^2) \quad (26)$$

Where $\varepsilon(t)$ is a zero mean noise process with time varying variance $\sigma^2$ that represent the combined effect of natural process variations and measurement noise. Here, the white noise or uniform spectral scaling is assumed to establish the fundamental concept and theoretical analysis that variance shifts can be detected through spectral analysis. For an incipient fault, the variance drift slowly

$$\sigma^2(t) = \sigma_0^2 + \beta t \quad (27)$$

Here, $\beta > 0$ is the drift rate. In the same way like discussed for mean shift, greater values for $\beta$ can cause abrupt fault. The Power Spectral Density (PSD) of the signal under normal conditions is

$$S_n(\omega_k) = \frac{1}{M}\sum_{t=1}^{M} |\chi_i(\omega_k)|^2 \quad (28)$$

Under a variance shift, the signal energy changes uniformly across frequencies (for white noise). The PSD becomes

$$S_x(\omega_k, t) = \frac{\sigma^2(t)}{\sigma_0^2} S_n(\omega_k) \quad (29)$$

The SSI at time $t$ is

$$SSI(t) = \sum_{k=k_{min}}^{k_{max}} |S_x(\omega_k, t) - S_n(\omega_k)|\,\Delta\omega \quad (30)$$

Substituting the variance-scaled PSD

$$SSI(t) = \sum_{k=k_{min}}^{k_{max}} \left|\frac{\sigma^2(t)}{\sigma_0^2} S_n(\omega_k) - S_n(\omega_k)\right| \Delta\omega = \left|\frac{\sigma^2(t)}{\sigma_0^2} - 1\right| \sum_{k=k_{min}}^{k_{max}} S_n(\omega_k)\,\Delta\omega \quad (31)$$

Define

$$P_n = \sum_{k=k_{min}}^{k_{max}} S_n(\omega_k)\,\Delta\omega \quad (32)$$

which is the total power in the reference band. Then

$$SSI(t) = \left|\frac{\sigma^2(t)}{\sigma_0^2} - 1\right| P_n \quad (33)$$

A variance shift is detected when

$$SSI(t) > T_{ssi} \quad (34)$$

Where $T_{ssi}$ is a predefined threshold. Substituting the drift model yields

$$\left|\frac{\beta t}{\sigma_0^2}\right| P_n > T_{ssi} \;\rightarrow t = t_{ssi} > \frac{T_{ssi}\sigma_0^2}{P_n\beta} \quad (35)$$

Here, $t_{ssi}$ is the detection time in the frequency domain. In the time domain, the sample variance in a window of length $N$ is

$$\hat{\sigma}^2(N) = \frac{1}{N}\sum_{t=1}^{N} (x(t) - \mu_0)^2 \quad (36)$$

For the drift $\sigma^2(t) = \sigma_0^2 + \beta t$ , the expected sample variance is

$$E[\hat{\sigma}^2(N)] \approx \sigma_0^2 + \beta \frac{N+1}{2} \quad (37)$$

Setting a threshold $T_\sigma$ for variance detection

$$T_\sigma = \sigma_0^2 + \beta \frac{N+1}{2} \rightarrow t_\sigma \approx \frac{2(T_\sigma - \sigma_0^2)}{\beta} \quad (38)$$

Compare the detection times

$$\frac{t_\sigma}{t_{ssi}} = \left( \frac{\frac{2(T_\sigma - \sigma_0^2)}{\beta}}{\frac{T_{ssi}\sigma_0^2}{P_n \beta}} \right) = \frac{2P_n(T_\sigma - \sigma_0^2)}{\frac{T_{ssi}\sigma_0^2}{P_n \beta}} \quad (39)$$

For small drift rates $\beta$, the ratio is constant and does not depend on $\beta$. However, SSI can still provide earlier detection if

The total power $P_n$ is large (i.e., the signal is broadband),

The threshold $T_{ssi}$ is set appropriately relative to $t_\sigma$

The variance change is reflected across multiple frequency bands.

SSI is sensitive to variance shifts through uniform scaling of the PSD. SSI can outperform time domain variance detection when the fault manifests across a wide frequency range, especially for incipient faults. This approach extends the SSI framework beyond mean shifts to variance-based fault detection, maintaining the advantages of frequency-domain analysis for early fault identification.

