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An Enhanced Conjugate Gradient Algorithm Based on Perry's Condition for Efficient Image Restoration

Saif A. Hussein[1], Ayad Abdulaziz Mahmood[2] and Basim Abbas Hassan[3,*]

[1]Salladdin in Education Directorate, Tikrit, Iraq.

[2]Department of Statistics and Information Techniques, Technical College of Management, Northern Technical University, Mosul.

[3]Department of Mathematics, College of Computer Sciences and Mathematics, University of Mosul, Iraq.

Abstract

Traditional conjugate gradient setups require the conjugate parameter to be determined. In order to address the complexities of image restoration, this study introduces a unique conjugate coefficient that is obtained using Perry's conjugacy condition in conjunction with a quadratic model. The proposed approaches provide global convergence while preserving the descent feature. Empirical numerical evaluations verify that the newly suggested approach significantly enhances performance when compared to the traditional FR conjugate gradient technology. In conclusion, the suggested approach demonstrates improved efficiency and robustness in the realm of image restoration attempts.

Keywords. Enhanced Parameter gradient, Global convergence, Image Restoration.

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1. Introduction

As indicated in [3], the reconstruction of an image from corrupted observations constitutes a fundamental obstacle within the field of image processing. The index domain $A = \{1, 2, 3, ...M\} \times \{1, 2, 3, ...N\}$ represents the original picture x, which is made up of M×N pixels. The median-filtered result is represented by $\widetilde{y}$, while the picture degraded by salt-and-pepper noise is represented by y. The estimated pixel value, represented by $\widetilde{y}_{i,j}$, is used to refine each pixel $x_{i,j}$. We use a median filter that captures the dynamic range $[smax_{\min}$ of the picture and define the neighborhood $V_{i,j}$ surrounding each pixel to reduce impulsive noise [13]. $N = \left\{(i,j) \in A/\widetilde{y}_{i,j} \neq y_{i,j}, y_{i,j} = s_{max} \text{ or } s_{min}\right\}$, $N^c = \{(i,j) \in A/(i,j) \notin N\}$ represents the noisy pixel set and its complement. Noisy pixels are therefore identified when $(i,j) \in N$. The intensities of the $c = |N|$ noisy pixels create a vector $u = \left[u_{i,j}\right]_{(i,j)\in \mathrm{N}} \in R^c$. $S^1_{i,j} = 2\sum_{(m,n)\in V_{i,j}\cap \mathrm{N}^c} \phi_\omega(u_{i,j} - y_{m,n}), S^2_{i,j} = \sum_{(m,n)\in V_{i,j}\cap \mathrm{N}} \phi_\omega(u_{i,j} - y_{m,n})$, are defined by the edge-preserving function ϕ_ω. Image denoising is expressed as the following optimization problem, as in [22]:

$$Minf_\omega(u) = \sum_{(i,j)\in \mathrm{N}} \left[\left|u_{i,j} - y_{i,j}\right| + \frac{\gamma}{2}(2S^1_{i,j} + S^2_{i,j})\right]. \tag{1.1}$$

where regularization strength is controlled by $\gamma > 0$. While eliminating non-smooth artefacts, the smooth approximation term $\phi_\omega = \sqrt{u + \omega^2}, \omega > 0$guarantees edge retention. Problems of this nature frequently manifest in non-smooth regularization contexts, wherein $F_\alpha(u)$ adheres to the formulation in (1.1), characterized by the term $S^1_{i,j} + S^2_{i,j}$ being

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* Corresponding author. Email:basimah@uomosul.edu.iq.

smooth, while $\left|u_{i,j} - y_{i,j}\right|$ demonstrates non-smooth behavior at the zero point. The main objective is to minimize a half-quadratic smooth approximation of the function $F_\alpha(u)$ as described below, as stated in [12]:

$$f_\alpha(u) = \sum_{(i,j)\in \mathrm{N}} \left[(2 \times S^1_{i,j} + S^2_{i,j}\right]. \tag{1.2}$$

