

Modulation instability and solitary wave analysis of the Ivancevic option pricing model via Novel mathematical techniques

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Abstract

In academia, financial engineering challenges are very important, and there is an ongoing demand for effective ways to examine and evaluate these models. The new extended direct algebraic approach and the new auxiliary equation method have been used in this study to investigate the analytical solutions of the economically significant model known as the Ivancevic option pricing model (IOPM) in the sense of a new definition of M-truncated derivative. Additionally, we will examine the equation's modulation instability (MI) using linear stability approach that demonstrates the accuracy and stability of all combined solutions. The analytical method of this model is infrequently observed in the literature, despite the fact that numerous scholars have examined its applicability and feasibility. The regulated Brownian motion associated with a non-linear Schrödinger type equation is described by this model. Mathematical approaches are applied to generate the solution to understand the fluctuations in market prices for the proposed model. The reaction of system to pulse propagation is explained through the use of graphical representations in 2D, contour and 3D forms. Anticipating suitable parameter values that correspond with the observed data is made possible by these illustrations. The sensitivity analysis predicts the model's dependence on the initial conditions. The findings of this study indicated that the suggested approaches preserved the physical characteristics present in realistic processes while providing a very dependable and adaptable substitute for problem-solving. Furthermore, we guarantee that all of the results demonstrate that these techniques are a useful mathematical tool for identifying precise solitary wave solutions to nonlinear models that are present in various scientific and technical domains.

Keywords. Ivancevic option pricing model, M-truncated derivative, New extended direct algebraic method, New auxiliary equation method, Exact solutions, Modulation instability analysis.

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1. INTRODUCTION

Differential equations (DEs) have been widely used for mathematical modeling of physical and biological phenomena for over 150 years, and their use is only increasing. The majority of occurrences in the real world are described by nonlinear partial differential equations NLPDEs. Nonlinear processes present the most challenging problems to resolve

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due to their nonlinear characteristics. It is also challenging to control nonlinear processes in a system that can change quickly with only slight modifications to the appropriate parameters.

In spite of this difficulty, scientists have developed a number of effective approaches to determine distinct and feasible solutions to NLPDEs, such as the mapping approach [26, 38], modified Sardar Sub equation technique [20, 33], the simple equation approach [32, 46], the homotopy perturbation technique [12, 17], the Jacobi elliptic function approach [15, 49], the extended tanh function approach [2, 19, 36], the direct algebraic technique [16, 29, 45], the Hirota bilinear approach [41, 48], the Riccati-Bernoulli sub-ODE scheme [4, 18, 23], the Kudryashov method [5], and many more [7–9, 24, 25, 47]. More than three hundred years have passed with the discovery of fractional derivatives and integrals, although this distinction has not always been novel. Recently, the calculus of nonlinear fractional partial differential equations (NLFPDEs) has emerged to be one of the most difficult fields in nonlinear dynamics. NLFPDEs have garnered significant attention in recent decades due to their widespread applications in various fields, including fluid mechanics, fractional dynamics, geochemistry, manipulate theory, astrophysics, and many other [31, 34, 35]. NLFPDEs may be utilized to accurately evaluate the engineering aspect phrases dispersion, convection, diffusion, and response, which are closely related to the subjects mentioned above. The fundamental method of standard differentiation and integration in any order is fractional calculus, which can be applied to construct and comprehend the various physical aspects of a continuous transition from stagnant to oscillating phenomena.

Nowadays, fractional non linear evaluation equations (FNLEEs) are quite well-known for conducting extensive research in a variety of engineering and physical science related fields. For option pricing, the Black-Scholes (BS) model is a widely used mathematical model, as described in [10]. By understanding the mathematics behind the BS model, traders can gain a better grasp of the risks and benefits associated with trading options. For option pricing, one more mathematical model is the binomial model [39]. This model determines an option's value via a series of calculations that consider the fundamental asset's price, fluctuations, and time to expiration. The most important and pressing research subject at the time is how to maintain and handle changes in market prices in the financial system and markets. A vast array of technological and scientific tools can be developed by professionals and scientists to facilitate the creation of more sophisticated goods. Several kinds of new analytical solutions were produced by using effective schemes [22, 40]. Raheel *et al.* [37] developed soliton solutions using three distinct approaches and provided a graphic representation of their propagation.

In finance, stochastic differential equations are frequently employed, and their stability is examined [43]. The developers of a continuous-time financial model determined the best expenditure and portfolio policies [6, 51]. By modeling a worldwide financial market and its constituent parts, a global information system is created [27]. The authors developed the complex schematics for the robust economic model [13]. An algorithm for identifying which economic zone should be focused has been provided, and the idea of risk factors has been included into the control plan. Adomian [1] and numerous others [28, 52] developed and evaluated a national economic model made up of connected nonlinear equations. In this research, we employed EDAM and NAEM to find the analytical solutions of fractional IOPM. The methods used in this study offer several advantages. They provide exact analytical solutions, allowing deeper insight into the behavior of the Ivancevic option pricing mode.

However, the methods also have some disadvantages. They involve complex algebraic computations and may not generalize well to more complex models. Their effectiveness depends on suitable parameter selection, and their limited use in existing literature may restrict opportunities for comparative validation. The BS model and the M-truncated derivative IOPM are two of the most popular financial models for option pricing. Both models can be used to calculate an option's theoretical price, but they differ in their assumptions and techniques. M-fractional IOPM, which relies on fractional calculus, is utilized to price options with non-constant volatility. This model takes into account the fluctuating volatility of the underlying asset, making it more realistic than the BS model. The more traditional BS model is based on the notion that the volatility of the underlying asset is constant.

It is among the most important and fascinating economic model under investigation. In the literature, a number of exact solutions have been discovered by applying this model in different ways. For example, new solutions to IOPM have been determined by Edeki *et al* [14]. The trial function technique is used to create rogue wave solitons for the governing model, as explained in [30]. The IOPM in the sense of M-truncated derivative is described as follows [11]:

$$iD_{M,t}^{\alpha,\beta}v + \frac{\varepsilon}{2}D_{M,x}^{2\alpha,\beta}v + \Omega|v|^2v = 0, \quad i = \sqrt{-1}, \quad (1.1)$$



where

$$D_{M,t}^{\alpha,\beta} v = \lim_{\tau \rightarrow 0} v(tE_\beta(\tau t^{1-\epsilon})) - v(t)/\tau, 0 < \alpha < 1, \beta > 0.$$

Here, $v = v(x, t)$ is the option price wave profile, t is the time variable, and x represents the model's asset price. The volatility, denoted by parameters ε , indicates either a constant or the stochastic process itself. Where the Landau coefficient, denoted by $\Omega = \Omega(r, w)$, characterizes adaptive market potential. Ω and r become equal in the nonadaptive simplest case, indicating the interest rate, however in the adaptive case, $\Omega(r, w)$ may be related to market temperature and is dependent on the set of tractable constants $\{W_i\}$. The potential field is shown by the probability density function in the third term, which is represented by $|v|^2$. In our research, we also examined MI to analyze the stability and dynamics of the obtained analytical solutions of the IOPM. As far as we are aware, these solutions are not found by utilizing new extended direct algebraic method (nEDAM) and new auxiliary equation method (NAEM) for IOPM. For financial professionals, the calculated solutions offer a reliable and precise way to value alternatives. The document has been divided into multiple sections. The description of techniques can be found in Section 2. The mathematical computation and analytical solutions of the proposed model are provided in Section 3. There is significant explanation of the graphs in Section 4. The sensitivity analysis research is covered in Section 5. Section 6 extracts the assessment of modulation instability. The conclusion of the paper is eventually laid out in Section 7.

2. DESCRIPTION OF METHODOLOGIES

2.1. The New Extended Direct Algebraic Method. Here, we will outline the primary principles of the recently developed EDAM [50] for obtaining soliton solutions to NLPDEs.

Step 1:

Let us considered the NLPDE of the form

$$G(z, z_t, z_x, z_{tt}, z_{xx}, \dots) = 0. \quad (2.1)$$

By applying the wave transformation, we obtain

$$z(x, t) = Q(\eta), \eta = x - ct, \quad (2.2)$$

where $c \neq 0$, and the subsequent ordinary differential equation (ODE) is obtained.

$$H(z, z', z'', \dots) = 0. \quad (2.3)$$

Step 2:

Suppose solution of the ODE as:

$$Q = Q(\eta) = \sum_{i=0}^M a_i Z^i(\eta), \quad (2.4)$$

where a_i ($0 \leq i \leq M$) are constants to be found and $Z(\eta)$ satisfies the ODE in the form of

$$Z'(\eta) = Ln(\rho) (\mu + \nu W(\eta) + \zeta W^2(\eta)), \rho \neq 0, 1. \quad (2.5)$$

Here, ν , μ and ζ represent the real constants and their relation to the parameters found in the auxiliary equation.