### *3.4. FAR and MAR*

Connecting the SSI-based detection approach to False Alarm Rate (FAR) and Missed Alarm Rate (MAR) is crucial for practical implementation. Under normal operation ($H_0$), the SSI follows some distribution due to natural variability. The probability of false alarms is

$$FAR = P(SSI > T_{ssi} | H_0) \quad (40)$$

Under $H_0$, the SSI statistic becomes

$$SSI(t)|_{H_0} = \sum_{k=k_{min}}^{k_{max}} |S_x(\omega_k) - S_n(\omega_k)| \, \Delta\omega \quad (41)$$

Due to natural variability around $\mu_0$, $SSI(t)|_{H_0}$ follows some distribution $f_0(SSI)$ with mean $E[SSI(t)|_{H_0}] = 0$ and variance $\sigma_0^2$ approximately. Therefore

$$FAR = 1 - \phi\left(\frac{T_{ssi}}{\sigma_0}\right) \quad (42)$$

And for a target FAR, the threshold can be selected as

$$T_{ssi} = \sigma_0 \phi^{-1}(1 - FAR_{target}) \quad (43)$$

Under abnormal condition ($H_1$), for mean shift $x(t) = \mu_0 + \Delta\mu$, the DC component becomes

$$S_x(0) = N^2(\mu_0 + \Delta\mu)^2 = N^2(\mu_0^2 + 2\mu_0\Delta\mu + \Delta\mu^2) \quad (44)$$

The deviation from normal is

$$|S_x(0) - S_n(0)| = N^2(2\mu_0\Delta\mu + \Delta\mu^2) \quad (45)$$

For small shifts ($\Delta\mu \ll \mu_0$)

$$E[SSI|H_1] \approx N^2|2\mu_0\Delta\mu| \quad (46)$$

Under abnormal condition ($H_1$), for variance shift $x(t) = \mu_0 + \varepsilon(t)$ with $var[\varepsilon(t)] = \sigma_0^2 + \Delta\sigma^2$, the expected SSI becomes

$$E[SSI|H_1] = P_n \left| \frac{\Delta\sigma^2}{\sigma_0^2} \right| \quad (47)$$

Therefore

$$MAR = P(SSI \le T_{ssi}|H_1) = \phi\left(\frac{T_{ssi} - E[SSI|H_1]}{\sigma_1}\right) \quad (48)$$

Actually here, the SSI under $H_1$ is modeled as a normal distribution with mean $E[SSI|H_1]$ and standard deviation $\sigma_1$ under $H_1$.

The SSI allows a structured design of the alarm system where the threshold is first set to achieve a target FAR by making use of the distribution of SSI values from normal operation data and, subsequently, the corresponding MAR becomes mathematically predictable based on fault development characteristics. This approach will leverage SSI's inherent sensitivity to incipient faults in the frequency domain and provides earlier detection compared to time-domain methods. Consequently, for the same FAR requirement, SSI achieves significantly lower MAR especially for slow-developing faults since the amplified frequency-domain signatures allow faults to cross the detection threshold sooner, creating a superior trade-off between false and missed alarms without compromising either metric.

### *3.5. Complexity of the SSI*

The computational complexity of the Spectral Stability Index method can be analysed through its constituent operations: the Fast Fourier Transform (FFT) operations required for each data window of length N are of $O(NlogN)$ complexity. For real-time implementation, this FFT computation must be performed on each new window, typically with some overlap between consecutive windows. Additional operations are power spectral density calculation $O(N)$, spectral summation across frequency bins $O(k)$ where k is the number of frequency components, and comparison with reference spectra. For a system processing M windows with N samples each, the overall complexity scales as $O(NlogN)$.

For a network comprising S independent sensors monitored simultaneously, the total system complexity is $O(S \times NlogN)$. This linear scaling with respect to the number of sensors is optimal for any method that processes each signal independently. The SSI method requires the following memory per sensor:

- Current data window: N floating-point values (8N bytes)
- Reference power spectral density: K floating-point values (8K bytes)
- Working arrays for FFT: Approximately 2N complex values (32N bytes)

Total per sensor: ≈ 40N + 8K bytes, typically < 10 KB for N = 256.

For a large plant with 10,000 sensors, this represents approximately 100 MB of memory well within the capabilities of modern industrial servers.