The conjugate gradient method was used to achieve the minimization of the function shown in (1.2):

$$f(u^*) = \min_{x\in R^N} f(u). \tag{1.3}$$

Conjugate gradient techniques often update the iterates according to the following approach, assuming that f is continuously differentiable (see [15]).

$$u_{k+1} = u_k + \alpha_k d_k. \tag{1.4}$$

In this context, d_k denotes the search direction, and, in alignment with [18], the step length is customarily determined through a suitable line search, represented as:

$$\alpha_k = -\frac{\mathcal{G}_k^T d_k}{d_k^T Q d_k}. \tag{1.5}$$

In order to guarantee enough reduction and convergence, this is often calculated in compliance with the Wolfe criteria [5, 19], as stated below:

$$f(u_k + \alpha_k d_k) \le f(u_k) + \delta \alpha_k \mathcal{G}_k^T d_k. \tag{1.6}$$

$$d_k^T \mathcal{G}(u_k + \alpha_k d_k) \ge \sigma\, d_k^T \mathcal{G}_k. \tag{1.7}$$

where $0 < \delta < \sigma < 1$. The convergence and implementation of CG algorithms have been the subject of extensive research; for more information, see [7, 12]. The search direction defined by:

$$d_{k+1} = -\mathcal{G}_{k+1} + \beta_k \nu_k. \tag{1.8}$$

Different choices of the parameter β_k in the suggested framework provide different computational results. The frequently used values for this parameter are described in Reference [6]:

$$\beta_k^{FR} = \frac{\|\mathcal{G}_{k+1}\|^2}{\|\mathcal{G}_k\|^2}. \tag{1.9}$$

The choice of conjugate gradient coefficients affects the method's performance. The traditional FR technique may yield subpar numerical results if the search direction is inadequate or the step size is too small.

Extensive research, initiated by Zoutendijk, has shown that the FR technique achieves global convergence under specific line search conditions [24]. Several scholars have proposed novel CG coefficient formulations that enhance convergence speed, preserve the descent property, and exhibit noteworthy numerical efficiency, building on this foundational work and perhaps assisting in the achievement of global solutions. The following examples of such approaches are provided by Wu and Chen [20]:

$$\beta_k^{WC} = \frac{y_{k+1}^T \mathcal{G}_{k+1}}{\nu_k^T y_k} + \frac{2(f_k - f_{k+1}) + \mathcal{G}_k^T \nu_k}{\nu_k^T y_k}. \tag{1.10}$$

These methods perform well in both theoretical research and practical applications. One of the primary distinctions between Wu and Chen's method and traditional CG methods is the computation of the search direction. For further details on these optimization techniques, the reader is directed to references [9, 10, 14]. Several CG versions that adhere to the aforementioned guidelines have been created. Using a quadratic model, it is feasible to create a new conjugate parameter with precise characteristics for specifying the search direction in order to enhance convergence and numerical efficiency.

In summary, the developments in conjugate gradient approaches-which include improved coefficient formulations, dependable line search strategies, and meticulous parameter optimization-offer a strong foundation for effective and trustworthy picture restoration.

2. A New Parameter For β_k

We develop the new conjugate gradient equation by using the Taylor series expansion, which may be written as follows:

$$f(u) = f(u_{k+1}) - \mathcal{G}_{k+1}^T \nu_k + \frac{1}{2} \nu_k^T Q(u_{k+1}) \nu_k. \tag{2.1}$$

where Q is Hessian matrix. The derivative is calculated by applying the following calculation:

$$\mathcal{G}_{k+1} = \mathcal{G}_k + Q(u_{k+1}) \nu_k. \tag{2.2}$$

By substituting Equation (2.2) into Equation (2.1), we arrive at:

$$\nu_k^T Q(u_k) \nu_k = 1/2 y_k^T \nu_k + (f_{k+1} - f_k) - \mathcal{G}_k^T \nu_k. \tag{2.3}$$

As a result, after several algebraic operations, we arrive at:

$$\nu_k^T Q(u_k) \nu_k = (\mathcal{G}_k^T \nu_k)^2 / (1/2 y_k^T \nu_k + (f_{k+1} - f_k) - \mathcal{G}_k^T \nu_k). \tag{2.4}$$