Family-1:

When $\phi < 0$ and $\zeta \neq 0$,

$$\begin{cases} Z_1(\eta) = -\frac{\nu}{2\zeta} + \frac{\sqrt{-\phi}}{2\zeta} \tan_\rho \left(\frac{\sqrt{-\phi}}{2} \eta \right), \\ Z_2(\eta) = -\frac{\nu}{2\zeta} - \frac{\sqrt{-\phi}}{2\zeta} \cot_\rho \left(\frac{\sqrt{-\phi}}{2} \eta \right), \\ Z_3(\eta) = -\frac{\nu}{2\zeta} + \frac{\sqrt{-\phi}}{2\zeta} (\tan_\rho(\sqrt{-\phi}\eta) \pm \sqrt{mn} \sec_\rho(\sqrt{-\phi}\eta)), \\ Z_4(\eta) = -\frac{\nu}{2\zeta} + \frac{\sqrt{-\phi}}{2\zeta} (\cot_\rho(\sqrt{-\phi}\eta) \pm \sqrt{mn} \csc_\rho(\sqrt{-\phi}\eta)), \\ Z_5(\eta) = -\frac{\nu}{2\zeta} + \frac{\sqrt{-\phi}}{4\zeta} \left(\tan_\rho \left(\frac{\sqrt{-\phi}}{4} \eta \right) - \cot_\rho \left(\frac{\sqrt{-\phi}}{4} \eta \right) \right). \end{cases}$$



Family-2:

When $\phi > 0$ and $\zeta \neq 0$,

$$\begin{cases} Z_6(\eta) = -\frac{\nu}{2\zeta} - \frac{\sqrt{\phi}}{2\zeta} \tanh_{\rho} \left(\frac{\sqrt{\phi}}{2} \eta \right), \\ Z_7(\eta) = -\frac{\nu}{2\zeta} - \frac{\sqrt{\phi}}{2\zeta} \coth_{\rho} \left(\frac{\sqrt{\phi}}{2} \eta \right), \\ Z_8(\eta) = -\frac{\nu}{2\zeta} + \frac{\sqrt{\phi}}{2\zeta} (-\tanh_{\rho}(\sqrt{\phi}\eta) \pm i\sqrt{mn} \operatorname{sech}_{\rho}(\sqrt{\phi}\eta)), \\ Z_9(\eta) = -\frac{\nu}{2\zeta} + \frac{\sqrt{\phi}}{2\zeta} (-\coth_{\rho}(\sqrt{\phi}\eta) \pm \sqrt{mn} \operatorname{csch}_{\rho}(\sqrt{\phi}\eta)), \\ Z_{10}(\eta) = -\frac{\nu}{2\zeta} - \frac{\sqrt{\phi}}{4\zeta} \left(\tanh_{\rho} \left(\frac{\sqrt{\phi}}{4} \eta \right) + \coth_{\rho} \left(\frac{\sqrt{\phi}}{4} \eta \right) \right). \end{cases}$$

Family-3:

When $\mu\zeta > 0$ and $\nu = 0$,

$$\begin{cases} Z_{11}(\eta) = \sqrt{\frac{\mu}{\zeta}} \tan_{\rho}(\sqrt{\mu\zeta}\eta), \\ Z_{12}(\eta) = -\sqrt{\frac{\mu}{\zeta}} \cot_{\rho}(\sqrt{\mu\zeta}\eta), \\ Z_{13}(\eta) = \sqrt{\frac{\mu}{\zeta}} (\tan_{\rho}(2\sqrt{\mu\zeta}\eta) \pm \sqrt{mn} \sec_{\rho}(2\sqrt{\mu\zeta}\eta)), \\ Z_{14}(\eta) = \sqrt{\frac{\mu}{\zeta}} (-\cot_{\rho}(2\sqrt{\mu\zeta}\eta) \pm \sqrt{mn} \csc_{\rho}(2\sqrt{\mu\zeta}\eta)), \\ Z_{15}(\eta) = \frac{1}{2} \sqrt{\frac{\mu}{\zeta}} \left(\tan_{\rho} \left(\frac{\sqrt{\mu\zeta}}{2} \eta \right) - \cot_{\rho} \left(\frac{\sqrt{\mu\zeta}}{2} \eta \right) \right). \end{cases}$$

Family-4:

When $\mu\zeta < 0$ and $\nu = 0$,

$$\begin{cases} Z_{16}(\eta) = -\sqrt{-\frac{\mu}{\zeta}} \tanh_{\rho}(\sqrt{-\mu\zeta}\eta), \\ Z_{17}(\eta) = -\sqrt{-\frac{\mu}{\zeta}} \coth_{\rho}(\sqrt{-\mu\zeta}\eta), \\ Z_{18}(\eta) = \sqrt{-\frac{\mu}{\zeta}} (-\tanh_{\rho}(2\sqrt{-\mu\zeta}\eta) \pm i\sqrt{mn} \operatorname{sech}_{\rho}(2\sqrt{-\mu\zeta}\eta)), \\ Z_{19}(\eta) = \sqrt{-\frac{\mu}{\zeta}} (-\coth_{\rho}(2\sqrt{-\mu\zeta}\eta) \pm \sqrt{mn} \operatorname{csch}_{\rho}(2\sqrt{-\mu\zeta}\eta)), \\ Z_{20}(\eta) = -\frac{1}{2} \sqrt{-\frac{\mu}{\zeta}} \left(\tanh_{\rho} \left(\frac{\sqrt{-\mu\zeta}}{2} \eta \right) + \coth_{\rho} \left(\frac{\sqrt{-\mu\zeta}}{2} \eta \right) \right). \end{cases}$$

Family-5:

When $\mu = \zeta$, and $\nu = 0$,

$$\begin{cases} Z_{21}(\eta) = \tan_{\rho}(\mu\eta), \\ Z_{22}(\eta) = -\cot(\mu\eta), \\ Z_{23}(\eta) = \tan_{\rho}(2\mu\eta) \pm \sqrt{mn} \sec_{\rho}(2\mu\eta), \\ Z_{24}(\eta) = -\cot_{\rho}(2\mu\eta) \pm \sqrt{mn} \csc_{\rho}(2\mu\eta), \\ Z_{25}(\eta) = \frac{1}{2} \left(\tan_{\rho} \left(\frac{\mu}{2} \eta \right) - \cot_{\rho} \left(\frac{\mu}{2} \eta \right) \right). \end{cases}$$

Family-6:

When $\mu = -\zeta$, and $\nu = 0$,

$$\begin{cases} Z_{26}(\eta) = -\tanh_{\rho}(\mu\eta), \\ Z_{27}(\eta) = -\coth_{\rho}(\mu\eta), \\ Z_{28}(\eta) = -\tanh_{\rho}(2\mu\eta) \pm i\sqrt{mn} \operatorname{sech}_{\rho}(2\mu\eta), \\ Z_{29}(\eta) = -\coth_{\rho}(2\mu\eta) \pm \sqrt{mn} \operatorname{csch}_{\rho}(2\mu\eta), \\ Z_{30}(\eta) = -\frac{1}{2} \tanh_{\rho} \left(\frac{\mu}{2} \eta \right) + \coth_{\rho} \left(\frac{\mu}{2} \eta \right). \end{cases}$$



Family-7:

When $\nu^2 = 4\mu\zeta$,

$$Z_{31}(\eta) = \frac{-2\mu(\nu\eta \ln \rho + 2)}{\nu^2 \eta \ln \rho}.$$

Family-8:

When $\nu = p, \mu = pq, (q \neq 0)$, and $\zeta = 0$,

$$Z_{32}(\eta) = \rho^{p\eta} - q.$$

Family-9:

When $\nu = \zeta = 0$,

$$Z_{33}(\eta) = \mu\eta \ln \rho.$$

Family-10:

When $\nu = \mu = 0$,

$$Z_{34}(\eta) = \frac{-1}{\zeta\eta \ln \rho}.$$

Family-11:

When $\mu = 0$, and $\nu \neq 0$,

$$\begin{cases} Z_{35}(\eta) = -\frac{m\nu}{\zeta(\cosh_\rho(\nu\eta) - \sinh_\rho(\nu\eta) + m)}, \\ Z_{36}(\eta) = -\frac{\nu(\sinh_\rho(\nu\eta) + \cosh_\rho(\nu\eta))}{\zeta(\sinh_\rho(\nu\eta) + \cosh_\rho(\nu\eta) + n)}. \end{cases}$$

Family-12:

When $\nu = p, \zeta = pq, (q \neq 0 \text{ and } \mu = 0)$,

$$Z_{37}(\eta) = -\frac{m\rho^{p\eta}}{m - qn\rho^{p\eta}}.$$

The trigonometric and hyperbolic functions are now provided as follows:

$$\begin{cases} \sinh_\psi(\eta) = \frac{m\psi^\eta - n\psi^{-\eta}}{2}, \cosh_\psi(\eta) = \frac{m\psi^\eta + n\psi^{-\eta}}{2}, \\ \tanh_\psi(\eta) = \frac{m\psi^\eta - n\psi^{-\eta}}{g\psi^\eta + h\psi^{-\eta}}, \coth_\psi(\eta) = \frac{m\psi^\eta + n\psi^{-\eta}}{m\psi^\eta - n\psi^{-\eta}}, \\ \operatorname{sech}_\psi(\eta) = \frac{2}{m\psi^\eta + n\psi^{-\eta}}, \operatorname{csch}_\psi(\eta) = \frac{2}{m\psi^\eta - n\psi^{-\eta}}, \\ \sin_\psi(\eta) = \frac{2i}{m\psi^{i\eta} - n\psi^{-i\eta}}, \cos_\psi(\eta) = \frac{2}{m\psi^{i\eta} + n\psi^{-i\eta}}, \\ \tan_\psi(\eta) = -i \frac{m\psi^{i\eta} - n\psi^{-i\eta}}{m\psi^{i\eta} + n\psi^{-i\eta}}, \cot_\psi(\eta) = i \frac{m\psi^{i\eta} + n\psi^{-i\eta}}{m\psi^{i\eta} - n\psi^{-i\eta}}, \\ \sec_\psi(\eta) = \frac{2}{m\psi^{i\eta} + n\psi^{-i\eta}}, \csc_\psi(\eta) = \frac{2i}{m\psi^{i\eta} - n\psi^{-i\eta}}. \end{cases}$$

where $m, n > 0$ are referred to as the parameters of deformation.

2.2. The New Auxiliary Equation Method. Here, we will describe the description of the NAEM [42].

Step 1:

Consider the Equations (2.1), (2.2), and (2.3).

Step 2:

Suppose that the analytical soliton solution to Equation (2.3) is:

$$q(\eta) = \sum_{i=0}^M a_i \beta^{if(\eta)}, \quad (2.6)$$

which satisfying the auxiliary equation of the subsequent form:

$$f'(\eta) = \frac{1}{\ln(\beta)} \left(\mu \beta^{-f(\eta)} + \lambda + v \beta^{f(\eta)} \right), \quad (2.7)$$



where $a_0, a_1, a_2, \dots, a_M$ are real constants such that $a_M \neq 0$.

Step 3:

We now use the balancing principle to determine M , which indicates that we are able to find value of M through comparison of the highest order derivative with the nonlinear terms of Equation (2.6). The following is the family of solutions for Equation (2.7):

Family-1:

When $\lambda^2 - 4\mu v < 0$, and $v \neq 0$,

$$\begin{cases} \beta^f(\eta) = \frac{-\lambda}{2\zeta} + \frac{\sqrt{4\mu v - \lambda^2}}{2\zeta} \tan\left(\frac{\sqrt{4\mu v - \lambda^2}}{2}\eta\right), \\ \beta^f(\eta) = \frac{-\lambda}{2\zeta} - \frac{\sqrt{4\mu v - \lambda^2}}{2\zeta} \cot\left(\frac{\sqrt{4\mu v - \lambda^2}}{2}\eta\right). \end{cases}$$

Family-2:

When $\lambda^2 - 4\mu v > 0$, and $v \neq 0$,

$$\begin{cases} \beta^f(\eta) = \frac{-\lambda}{2\zeta} - \frac{\sqrt{\lambda^2 - 4\mu v}}{2\zeta} \tanh\left(\frac{\sqrt{\lambda^2 - 4\mu v}}{2}\eta\right), \\ \beta^f(\eta) = \frac{-\lambda}{2\zeta} - \frac{\sqrt{\lambda^2 - 4\mu v}}{2\zeta} \coth\left(\frac{\sqrt{\lambda^2 - 4\mu v}}{2}\eta\right). \end{cases}$$

Family-3:

When $\lambda^2 + 4\mu^2 < 0$, $v \neq 0$, and $v = -\mu$,

$$\begin{cases} \beta^f(\eta) = \frac{\lambda}{2\mu} - \frac{\sqrt{-4\mu^2 - \lambda^2}}{2\mu} \tan\left(\frac{\sqrt{-4\mu^2 - \lambda^2}}{2}\eta\right), \\ \beta^f(\eta) = \frac{\lambda}{2\mu} + \frac{\sqrt{-4\mu^2 - \lambda^2}}{2\mu} \cot\left(\frac{\sqrt{-4\mu^2 - \lambda^2}}{2}\eta\right). \end{cases}$$

Family-4:

When $\lambda^2 + 4\mu^2 > 0$, $v \neq 0$, and $v = -\mu$,

$$\begin{cases} \beta^f(\eta) = \frac{\lambda}{2\mu} + \frac{\sqrt{4\mu^2 + \lambda^2}}{2\mu} \tanh\left(\frac{\sqrt{4\mu^2 + \lambda^2}}{2}\eta\right), \\ \beta^f(\eta) = \frac{\lambda}{2\mu} + \frac{\sqrt{4\mu^2 + \lambda^2}}{2\mu} \coth\left(\frac{\sqrt{4\mu^2 + \lambda^2}}{2}\eta\right). \end{cases}$$

Family-5:

When $\lambda^2 + 4\mu^2 < 0$, and $v = \mu$,

$$\begin{cases} \beta^f(\eta) = \frac{-\lambda}{2\mu} + \frac{\sqrt{4\mu^2 - \lambda^2}}{2\mu} \tan\left(\frac{\sqrt{4\mu^2 - \lambda^2}}{2}\eta\right), \\ \beta^f(\eta) = \frac{-\lambda}{2\mu} - \frac{\sqrt{4\mu^2 - \lambda^2}}{2\mu} \cot\left(\frac{\sqrt{4\mu^2 - \lambda^2}}{2}\eta\right). \end{cases}$$

Family-6:

When $\lambda^2 + 4\mu^2 > 0$, and $v = \mu$,

$$\begin{cases} \beta^f(\eta) = \frac{-\lambda}{2\mu} - \frac{\sqrt{-4\mu^2 + \lambda^2}}{2\mu} \tanh\left(\frac{\sqrt{-4\mu^2 + \lambda^2}}{2}\eta\right), \\ \beta^f(\eta) = \frac{-\lambda}{2\mu} - \frac{\sqrt{-4\mu^2 + \lambda^2}}{2\mu} \coth\left(\frac{\sqrt{-4\mu^2 + \lambda^2}}{2}\eta\right). \end{cases}$$

Family-7:

When $\lambda^2 = 4\mu v$,

$$\beta^f(\eta) = -\frac{2 + \lambda\eta}{2\zeta\eta}.$$



Family-8:

When $\mu v < 0$, $\lambda = 0$, and $v \neq 0$,

$$\begin{cases} \beta^{f(\eta)} = -\sqrt{\frac{-\mu}{v}} \tanh(\sqrt{-\mu v} \eta), \\ \beta^{f(\eta)} = -\sqrt{\frac{-\mu}{v}} \coth(\sqrt{-\mu v} \eta). \end{cases}$$

Family-9:

When $\lambda = 0$, and $\mu = -v$,

$$\beta^{f(\eta)} = \frac{1 + e^{-2v\eta}}{-1 + e^{-2v\eta}}.$$

Family-10:

When $\mu = v = 0$,

$$\beta^{f(\eta)} = \cosh(\lambda\eta) + \sinh(\lambda\eta).$$

Family-11:

When $\mu = \lambda = K$, and $v = 0$,

$$\beta^{f(\eta)} = e^{K\eta} - 1.$$

Family-12:

When $\mu = \lambda = K$, and $\mu = 0$,

$$\beta^{f(\eta)} = \frac{e^{K\eta}}{1 - e^{K\eta}}.$$

Family-13:

When $\lambda = \mu + v$,

$$\beta^{f(\eta)} = -\frac{1 - \mu e^{(\mu-v)\eta}}{1 - \zeta e^{(\mu-v)\eta}}.$$

Family-14:

When $\lambda = -(\mu + v)$,

$$\beta^{f(\eta)} = \frac{\mu - e^{(\mu-v)\eta}}{v - e^{(\mu-v)\eta}}.$$

Family-15:

When $\mu = 0$,

$$\beta^{f(\eta)} = \frac{\lambda e^{\lambda\eta}}{1 - v e^{\lambda\eta}}.$$

Family-16:

When $\lambda = \mu = v \neq 0$,

$$\beta^{f(\eta)} = \frac{1}{2} \left[\sqrt{3} \tan \left(\frac{\sqrt{3}}{2} \mu \eta \right) - 1 \right].$$

Family-17:

When $\lambda = v = 0$,

$$\beta^{f(\eta)} = \mu \eta.$$

Family-18:

When $\lambda = \mu = 0$,

$$\beta^{f(\eta)} = -\frac{1}{v\eta}.$$



Family-19:

When $\mu = v$, and $\lambda = 0$,

$$\beta^{f(\eta)} = \tan(\mu\eta).$$

Family-20:

When $v = 0$,

$$\beta^{f(\eta)} = e^{\lambda\eta} - \frac{g}{h}.$$

3. MATHEMATICAL COMPUTATION OF THE MODEL AND ITS SOLUTIONS

Consider the transformation

$$q(x, t) = Q(\eta) e^{i\left(\frac{\Gamma(\beta+1)}{\alpha}(\omega x^\alpha + \gamma t^\alpha)\right)}, \quad \eta = \frac{\Gamma(\beta+1)}{\alpha}(\sigma x^\alpha + \tau t^\alpha). \quad (3.1)$$

where γ and τ stand for temporal velocity and $Q(\eta)$ for wave function amplitude. By applying Equation (3.1) to Equation (1.1) and separating the outcomes into real and imaginary parts we arrive to

Real part:

$$2\Omega Q^3 + \varepsilon\sigma^2 Q'' - (2\gamma + \varepsilon\omega^2) Q = 0, \quad (3.2)$$

Imaginary part:

$$(\varepsilon\omega\sigma + \tau) Q' = 0.$$

The velocity of the wave function is obtained from Equation (3.2) as follows:

$$\tau = -\varepsilon\omega\sigma. \quad (3.3)$$

3.1. Application of New Extended Direct Algebraic Method. The following solution can be written by taking into account the homogeneous balance between Q'' and Q^3 in Equation (3.2), which yields $N = 1$.

$$Q(\eta) = a_0 + a_1 Z(\eta), \quad (3.4)$$

where a_0 and a_1 are real constants. The set of algebraic equations involving a_0 , a_1 , and other constants can be obtained by utilizing Equations (3.5) and (2.5) into Equation (3.2) and adding up all of the coefficients of various powers of $Z(\eta)$. Utilizing certain computational packet algorithms to solve the resulting collection of algebraic equations, we determine

$$a_0 = \frac{\Delta\nu}{2}, a_1 = 2\zeta, \gamma = -\frac{1}{4}\varepsilon(2\omega^2 + \sigma^2\phi), \quad \phi = \nu^2 - 4\mu\zeta, \quad (3.5)$$

where $\Delta = \pm \frac{i\sqrt{\varepsilon}\sigma}{\sqrt{\Omega}}$. By utilizing the Equation (3.5), we obtained the subsequent solutions for the Equation (2.5).

Set 1:

When $\phi < 0$ and $\zeta \neq 0$,

$$q_1(x, t) = \frac{\Delta}{2} \sqrt{-\phi} \tan_\rho \left(\frac{\sqrt{-\phi}}{2} \eta \right) e^{i\phi(x, t)}, \quad (3.6)$$

where

$$\begin{aligned} \phi(x, t) &= \frac{\Gamma(\beta+1)}{\alpha}(\omega x^\alpha + \gamma t^\alpha). \\ q_2(x, t) &= -\frac{\Delta}{2} \sqrt{-\phi} \cot_\rho \left(\frac{\sqrt{-\phi}}{2} \eta \right) e^{i\phi(x, t)}, \end{aligned} \quad (3.7)$$

$$q_3(x, t) = \frac{\Delta}{2} \sqrt{-\phi} \left(\tan_\rho \left(\sqrt{-\phi} \eta \right) \pm \sqrt{mn} \operatorname{sech}_\rho \left(\sqrt{-\phi} \eta \right) \right) e^{i\phi(x, t)}, \quad (3.8)$$



$$q_4(x, t) = \frac{\Delta}{2} \sqrt{-\phi} \left(\cot_{\rho} \left(\sqrt{-\phi} \eta \right) \pm \sqrt{mn} \csc_{\rho} \left(\sqrt{-\phi} \eta \right) \right) e^{i\phi(x, t)}, \quad (3.9)$$

$$q_5(x, t) = \Delta \frac{\sqrt{-\phi}}{4} \left(\tan_{\rho} \left(\frac{\sqrt{-\phi}}{4} \eta \right) - \cot_{\rho} \left(\frac{\sqrt{-\phi}}{4} \eta \right) \right) e^{i\phi(x, t)}. \quad (3.10)$$

Set 2:

When $\phi > 0$ and $\zeta \neq 0$,

$$q_6(x, t) = -\frac{\Delta}{2} \sqrt{\phi} \tanh_{\rho} \left(\frac{\sqrt{\phi}}{2} \eta \right) e^{i\phi(x, t)}, \quad (3.11)$$

$$q_7(x, t) = -\frac{\Delta}{2} \sqrt{\phi} \coth_{\rho} \left(\frac{\sqrt{\phi}}{2} \eta \right) e^{i\phi(x, t)}, \quad (3.12)$$

$$q_8(x, t) = \frac{\Delta}{2} \sqrt{\phi} \left(-\tanh_{\rho} \left(\sqrt{\phi} \eta \right) \pm i \sqrt{mn} \sec h_{\rho} \left(\sqrt{\phi} \eta \right) \right) e^{i\phi(x, t)}, \quad (3.13)$$

$$q_9(x, t) = \frac{\Delta}{2} \sqrt{\phi} \left(-\coth_{\rho} \left(\sqrt{\phi} \eta \right) \pm \sqrt{mn} \operatorname{cosech}_{\rho} \left(\sqrt{\phi} \eta \right) \right) e^{i\phi(x, t)}, \quad (3.14)$$

$$q_{10}(x, t) = \frac{-\Delta \sqrt{\phi}}{4} \left(\tanh_{\rho} \left(\frac{\sqrt{\phi}}{4} \eta \right) + \coth_{\rho} \left(\frac{\sqrt{\phi}}{4} \eta \right) \right) e^{i\phi(x, t)}. \quad (3.15)$$

Set 3:

When $\mu\zeta > 0$ and $\nu = 0$,

$$q_{11}(x, t) = \Delta \sqrt{\mu\zeta} \tan_{\rho} \left(\sqrt{\mu\zeta} \eta \right) e^{i\phi(x, t)}, \quad (3.16)$$

$$q_{12}(x, t) = -\Delta \sqrt{\mu\zeta} \cot_{\rho} \left(\sqrt{\mu\zeta} \eta \right) e^{i\phi(x, t)}, \quad (3.17)$$

$$q_{13}(x, t) = \Delta \sqrt{\mu\zeta} \left(\tan_{\rho} \left(2\sqrt{\mu\zeta} \eta \right) \pm \sqrt{mn} \sec_{\rho} \left(2\sqrt{\mu\zeta} \eta \right) \right) e^{i\phi(x, t)}, \quad (3.18)$$

$$q_{14}(x, t) = \Delta \sqrt{\mu\zeta} \left(-\cot_{\rho} \left(2\sqrt{\mu\zeta} \eta \right) \pm \sqrt{mn} \operatorname{cosec}_{\rho} \left(2\sqrt{\mu\zeta} \eta \right) \right) e^{i\phi(x, t)}, \quad (3.19)$$

$$q_{15}(x, t) = \frac{\Delta}{2} \sqrt{\mu\zeta} \left(\tan_{\rho} \left(\frac{\sqrt{\mu\zeta}}{2} \eta \right) - \cot_{\rho} \left(\frac{\sqrt{\mu\zeta}}{2} \eta \right) \right) e^{i\phi(x, t)}. \quad (3.20)$$