While the SSI introduces greater computational complexity relative to the traditional time-domain alarm approach, its implementation is nevertheless highly feasible in typical industrial environments in which the sampling period is one second or longer. This processing window easily accommodates the FFT operations and spectral calculations necessitated by SSI when using modern programmable logic controllers or industrial computers. Against this marginal increase in computational demand must be weighed significant performance advantages: SSI offers much earlier detection of incipient faults, plus a superior trade-off between false and missed alarm rates. Accordingly, for

**Algorithm I:** Offline and Online procedure of SSI

Offline Training Phase:
Get normal window samples $X_i$
Compute DFT $\chi_i(\omega_k)$ for each window using (5)
Compute $S_i(\omega_k)$ for each window using (4)
Compute $S_{ref}(\omega_k)$ using (4)
Compute SSI for each window using (6)
Estimate empirical distribution of SSI values
Set threshold $T_{ssi}$ to achieve $FAR_{target}$ using (43)

Online Monitoring Phase:
**for** each new sample
Get new window
Compute DFT $\chi_i(\omega_k)$ using (5)
Compute instantaneous $S_x(\omega_k, t)$ using (7)
Calculate current SSI using (6) with $S_{ref}(\omega_k)$
  **if** $SSI(t) > T_{ssi}$ **then**
    Set alarm $a(t) = 1$
  **else**
    Set alarm $a(t) = 0$
  **end if**
Slide window forward by step size $L$
**end for**

**// end of algorithm**

critical applications in which early fault recognition translates into prevented downtime and major cost savings, the SSI framework offers a justifiable and practical enhancement over simpler, yet correspondingly less sensitive, time-domain alternatives.
The offline procedure and online monitoring phase of the proposed method is presented according to Algorithm I.

## 4. Experimental Results

In this section, two experiments have been considered to evaluate the performance of the proposed alarm system. In experiment 1, a numerical signal with small shifted in mean is considered to illustrate the design procedure of the alarm system based on the SSI. In experiment 2, the incipient fault is considered in the Tennessee Eastman (TE) process [25]. The TE process is a widely recognized benchmark in the fields of process monitoring and alarm system optimization.

### *4.1. Experiment 1*

In this numerical case study, a time series of 10,000 samples was generated from a Gaussian process to assess the performance of the SSI. The first 5,000 samples correspond to normal operation, and are drawn from a distribution with mean 4 and standard deviation 1; to model a small sudden fault, the subsequent 5,000 samples have a mean of 4.5 in Fig. 2 while retaining a standard deviation of 1. The fault was introduced at $t_f = 5000$ seconds, with a sampling interval $h = 1$ second. What this experiment intends to show is that, even for such a minor shift of 0.5 units, which corresponds to a 12.5% change in the mean, the SSI method can achieve an acceptable and well-balanced performance from the viewpoint of both the FAR and the MAR and thus prove its sensitivity for small but critical process deviations.

From normal samples the PDF of normal part of SSI can be estimated as illustrated in Fig. 3. Based on the target FAR ( $FAR_{target} = 0.005$ ), the threshold for SSI ($T_{ssi}$) can be computed from (43) as shown in Fig. 4. The alarm signals for two cases of shifted in mean ($\Delta\mu = 0.5$ and $\Delta\mu = 1$) are depicted in Fig. 7. The alarm parameters are considered as depicted in Table I. The choice of frequency resolution in SSI involves trading off spectral detail against estimation stability and computational efficiency. Higher resolution benefit's narrowband fault detection and precise frequency localization. Lower resolution benefits broadband detection, estimation stability, and computational efficiency. Mean shift detection is a special case benefiting from high resolution at DC (via zero-padding). Mean shift detection is a special case benefiting from high resolution at DC (via zero-padding). Spectral leakage is mitigated by windowing and is less concerning for detection (vs. estimation) applications. Systematic sensitivity analysis should guide final parameter selection for each application. In this experiment, the choice of 256 frequency bins with zero-padding provides high interpolated resolution around DC, enabling sensitive detection of the mean shift through the quadratic amplification described in equation (45). While the underlying window length N=100 limits the true resolution to 0.01 Hz, zero-padding aids visualization and does not affect detection performance as the DC estimate depends only on the window sum.
From Fig. 7(a), which represents a small shift of the signal mean, one sees that the proposed SSI-based alarm system presents a desirable and acceptable performance. Although there are both false and missed alarms, their rates are in practically tolerable limits, keeping in mind that this minor deviation from normality is hard to detect. The presented algorithm detects this minor fault condition early and reliably, where conventional time-domain methods might fail to trigger an alarm or do so with considerable time delay, confirming its practical utility and establishing a significant improvement in capability for incipient fault scenarios in alarm systems.
To assess the statistical reliability of the ROC analysis, we performed 1000 independent Monte Carlo simulations. In each run, a new dataset of 10,000 samples was generated independently using the same distributions. For each run, we computed the SSI values and derived the ROC curve by varying the threshold $T_{ssi}$. The mean ROC curve across all simulations is shown in Fig. 5, with the shaded region indicating the 95% confidence interval (2.5th to 97.5th percentiles). The narrow confidence bands confirm that the SSI method's detection performance is highly consistent and not dependent on a particular random seed.
Fig. 6 explains why SSI is able to achieve such a ROC performance - the variation in the frequency domain is not a simple proportional reflection of time-domain changes but is amplified via the mathematical relationship presented in equation (45), $\Delta S(0) = 2N^2\mu\Delta\mu + N^2\Delta\mu^2$ by the $N^2$ term in conjunction with the quadratic term $\Delta\mu^2$. Small changes are multiplied by large factors, thus becoming much more detectable. The results clearly show that SSI provides a fundamentally better characteristic of detection, it transforms the