The conjugate parameter is obtained as follows using Perry's conjugacy condition:

$$d_{k+1}^T y_k = -\nu_k^T \mathcal{G}_{k+1}. \tag{2.5}$$

Combining Equations (1.8), (2.4), and (2.5) yields:

$$\beta_k d_k^T y_k = \mathcal{G}_{k+1}^T y_k - ((\mathcal{G}_k^T \nu_k)^2 / (1/2 y_k^T \nu_k + (f_{k+1} - f_k) - \mathcal{G}_k^T \nu_k)) - \mathcal{G}_k^T \nu_k. \tag{2.6}$$

As a result, we obtain:

$$\beta_k^{BBV} = \frac{\mathcal{G}_{k+1}^T y_k}{\nu_k^T y_k} - \frac{(\mathcal{G}_k^T \nu_k)^2 / (1/2 y_k^T \nu_k + (f_{k+1} - f_k) - \mathcal{G}_k^T \nu_k)}{\nu_k^T y_k} - \frac{\nu_k^T \mathcal{G}_k}{\nu_k^T y_k}. \tag{2.7}$$

The resultant parameter from the aforementioned derivation is designated as BBV.

Algorithm 1 Algorithm BBV

Initialization: Given $u_0 \in \mathbb{R}^n$, set $k = 0$ and $d_0 = -\nabla f(u_0)$.
while true **do**
  if $\|\nabla f(u_k)\| \leq \varepsilon$ **then**
    stop.
  end if
  Estimate α_k using (1.6) and (1.7).
  Update $u_{k+1} = u_k + \alpha_k d_k$, and compute $\max(\beta_k, 0)$ using (2.7).
  Set $d_{k+1} = -\nabla f(u_{k+1}) + \beta_k d_k$.
  Set $k = k + 1$.
end while

3. Global convergence:

This section endeavors to scrutinize the global convergence characteristics of the methodologies. We initiate this investigation by conducting the following analysis.

1. The level set has a boundary for $\Omega = \{u : u \in R^n, f(u) \leq f(u_1)\}$.
2. In a neighborhood Λ in Ω, the gradient satisfies the Lipchitz criterion in the following manner:

$$\|g(\underset{,}{a}_1) - g(\underset{,}{a}_2)\| \leq L \|\underset{,}{a}_1 - \underset{,}{a}_2\|, \forall \underset{,}{a}_1, \underset{,}{a}_2 \in \Lambda. \tag{3.1}$$

In accordance with the initial assumption, there exists an $\mu > 0$ such that:

$$(\nabla f(\underset{,}{r}_1) - \nabla f(\underset{,}{r}_2))^T \geq \mu \|\underset{,}{r}_1 - \underset{,}{r}_2\|^2, \forall \underset{,}{r}_1, \underset{,}{r}_2 \in R^n. \tag{3.2}$$

See [4, 23].

Theorem 3.1. *Any classical two-term CG methods. If $\beta_k = \beta_k^{BBV}$, then:*

$$d_{k+1}^T \mathcal{G}_{k+1} \leq -c \left\| \mathcal{G}_{k+1} \right\|^2. \tag{3.3}$$

Proof. After that, $\mathcal{G}_0^T d_0 = -\left\| \mathcal{G}_0 \right\|^2$ if $k = 0$. For every k, let $d_k^T \mathcal{G}_k < 0$. When we multiply (1.8) by $\mathcal{G}_{k+1}$, we obtain:

$$d_{k+1}^T \mathcal{G}_{k+1} = -\mathcal{G}_{k+1}^T \mathcal{G}_{k+1} + \beta_k \nu_k^T \mathcal{G}_{k+1}. \tag{3.4}$$

By employing (2.2) and transforming (2.7) to (3.4), we derive:

$$d_{k+1}^T \mathcal{G}_{k+1} = -\left\| \mathcal{G}_{k+1} \right\|^2 + \left(\frac{\mathcal{G}_{k+1}^T y_k}{\nu_k^T y_k} - \frac{\nu_k^T \mathcal{G}_{k+1}}{\nu_k^T y_k} \right) \nu_k^T \mathcal{G}_{k+1}. \tag{3.5}$$