Set 4:

When $\mu\zeta < 0$ and $\nu = 0$,

$$q_{16}(x, t) = -i\Delta \sqrt{\mu\zeta} \tanh_{\rho} \left(\sqrt{-\mu\zeta} \eta \right) e^{i\phi(x, t)}, \quad (3.21)$$

$$q_{17}(x, t) = -i\Delta \sqrt{\mu\zeta} \coth_{\rho} \left(\sqrt{-\mu\zeta} \eta \right) e^{i\phi(x, t)}, \quad (3.22)$$

$$q_{18}(x, t) = i\Delta \sqrt{\mu\zeta} \left(-\tanh_{\rho} \left(2\sqrt{-\mu\zeta} \eta \right) \pm i \sqrt{mn} \operatorname{sech}_{\rho} \left(2\sqrt{-\mu\zeta} \eta \right) \right) e^{i\phi(x, t)}, \quad (3.23)$$



$$q_{19}(x, t) = i\Delta\sqrt{\mu\zeta} \left(-\coth_{\rho} \left(2\sqrt{-\mu\zeta}\eta \right) \pm \sqrt{mn} \operatorname{cosech}_{\rho} \left(2\sqrt{-\mu\zeta}\eta \right) \right) e^{i\phi(x, t)}, \quad (3.24)$$

$$q_{20}(x, t) = -\frac{\Delta}{2} i\sqrt{\mu\zeta} \left(\tanh_{\rho} \left(\frac{\sqrt{-\mu\zeta}}{2}\eta \right) + \coth_{\rho} \left(\frac{\sqrt{-\mu\zeta}}{2}\eta \right) \right) e^{i\phi(x, t)}. \quad (3.25)$$

Set 5:

When $\mu = \zeta$ and $\nu = 0$,

$$q_{21}(x, t) = \Delta\zeta \tanh_{\rho}(\mu\eta) e^{i\phi(x, t)}, \quad (3.26)$$

$$q_{22}(x, t) = -\Delta\zeta \cot_{\rho}(\mu\eta) e^{i\phi(x, t)}, \quad (3.27)$$

$$q_{23}(x, t) = 2\Delta\zeta \left(\tanh_{\rho}(2\mu\eta) \pm \sqrt{mn} \operatorname{sec}_{\rho}(2\mu\eta) \right) e^{i\phi(x, t)}, \quad (3.28)$$

$$q_{24}(x, t) = \Delta\zeta \left(-\cot_{\rho}(2\mu\eta) \pm \sqrt{mn} \operatorname{csc}_{\rho}(2\mu\eta) \right) e^{i\phi(x, t)}, \quad (3.29)$$

$$q_{25}(x, t) = \frac{\Delta}{2} \zeta \left(\tanh_{\rho} \left(\frac{\mu}{2}\eta \right) - \cot_{\rho} \left(\frac{\mu}{2}\eta \right) \right) e^{i\phi(x, t)}. \quad (3.30)$$

Set 6:

When $\zeta = -\mu$ and $\nu = 0$,

$$q_{26}(x, t) = \Delta\mu \tanh_{\rho}(\mu\eta) e^{i\phi(x, t)}, \quad (3.31)$$

$$q_{27}(x, t) = \Delta\mu \coth_{\rho}(\mu\eta) e^{i\phi(x, t)}, \quad (3.32)$$

$$q_{28}(x, t) = -\Delta\mu \left(-\tanh_{\rho}(2\mu\eta) \pm i\sqrt{mn} \operatorname{sech}_{\rho}(2\mu\eta) \right) e^{i\phi(x, t)}, \quad (3.33)$$

$$q_{29}(x, t) = -\Delta\mu \left(-\coth_{\rho}(2\mu\eta) \pm \sqrt{mn} \operatorname{csch}_{\rho}(2\mu\eta) \right) e^{i\phi(x, t)}, \quad (3.34)$$

$$q_{30}(x, t) = -\frac{\Delta}{2} \mu \left(\frac{1}{2} \tanh_{\rho} \left(\frac{\mu}{2}\eta \right) \pm \coth_{\rho} \left(\frac{\mu}{2}\eta \right) \right) e^{i\phi(x, t)}. \quad (3.35)$$

Set 7:

When $\nu^2 = 4\mu\zeta$,

$$q_{31}(x, t) = \Delta\sqrt{\mu\zeta} \left(1 - \frac{2\sqrt{\mu\zeta}(\nu\eta \ln \rho + 2)}{\nu^2 \eta \ln \rho} \right) e^{i\phi(x, t)}. \quad (3.36)$$

Set 8:

When $\nu = \mu = 0$,

$$q_{34}(x, t) = \frac{-\Delta}{\eta \ln \rho} e^{i\phi(x, t)}. \quad (3.37)$$

Set 9:

When $\mu = 0$ and $\nu \neq$,

$$q_{35}(x, t) = \Delta v \left(\frac{1}{2} - \frac{m}{(\cosh_{\rho}(\nu\eta) - \sinh_{\rho}(\nu\eta) + m)} \right) e^{i\phi(x, t)}, \quad (3.38)$$



$$q_{36}(x, t) = \Delta v \left(\frac{2(\sinh_{\rho}(\nu\eta) + \cosh_{\rho}(\nu\eta))}{(\sinh_{\rho}(\nu\eta) + \cosh_{\rho}(\nu\eta) + n)} \right) e^{i\phi(x, t)}. \quad (3.39)$$

Set 10:

When $\nu = p, \zeta = pq, (q \neq 0, \mu = 0)$,

$$q_{37}(x, t) = \Delta p \left(1 - \frac{qm\rho^{p\eta}}{m - qn\rho^{p\eta}} \right) e^{i\phi(x, t)}. \quad (3.40)$$

3.2. Application of New Auxiliary Equation Method. The homogeneous balancing method yields $N = 1$ when applied to Equation (3.2). The reduction of Equation (2.6) for $N = 1$ is:

$$q(\eta) = a_0 + a_1\beta^{f(\eta)}, \quad (3.41)$$

where a_0, a_1 are real constants. A collection of algebraic equations containing a_0, a_1 , and other parameters is obtained by substituting Equation (3.41) in Equation (3.2) with Equation (2.7) and adding up all coefficients of various powers of $Z(\eta)$. Utilizing certain computational packet algorithms to solve the resulting collection of algebraic equations, we acquire

$$a_0 = \frac{\Pi\lambda}{2}, a_1 = \Pi v, \gamma = -\frac{\varepsilon(\sigma^2(\lambda^2 - 4v\mu) + 2\omega^2)}{4}, \quad (3.42)$$

where $\Pi = \frac{i\sqrt{\varepsilon}\sigma}{\sqrt{\Omega}}$. Now, by replacing the acquired solutions into Equation (3.41), we obtain

$$q(\eta) = \Pi \left\{ \frac{\lambda}{2} + v\beta^{f(\eta)} \right\}. \quad (3.43)$$

The solutions obtained by integrating the solution given by Equation (2.7) into Equation (3.43) are:

Family 1:

When $\lambda^2 - 4\mu v < 0$, and $v \neq 0$,

$$q_{1,1}(x, t) = \Pi \frac{\sqrt{4\mu v - \lambda^2}}{2} \tan \left(\frac{\sqrt{4\mu v - \lambda^2}}{2} \eta \right) e^{i\phi(x, t)}, \quad (3.44)$$

$$q_{1,2}(x, t) = -\Pi \frac{\sqrt{4\mu v - \lambda^2}}{2} \cot \left(\frac{\sqrt{4\mu v - \lambda^2}}{2} \eta \right) e^{i\phi(x, t)}, \quad (3.45)$$

where $\phi(x, t) = \frac{\Gamma(\beta+1)}{\alpha} (\omega x^{\alpha} + \gamma t^{\alpha})$.