difficult problem of the detection of small time-domain changes into the easier problem of detecting large frequency-domain changes. This mathematical advantage translates directly to the superior ROC performance observed rendering SSI particularly valuable in applications where early detection of developing faults is critical, even in the presence of the inevitable trade-off between false and missed alarms. Sensitivity analysis quantitatively confirms that small defects which are nearly invisible in the time-domain become clearly detectable in the frequency-domain - this explains why SSI can achieve an acceptable performance even in challenging detection scenarios with minimal signal changes.

Table I: Alarm parameters in experiment 1

| Parameter | Value |
|---|---|
| Window Length | 100 |
| Frequency Resolution | 256 frequency bins |
| Target FAR | 0.005 |
| Sampling Time | 1 sec |

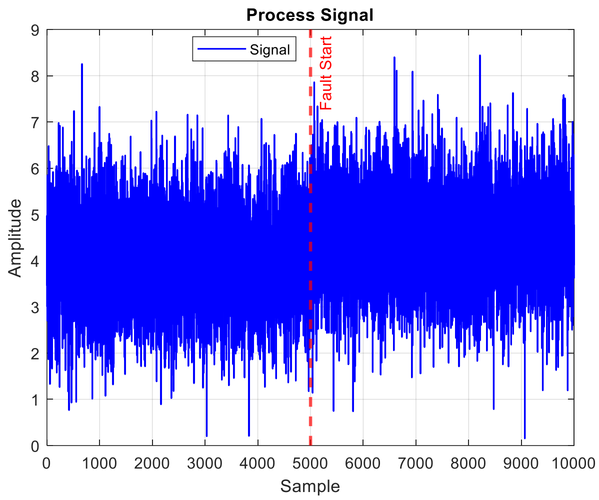



Fig. 2: a process variable under abrupt fault with mean shift deviation $\Delta\mu = 0.5$ at time 5000 sec.