It implies:

$$d_{k+1}^T \mathcal{G}_{k+1} = -\left\| \mathcal{G}_{k+1} \right\|^2 + \frac{\mathcal{G}_{k+1}^T y_k \nu_k^T \mathcal{G}_{k+1}}{\nu_k^T y_k} - \frac{(\nu_k^T \mathcal{G}_{k+1})^2}{\nu_k^T y_k}. \tag{3.6}$$

The Cauchy-Schwartz inequality is used $w^T s \leq \frac{1}{2}(\|w\|^2 + \|s\|^2$, where $w = (y_k^T \nu_k) \mathcal{G}_{k+1}$ and $s = (\nu_k^T \mathcal{G}_{k+1}) y_k$ we obtain:

$$\frac{\mathcal{G}_{k+1}^T y_k \nu_k^T \mathcal{G}_{k+1}}{\nu_k^T y_k} \leq \frac{\frac{1}{2}\left[\left\| \mathcal{G}_{k+1} \right\|^2 (y_k^T \nu_k)^2 + (\nu_k^T \mathcal{G}_{k+1})^2 \left\| y_k \right\|^2 \right]}{(\nu_k^T y_k)^2}. \tag{3.7}$$

Establishing a connection between (3.7) and (3.6) yields the following result:

$$d_{k+1}^T \mathcal{G}_{k+1} \leq -\left\| \mathcal{G}_{k+1} \right\|^2 + \frac{1/2\left[\left\| \mathcal{G}_{k+1} \right\|^2 (y_k^T \nu_k)^2 + (\nu_k^T \mathcal{G}_{k+1})^2 \left\| y_k \right\|^2 \right]}{(\nu_k^T y_k)^2} - \frac{(\nu_k^T \mathcal{G}_{k+1})^2}{\nu_k^T y_k}. \tag{3.8}$$

It ensures, by using (3.1) in (3.8).

$$d_{k+1}^T \mathcal{G}_{k+1} \leq -\frac{1}{2} \left\| \mathcal{G}_{k+1} \right\|^2 + \left[\frac{1}{2} L - 1 \right] \frac{(\nu_k^T \mathcal{G}_{k+1})^2}{\nu_k^T y_k}. \tag{3.9}$$

Therefore, as follows:

$$d_{k+1}^T \mathcal{G}_{k+1} \leq -c \left\| \mathcal{G}_{k+1} \right\|^2. \tag{3.10}$$

As a result, it is proven. □

Convergence can be attained with any conjugate gradient technique in conjunction with a Wolfe line search. The Zoutendijk condition [24] suffices for it to be considered weak.

Lemma 3.2. *Any iterative strategy yielding α_k results may be acquired via the application of Wolfe conditions with descent search direction. Thus:*

$$\sum_{k \geq 0} \frac{1}{\left\| d_{k+1} \right\|^2} = \infty. \tag{3.11}$$

Then

$$\lim_{k \to \infty} \inf \left\| \mathcal{G}_k \right\| = 0. \tag{3.12}$$

Theorem 3.3. *Let d_{k+1} be given by (1.8) and (2.7). Then, the result:*

$$\lim_{k \to \infty} \inf \left\| \mathcal{G}_k \right\| = 0. \tag{3.13}$$

Proof. Nevertheless, it remains valid from (1.8) that:

$$\|d_{k+1}\| = \left\|-\mathcal{G}_{k+1} + \beta_k^{\mathrm{BBV}} \nu_k\right\|. \tag{3.14}$$

By utilizing (2.2), we may substitute (2.7) into (3.14), which implies:

$$\|d_{k+1}\| \;= \left\|-\mathcal{G}_{k+1} + \frac{\mathcal{G}_{k+1}^T y_k}{\nu_k^T y_k}\nu_k - \frac{\nu_k^T \mathcal{G}_{k+1}}{\nu_k^T y_k}\nu_k\right\|. \tag{3.15}$$