Family 2:

When $\lambda^2 - 4\mu v > 0$, and $v \neq 0$,

$$q_{2,1}(x, t) = -\Pi \frac{\sqrt{\lambda^2 - 4\mu v}}{2} \tanh \left(\frac{\sqrt{\lambda^2 - 4\mu v}}{2} \eta \right) e^{i\phi(x, t)}, \quad (3.46)$$

$$q_{2,2}(x, t) = -\Pi \frac{\sqrt{\lambda^2 - 4\mu v}}{2} \coth \left(\frac{\sqrt{\lambda^2 - 4\mu v}}{2} \eta \right) e^{i\phi(x, t)}. \quad (3.47)$$

Family 3:

When $\lambda^2 - 4\mu v < 0$, $v \neq 0$, and $v = -\mu$,

$$q_{3,1}(x, t) = \Pi \frac{\sqrt{-4\mu^2 - \lambda^2}}{2} \tan \left(\frac{\sqrt{-4\mu^2 - \lambda^2}}{2} \eta \right) e^{i\phi(x, t)}, \quad (3.48)$$



$$q_{3,2}(x, t) = \Pi \frac{\sqrt{-4\mu^2 - \lambda^2}}{2} \cot \left(\frac{\sqrt{-4\mu^2 - \lambda^2}}{2} \eta \right) e^{i\phi(x, t)}. \quad (3.49)$$

Family 4:

When $\lambda^2 - 4\mu v > 0$, $v \neq 0$, and $v = -\mu$,

$$q_{4,1}(x, t) = -\Pi \frac{\sqrt{4\mu^2 + \lambda^2}}{2} \tanh \left(\frac{\sqrt{4\mu^2 + \lambda^2}}{2} \eta \right) e^{i\phi(x, t)}, \quad (3.50)$$

$$q_{4,2}(x, t) = -\Pi \frac{\sqrt{4\mu^2 + \lambda^2}}{2} \coth \left(\frac{\sqrt{4\mu^2 + \lambda^2}}{2} \eta \right) e^{i\phi(x, t)}. \quad (3.51)$$

Family 5:

When $\lambda^2 - 4\mu^2 < 0$, and $v = \mu$,

$$q_{5,1}(x, t) = \Pi \frac{\sqrt{4\mu^2 - \lambda^2}}{2} \tan \left(\frac{\sqrt{4\mu^2 - \lambda^2}}{2} \eta \right) e^{i\phi(x, t)}, \quad (3.52)$$

$$q_{5,2}(x, t) = -\Pi \frac{\sqrt{4\mu^2 - \lambda^2}}{2} \cot \left(\frac{\sqrt{4\mu^2 - \lambda^2}}{2} \eta \right) e^{i\phi(x, t)}. \quad (3.53)$$

Family 6:

When $\lambda^2 - 4\mu^2 > 0$, and $v = \mu$,

$$q_{6,1}(x, t) = -\Pi \frac{\sqrt{-4\mu^2 + \lambda^2}}{2} \tanh \left(\frac{\sqrt{-4\mu^2 + \lambda^2}}{2} \eta \right) e^{i\phi(x, t)}, \quad (3.54)$$

$$q_{6,2}(x, t) = -\Pi \frac{\sqrt{-4\mu^2 + \lambda^2}}{2} \coth \left(\frac{\sqrt{-4\mu^2 + \lambda^2}}{2} \eta \right) e^{i\phi(x, t)}. \quad (3.55)$$

Family 7:

When $\lambda^2 = 4\mu v$,

$$q_7(x, t) = \frac{-\Pi}{\eta} e^{i\phi(x, t)}. \quad (3.56)$$

Family 8:

When $\mu v < 0$, $\lambda = 0$, and $v \neq 0$,

$$q_{8,1}(x, t) = -\Pi \sqrt{-\mu v} \tanh(\sqrt{-\mu v} \eta) e^{i\phi(x, t)}, \quad (3.57)$$

$$q_{8,2}(x, t) = -\Pi \sqrt{-\mu v} \coth(\sqrt{-\mu v} \eta) e^{i\phi(x, t)}. \quad (3.58)$$

Family 9:

When $\lambda = 0$, and $\mu = -v$,

$$q_9(x, t) = \Pi v \left(\frac{1 + e^{-2v\eta}}{-1 + e^{-2v\eta}} \right) e^{i\phi(x, t)}. \quad (3.59)$$

Family 10:

When $v = \lambda = K$, and $\mu = 0$,

$$q_{12}(x, t) = \frac{\Pi}{2} \left[K + 2K \left(\frac{e^{K\eta}}{1 - e^{K\eta}} \right) \right] e^{i\phi(x, t)}. \quad (3.60)$$



Family 11:When $\lambda = \mu + v$,

$$q_{13}(x, t) = \frac{\Pi}{2} \left[\mu + v - 2v \left(\frac{1 - \mu e^{(\mu-v)\eta}}{1 - v e^{(\mu-v)\eta}} \right) \right] e^{i\phi(x, t)}. \quad (3.61)$$

Family 12:When $\lambda = -(\mu + v)$,

$$q_{14}(x, t) = \frac{\Pi}{2} \left[-\mu - v + 2v \left(\frac{\mu - e^{(\mu-v)\eta}}{v - e^{(\mu-v)\eta}} \right) \right] e^{i\phi(x, t)}. \quad (3.62)$$

Family 13:When $\mu = 0$,

$$q_{15}(x, t) = \Pi \left[\frac{\lambda}{2} + v \left(\frac{\lambda e^{\lambda\eta}}{1 - v e^{\lambda\eta}} \right) \right] e^{i\phi(x, t)}. \quad (3.63)$$

Family 14:When $\lambda = \mu = v \neq 0$,

$$q_{16}(x, t) = \frac{\Pi}{2} \left[\lambda + v \left(\sqrt{3} \tan \left(\frac{\sqrt{3}}{2} \mu \eta \right) - 1 \right) \right] e^{i\phi(x, t)}. \quad (3.64)$$

Family 15:When $\lambda = \mu = 0$,

$$q_{18}(x, t) = -\frac{\Pi}{\eta} e^{i\phi(x, t)}. \quad (3.65)$$

Family 16:When $\mu = v$, and $\lambda = 0$,

$$q_{19}(x, t) = \Pi v \tan(\mu \eta) e^{i\phi(x, t)}. \quad (3.66)$$

4. RESULTS AND DISCUSSION

This section presents a number of the analytical results of the research in a graphic representation. The section mostly concentrates on past knowledge of particular findings that are investigated in this paper. Latest professional programming tools are used to build graphs for simple illustration. Additionally, each 2D, 3D and contour graphs are shown over a distinct time period. Figure 1 illustrates the periodic soliton structure of the solution $q_6(x, t)$, when $\alpha = 0.7, \beta = 1.9, \omega = 0.3, \sigma = 5, \tau = 10, \gamma = 0.7, m = 0.05, n = 0.05, \nu = 0.9, \Omega = -0.5$. The impact of fractional parameter on the dynamical behavior of solution $q_6(x, t)$ is demonstrated in Figure 2 and Figure 3 by selecting $\alpha = 0.85$ and $\alpha = 0.95$, respectively. Figure 4 illustrates the periodic soliton solution of $q_8(x, t)$, when $\alpha = 0.7, \beta = 1.9, \omega = 0.3, \sigma = 5, \tau = 10, \gamma = 0.7, m = 0.05, n = 0.05, \nu = 0.9, \Omega = -0.5$. Figures 5 and 6 describe impact of fractional parameter on solution $q_6(x, t)$ by selecting $\alpha = 0.85$ and $\alpha = 0.95$, respectively. The physical behavior of the solution q_{10} is represented in the Figures 7, 8, and 9 for $\beta = 0.9, \omega = 0.5, \sigma = 0.5, \tau = 0.5, \gamma = 0.9, \nu = -10, \Omega = 5$. The change in wave profile can be seen for different values of α as, $\alpha = 0.99, \alpha = 0.85$, and $\alpha = 0.3$ respectively. Figures 10, 11, and 12 represent the dark-type solution for the Equation (3.18) with the parameters $\beta = 0.9, \omega = 0.5, \sigma = 0.5, \tau = 0.5, \gamma = 0.9, \nu = -10, \Omega = 5$. The change in wave profile can be seen for different values of α as $\alpha = 0.99, \alpha = 0.89$, and $\alpha = 0.79$, respectively. Figures 13, 14, and 15 depict the bright-type solution for the Equation (3.25) with the parameters $\alpha = 0.99, \beta = 0.9, \omega = 0.5, \sigma = -0.5, \tau = 0.2, \gamma = -1, m = -4, n = 5, \nu = 5, \Omega = -0.5, \mu = -0.5, \xi = -0.5$. The change in wave profile can be seen by varying the values of fractional parameter as $\alpha = 0.99, \alpha = 0.80$, and $\alpha = 0.70$, respectively. Figure 16 depicts the periodic solution for the Equation (3.48) with the parameters $\alpha = 0.9, \beta = 0.1, \omega = 0.5, \sigma = -0.5, \tau = 0.2, \gamma = -1, \nu = 5, \Omega = 5, \mu = 0.5, \nu = 0.5, \lambda = 5$. Figures 17 and 18 show effect of fractional parameter by considering $\alpha = 0.80$, and $\alpha = 0.70$, respectively. Figure 19 displays the physical properties of the solution of Equation (3.56), which shows a periodic soliton with $\alpha = 0.9, \beta = 0.05, \omega =$