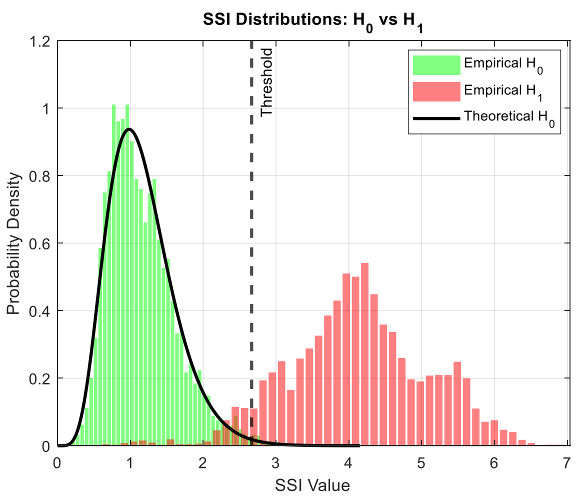



Fig. 3: the PDFs of normal ($H_0$) and abnormal ($H_1$) parts of SSI

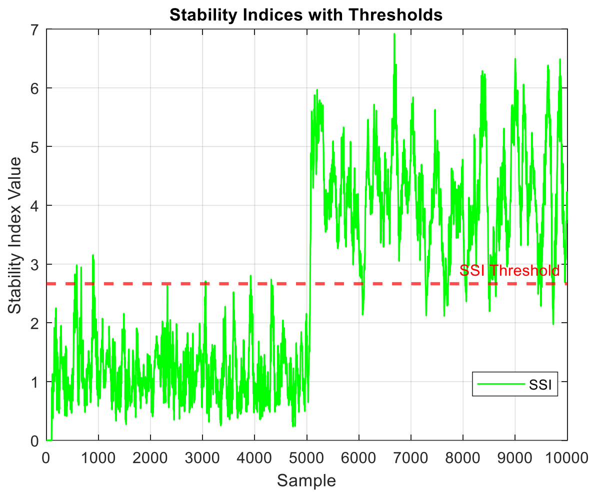



Fig. 4: the SSI values and thresholds in experiment 1.

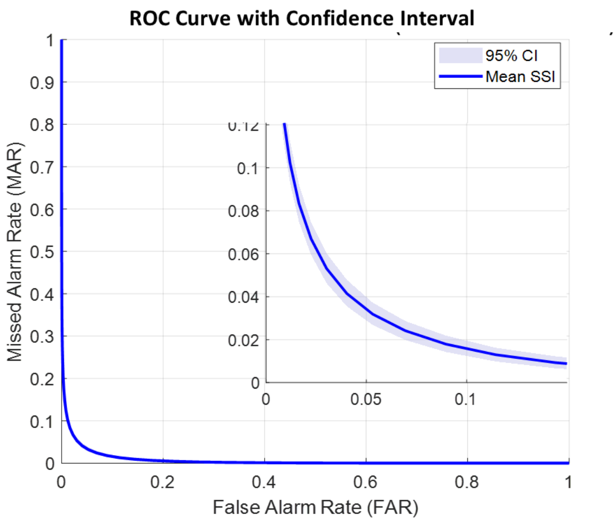



Fig. 5: the ROC curve for proposed method in experiment 1.

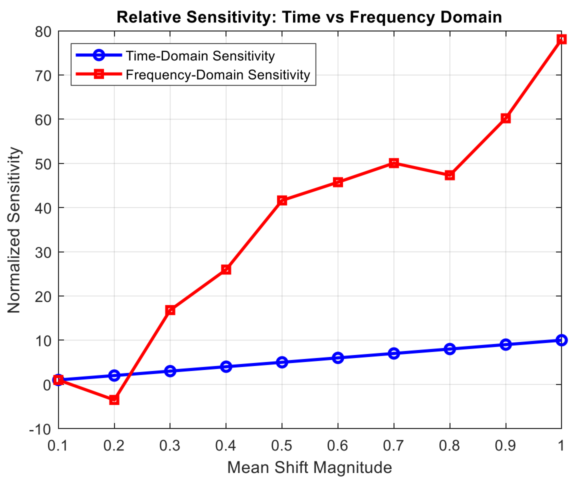



Fig. 6: the sensitivity analysis of time domain and proposed method for various shifted mean.