The integration of Equations (3.1) and (3.2) results in the ensuing conclusion:

$$\|d_{k+1}\| \;\leq \|\mathcal{G}_{k+1}\| + \frac{\|\mathcal{G}_{k+1}\|\, L\, \|\nu_k\|^2}{\mu\, \|\nu_k\|^2} + \frac{\|\mathcal{G}_{k+1}\|\, \|\nu_k\|^2}{\mu\, \|\nu_k\|^2} \leq \left(1 + \frac{L}{\mu} + \frac{1}{\mu}\right) \|\mathcal{G}_{k+1}\| \leq \left[\frac{\mu + L + 1}{\mu}\right] \|\mathcal{G}_{k+1}\|. \tag{3.16}$$

As a result,

$$\sum_{k\geq 1} \frac{1}{\|d_k\|^2} \geq \left(\frac{\mu}{\mu + L + 1}\right) \frac{1}{\Gamma} \sum_{k\geq 1} 1 = \infty. \tag{3.17}$$

By Lemma 3.2, this study determines that $\lim_{k\to\infty} \inf \|\mathcal{G}_k\| = 0$. □

4. Numerical Results

In this work, we present a set of numerical findings that validate the efficacy of the proposed strategy in mitigating impulsive noise resulting from disruptions generated by salt and pepper. An empirical analysis is conducted to compare the outcomes achieved from the innovative methodology with those obtained via the FR method. The computational program utilized for the whole procedure is MATLAB r2017a.

The generated codes are subsequently executed on a computer. The following parameters were utilized to determine the termination of both techniques:

$$\|f(u_k)\| \leq 10^{-4}(1 + |f(u_k)|) \text{ and } \frac{|f(u_k) - f(u_{k-1})|}{|f(u_k)|} \leq 10^{-4}. \tag{4.1}$$

The subjects of the test images include Elaine, Lena, House, and the cameraman. In a manner analogous to other studies, we assess the restoration performance qualitatively using the PSNR (peak signal-to-noise ratio) alongside the test text [21]. Restoration performance is defined as follows:

$$PSNR \;=\; 10\; \log_{10} \frac{255^2}{\frac{1}{MN} \sum\limits_{i,j} (u_{i,j}^r - u_{i,j}^*)^2}. \tag{4.2}$$

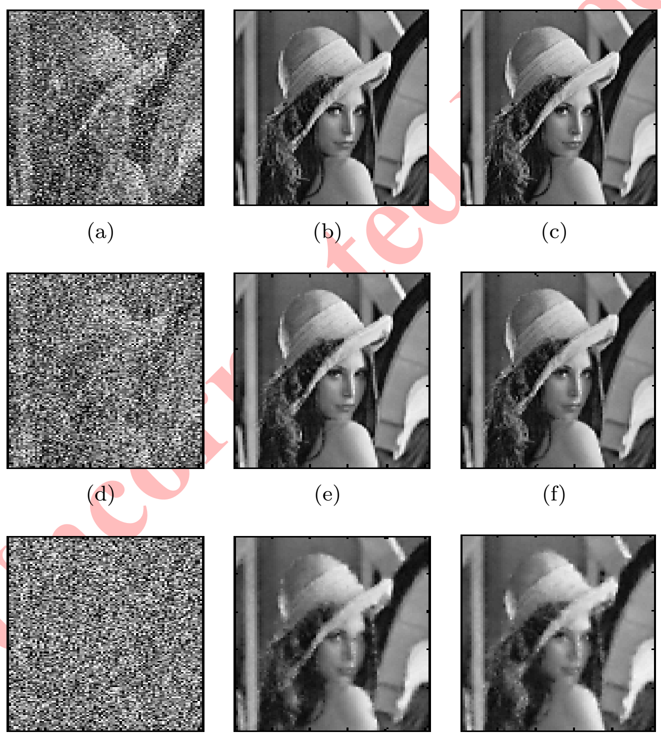
The original picture's pixel values and the restored image's pixel values are denoted by $u_{i,j}^r$ and $u_{i,j}^*$, respectively. This study examines the number of function evaluations (NF) and iterations (NI) required to complete the denoising process and the PSNR of the resulting picture. The FR way takes a substantial amount of time, while the new method finishes considerably faster. This may be seen from the data in Table 1. Additionally, there is no significant difference between the PSNR values produced with the innovative methodology and the FR technique. In an attempt to bolster the investigation's underlying theoretical framework, several studies have examined this topic from various perspectives [1, 2, 8, 11, 16, 17].