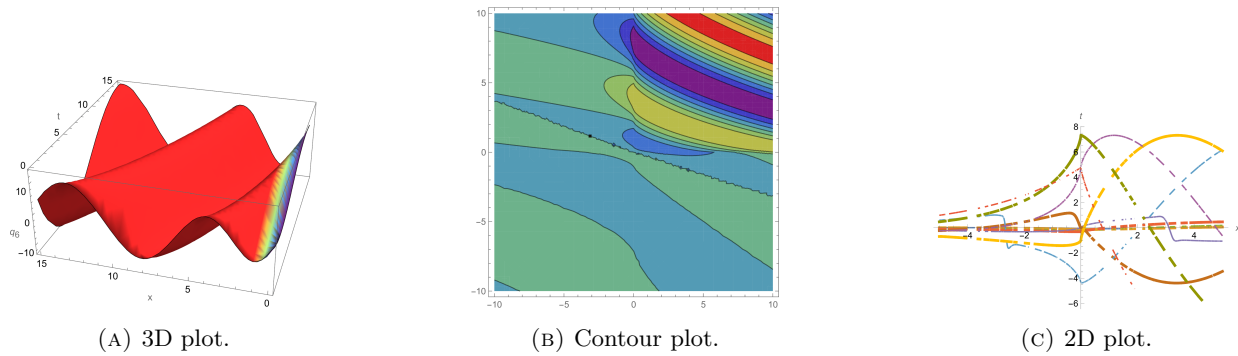


FIGURE 1. The visual depiction of the solution $q_6(x, t)$, when $\alpha = 0.7, \beta = 1.9, \omega = 0.3, \sigma = 5, \tau = 10, \gamma = 0.7, m = 0.05, n = 0.05, \Omega = -0.5$.

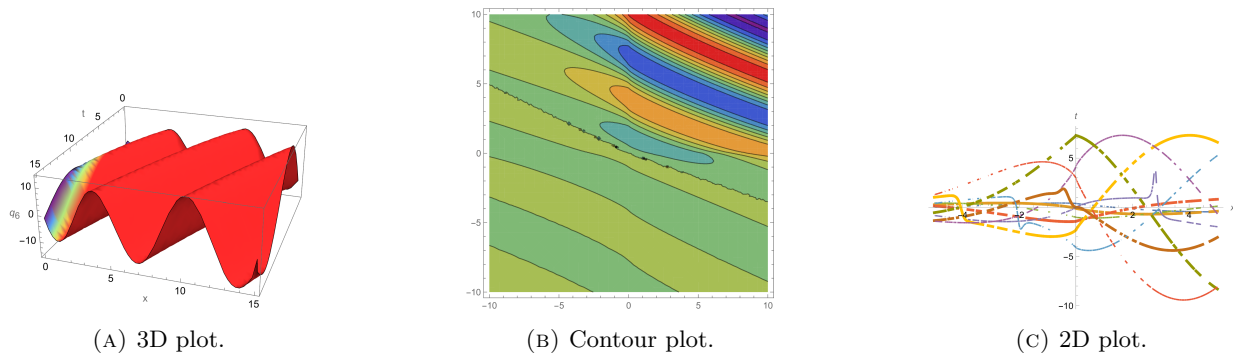


FIGURE 2. Effect of fractional parameter on the solution $q_6(x, t)$, when $\alpha = 0.85$.

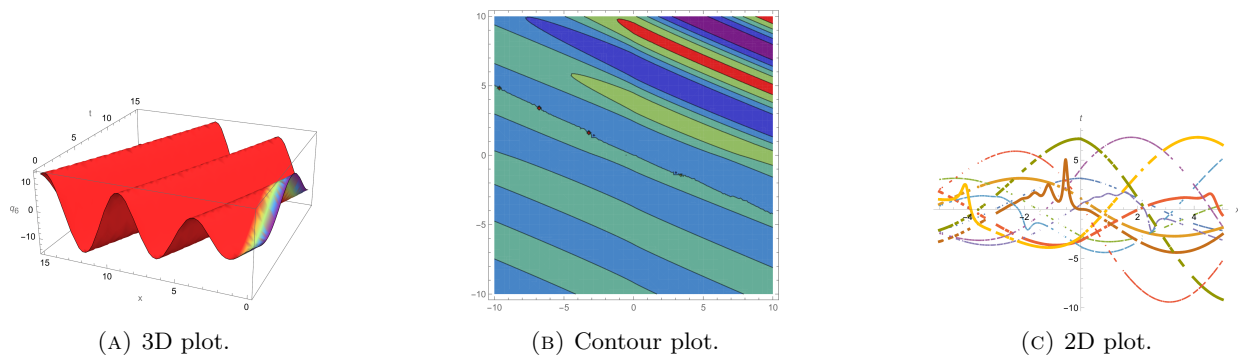


FIGURE 3. Effect of fractional parameter on the solution $q_6(x, t)$, when $\alpha = 0.95$.

1, $\sigma = 0.3, \tau = 3, \gamma = 1, \omega = 5, \Omega = 1, \mu = -0.5, \nu = 5, \lambda = -5$, and change in behaviour of solution is shown in Figures 20 and 21, when $\alpha = 0.7$, and $\alpha = 0.8$, respectively. Figures 22-24 depict the periodic soliton structure of the solution q_{12} , for parameters $\beta = 0.5, \omega = 4, \sigma = 0.3, \tau = 0.3, \gamma = 0.2, \Omega = -5, \Omega = -0.6, K = -2, \nu = -2, \lambda = -2$, at distinct values of α as $\alpha = 0.99, \alpha = 0.89$, and $\alpha = 0.79$.

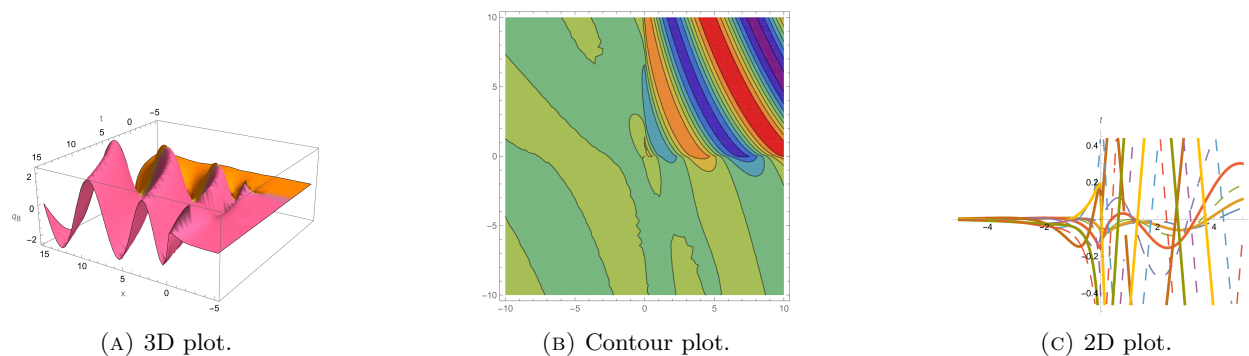


FIGURE 4. The visual representation of rogue wave solution $q_8(x, t)$, when $\alpha = 0.7, \beta = 0.5, \omega = 1.9, \sigma = 0.8, \tau = 0.5, \gamma = 0.9, m = 5, n = 5.5, -0.9, \Omega = 0.5$.