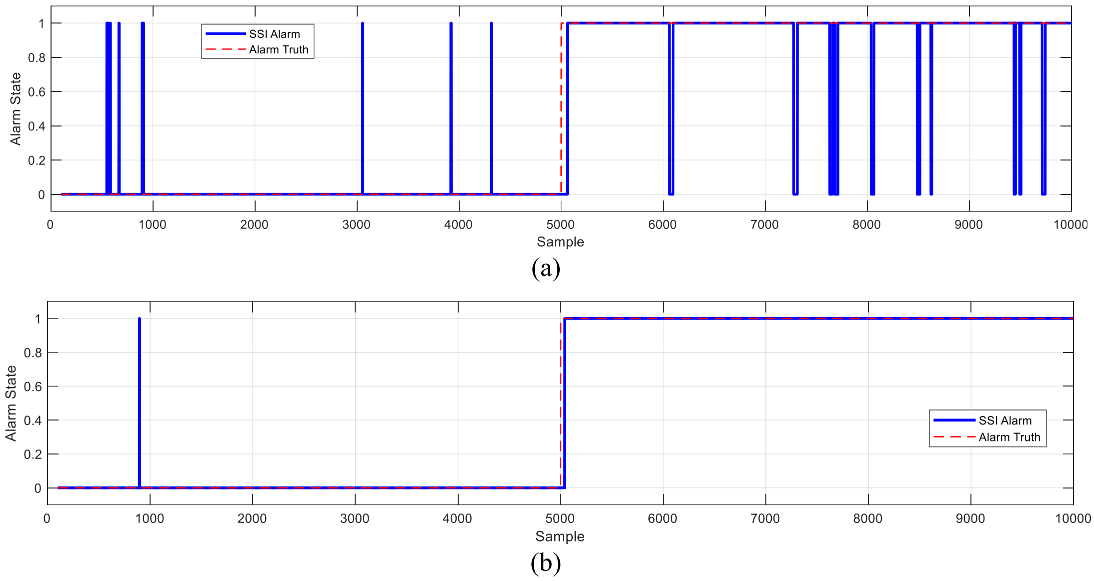



Fig. 7: (a) the alarm signal for $\Delta\mu = 0.5$ and, (b) the alarm signal for $\Delta\mu = 1$

*4.2. Experiment 2*

The TE process plant, illustrated in Fig. 8, comprises five major unit operations: a reactor, a condenser, a compressor, a vapor-liquid separator, and a stripper. The model provides 22 continuous process measurements, 19 composition measurements, and 12 manipulated variables. In this study, the Spectral Stability Index (SSI) is applied to monitor the E Feed (Stream 3) signal as shown in Fig. 9 under the conditions of Fault 2. This specific fault is a step disturbance in the composition of component B, while the feed rates of components A and C are held constant. The fault is introduced at a known time instant, $t_f = 5000$ seconds.

Fault 2 is implemented as a step change in the composition of component B in the feed stream. However, the E Feed signal (Stream 3) is located downstream of this disturbance source. The Tennessee Eastman process contains significant dynamic elements including reactor holdup, recycle loops, separator dynamics, and feedback control that collectively act as a low-pass filter, smoothing abrupt inputs into gradual transitions. The E Feed signal is chosen for this analysis because of its unique incipient fault behaviour in response to the disturbance. The signal does not experience an abrupt shift but, instead, undergoes a gradual, slow-developing change subsequent to the occurrence of the fault. This makes it an excellent candidate to verify the SSI method, which has been specifically developed for such subtle, progressive deviations at their earliest stage and thus clearly outperforms conventional time-domain alarm methods.

In order to comprehensively evaluate the performance of the proposed SSI algorithm, we conduct a comparative analysis against several basic and advanced established time domain alarm design methodologies documented in the literature. These include: a delayed timer-based penalty scenario [5], the dead band-based maximum amplitude deviation (MAD) method [19], the serial alarm configuration [17], the Cumulative Sum Control Chart (CUSUM) technique [26] and, cascaded alarm systems (CAS) proposed in [18]. The delayed timer approach with penalty activates an alarm only when n-consecutive samples exceed the threshold and deactivates it when $n$-consecutive samples fall below. A unique penalty mechanism is incorporated where if any intermediate sample violates the consecutive condition, the counter is penalized by a factor ($1 \le i < n$), thereby adding robustness against transient fluctuations. The MAD method employs a dead band strategy that utilizes the maximum deviation amplitude during false alarm events to determine the optimal dead band width $\delta_d$. Through Bayesian estimation, the method computes the cumulative probability $\Phi(\delta_d)$ and selects $\delta_d$ such that $\Phi(\delta_d) = \eta_0$, where $\eta_0$ represents the target false alarm rate. The CUSUM methodology implements sequential analysis for detecting mean shifts in processes with parameter drift $k_d$.

Because it accumulates deviations from a target mean value, the algorithm generates a signal when the cumulative deviations exceed a given threshold value; hence, it is significantly effective for detecting small and persistent process mean shifts. The serial alarm system combines dead band and delay timer elements serially. Indeed, all alarm deviations below a specified dead band width $A_d$ are eliminated by this design, while a delay timer having a factor $d$ filters out all alarm durations below threshold d, hence providing a unique dual stage filtering against nuisance alarms. The CAS method uses k-alarm subsystems in cascade in a sliding window ($w_s$) with damping threshold parameter $\gamma$ . For our comparative study, using the ROC curve analysis given in Fig. 10, the optimum threshold was obtained as $x_{tp} = 4459.5794$.