5. Conclusions

The conjugate gradient method known as BBV was thoroughly examined in this work, and a recently suggested conjugate gradient equation was developed and examined. This method was developed in response to the requirement to improve stability and computing efficiency in large-scale optimization issues, especially picture restoration-related ones. We made sure that the suggested strategy will globally converge in compliance with the Wolfe line search requirements by using well-crafted search parameters. Numerous experimental findings and comparative analysis showed that the BBV method may drastically cut down on the overall number of function evaluations and simulation

TABLE 1. Numerical results of FR and BBV algorithms.

Image	**Noise**	**FR-Method**			**BBV-Method**		
		NI	**NF**	**PSNR. (dB)**	**NI**	**NF**	**PSNR. (dB)**
El	90%	65	114	28.2019	40	79	28.4162
Ho		111	214	25.287	50	100	25.2720
Le		121	236	24.3962	51	103	24.9529
c512		108	211	22.8583	51	101	22.7446
El	70%	38	39	31.864	31	61	31.8691
Ho		63	116	31.2564	38	75	30.8690
Le		81	155	27.4824	52	104	27.3591
c512		78	142	30.6259	39	80	31.2461
El	50%	35	36	33.9129	27.0	53.0	33.9432
Ho		52	53	30.6845	38.0	77.0	34.6589
Le		82	153	30.5529	54.0	107.0	30.6755
c512		59	87	35.5359	34.0	70.0	35.6207



(a) (b) (c)

(d) (e) (f)

(g) (h) (i)

FIGURE 1. Shows the 256 × 256 Lena image's FR and BBV algorithm results.

iterations without sacrificing the quality of the reconstructed image. This demonstrates the suggested method's resilience and effectiveness in managing high-dimensional optimization.

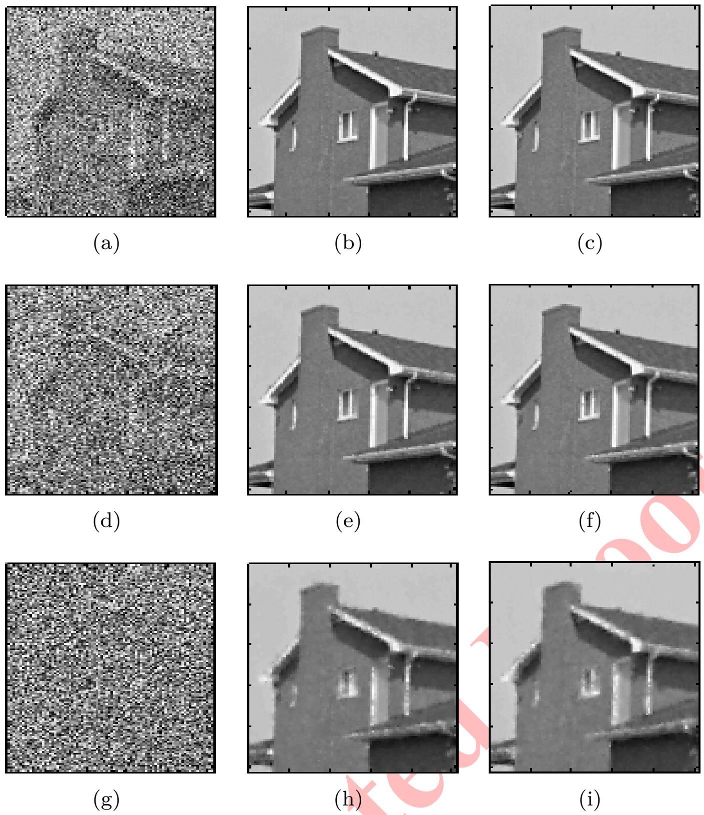
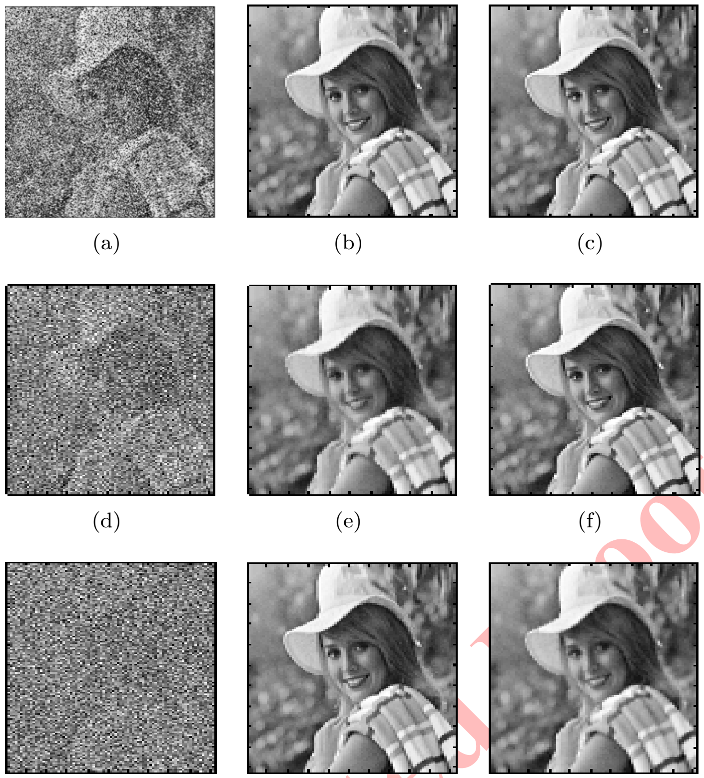


FIGURE 2. Shows the 256×256 House image's FR and BBV algorithm results.



(a) (b) (c)

(d) (e) (f)

(g) (h) (i)

FIGURE 3. Shows the 256 × 256 Elaine image's FR and BBV algorithm results.

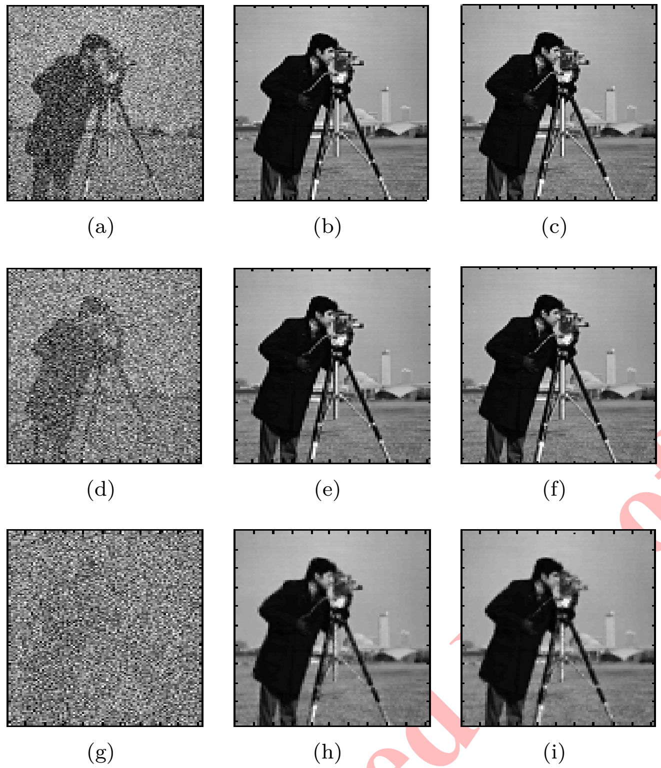


FIGURE 4. Shows the 256×256 Cameraman image's FR and BBV algorithm results.

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