The alarm parameters of all comparative methods were optimized by using a Genetic Algorithm framework, where the objective function $J = \theta_1 FAR + \theta_2 MAR$ where$\theta_1$and $\theta_2$ are weights and $\theta_1 + \theta_2 = 1$, with $\theta_1 = \theta_2 = 0.5$, ensures balanced performance over both false and missed alarm metrics. However, for $\theta_1 > 0.5$, false alarms are penalized more heavily; for $\theta_2 > 0.5$, missed alarms dominate.

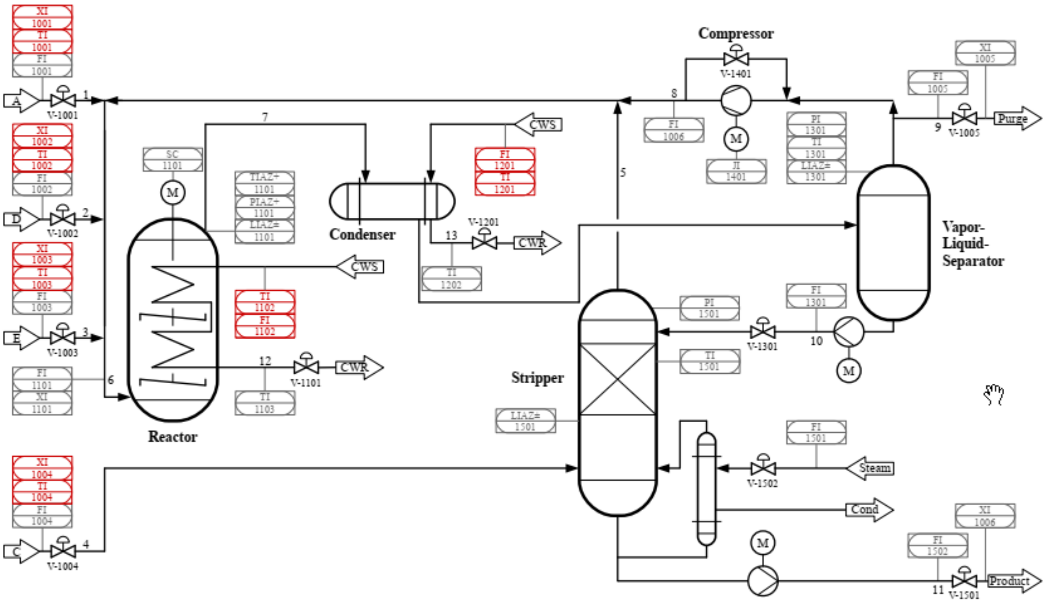



Fig. 8: the TE process [25]

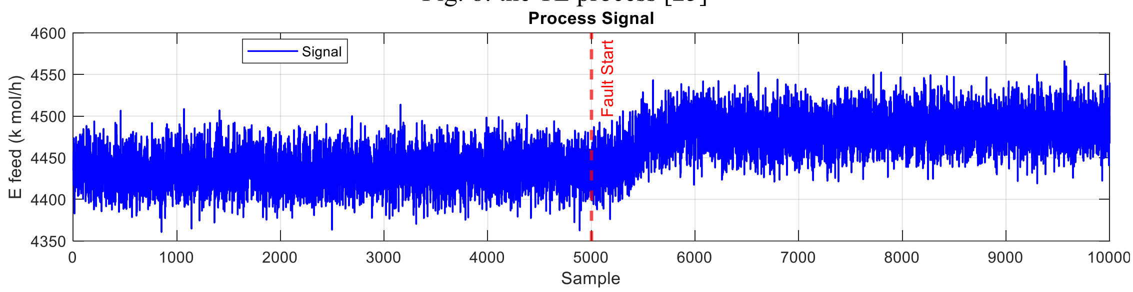



Fig. 9: the E Feed (Stream 3) signal in TE process with slow-developing change (Incipient fault).

The consistent outperformance of SSI over this optimized parameter space validates its effectiveness as a robust solution to industrial alarm systems, particularly in applications where early detection of developing faults is critical for operational safety and efficiency.

Numerical results are as follows: in conformity with the numerical results given in Table III, all benchmark methods demonstrate satisfactory performance over the control of false alarm rates. However, when confronted with the slow-developing and incipient nature of the fault scenario characterized by gradual variations of process variable, the performance of conventional methods deteriorates significantly in terms of missed alarm rate. In contrast, the proposed SSI method demonstrates superior performance by significantly minimizing missed alarms while retaining acceptable false alarm rates. This enhanced capability is related to its fundamental operating principle: SSI transforms subtle time-domain changes into amplified frequency domain signatures via spectral analysis. Hence, by monitoring the deviation in power spectral density from established normal operation patterns, SSI achieves much earlier detection of incipient faults that manifest themselves via gradual process characteristic variations. The alarm parameters for proposed method are taken as given in Table II. The TE process fault exhibits broadband characteristics, making fine frequency resolution unnecessary. The choice of 16 frequency bins aggregate spectral information across wider bands, reducing estimation variance and improving detection robustness for this incipient fault.

Table II: Alarm parameters in experiment 2

| Parameter | Value |
|---|---|
| Window Length | 100 |
| Frequency Resolution | 16 frequency bins |
| Target FAR | 0.015 |
| Sampling Time | 1 sec |

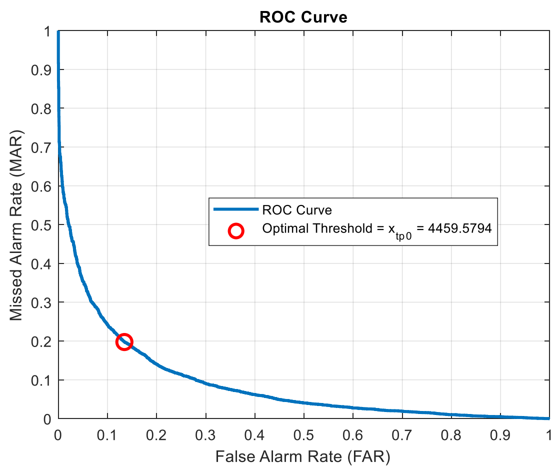



Fig. 10: The ROC curve for E feed flow samples and its optimal threshold.

Table III: the comparison between time domain alarm systems and proposed method

| Method | Optimal alarm parameters | | | | | | | | Number of alarms and value of $J$ | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | $k_d$ | $d$ | $A_d$ | $w$ | $\gamma$ | $n$ | $i$ | $N_m$ | False | Missed | $J$ |
| Serial | | 28.2710 | 8.1298 | - | - | - | - | - | 1 | 529 | 0.0530 |
| CAS | - | - | - | 20 | 7.2 | - | - | - | 1 | 380 | 0.0381 |
| Delay timer | - | - | - | - | - | 4 | 3 | - | 1 | 427 | 0.0428 |
| CUSUM | 0.5 | - | - | - | - | - | - | - | 1 | 595 | 0.0596 |
| Proposed SSI | Table II | | | | | | | | 8 | 312 | 0.0320 |
| Dead band-based MAD | - | - | - | - | - | - | - | 40.4837 | 6 | 2080 | 0.2086 |

## 5. Conclusion

This paper has introduced the Spectral Stability Index as a powerful framework for advanced industrial alarm system design and has analysed it rigorously. Overcoming the limitation of traditional time-domain methods, SSI utilized the frequency-domain analysis to realize a fundamental improvement in fault detection performance. We show mathematically that the SSI is able to detect an incipient fault earlier, with its advantage growing unbounded with the decrease in the fault development rate. The crux of the developed procedure is a systematic design that effectively decouples FAR and MAR: A detection threshold is set a priori, based exclusively on the distribution of the SSI under normal operating conditions, thereby guaranteeing a target FAR. Under this setting, the MAR becomes a predictable function of fault parameters and is inherently smaller than achievable using time-domain methods for the same value of FAR. This framework was generalized to both mean shifts and variance shift detection, yielding a unified spectral approach with respect to monitoring changes in a signal's fundamental statistics. The theoretical development in this paper assumes weakly stationary normal operation, enabling a fixed reference power spectral density. While this assumption holds for many continuous processes operating at steady-state, industrial practice often involves normal operations that are inherently non-stationary. Future work will focus on developing adaptive online change point detection algorithms specifically tailored to spectral features, enabling fully automated adaptation without risking fault contamination of the reference.

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