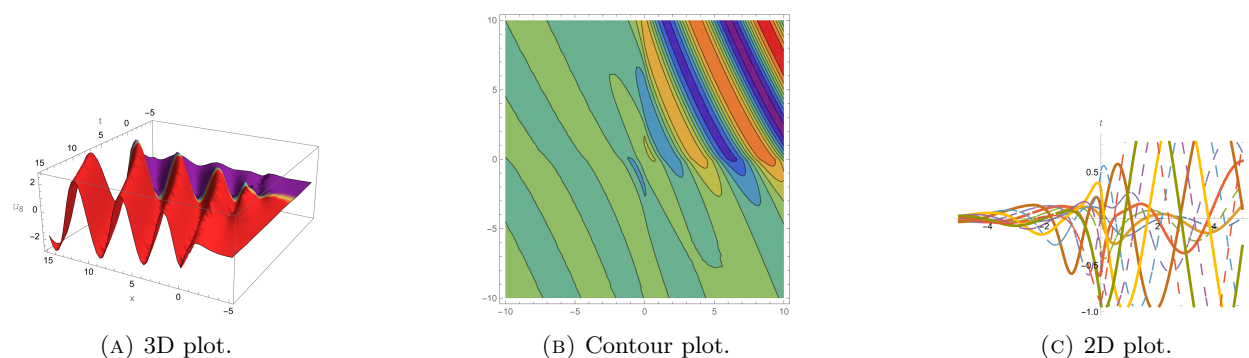


FIGURE 5. Effect of fractional parameter on the solution $q_8(x, t)$, when $\alpha = 0.85$.

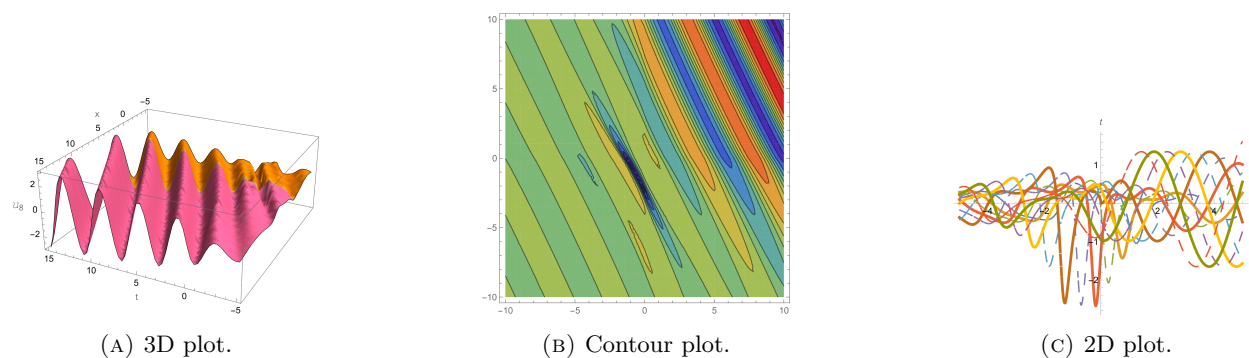


FIGURE 6. Effect of fractional parameter on the solution $q_8(x, t)$, when $\alpha = 0.95$.

5. SENSITIVE VISUALIZATION

This section utilizes the computational software MAPLE to perform a sensitivity analysis, examining the response of the IOPM. By applying the Galilean transformation to the ODE, the resulting dynamical system takes the following form.



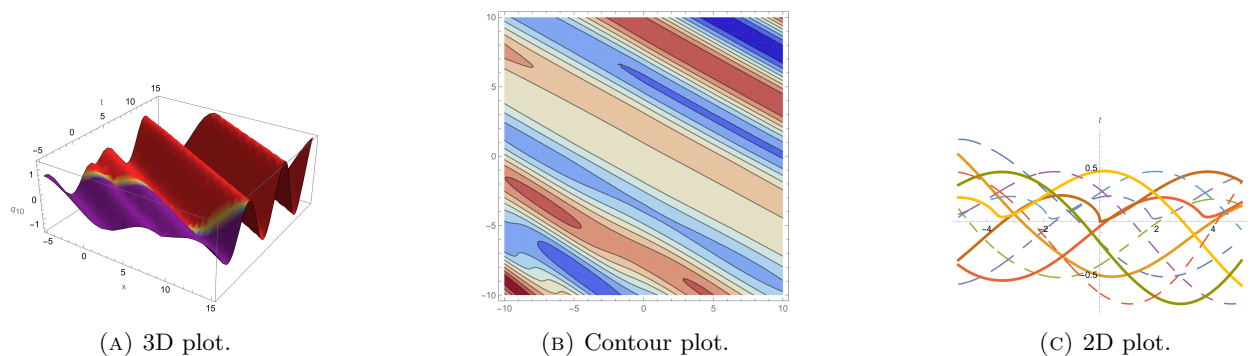


FIGURE 7. The visual representation of rogue wave solution q_{10} , $\alpha = 0.99, \beta = 0.9, \omega = 0.5, \sigma = 0.5, \tau = 0.5, \gamma = 0.9, = -10, \Omega = 5$.

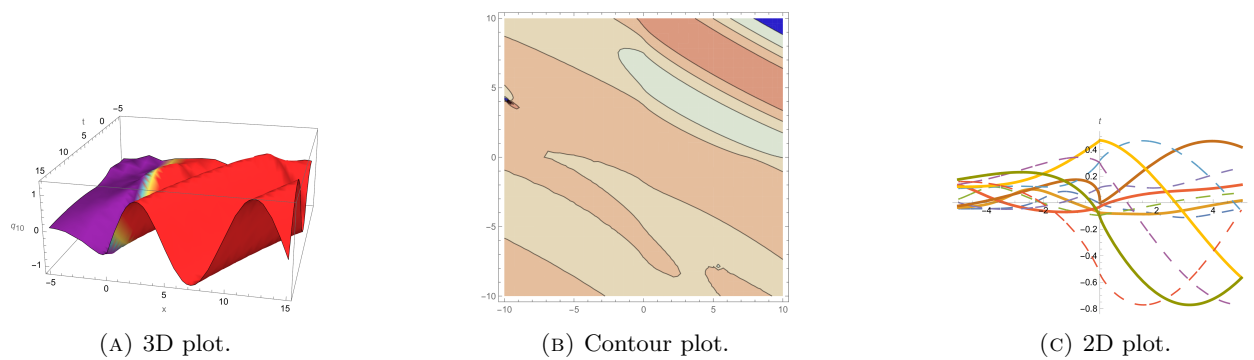


FIGURE 8. Effect of fractional parameter on the solution q_{10} , when $\alpha = 0.85$.

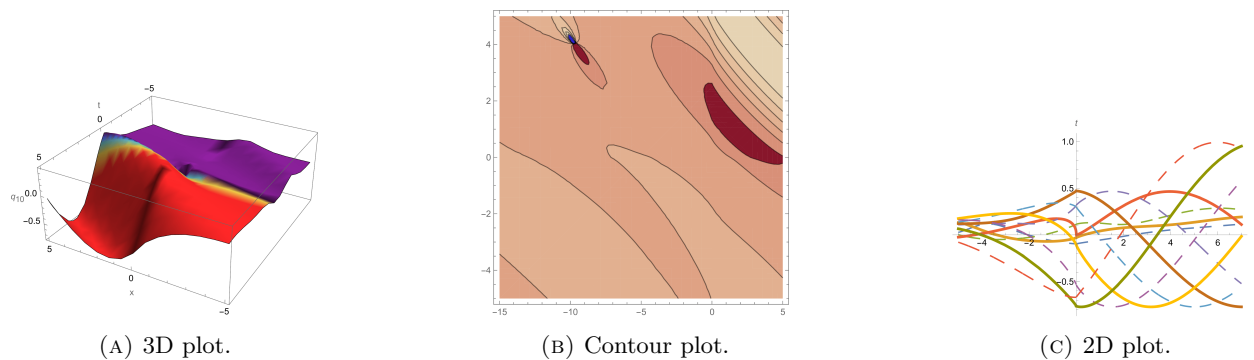


FIGURE 9. Effect of fractional parameter on the solution q_{10} , when $\alpha = 0.3$.

Let $Q'(\eta) = \lambda_2(\eta)$ and $Q(\eta) = \lambda_1(\eta)$,

$$\frac{d\lambda_2}{d\eta} = \frac{2\gamma + \epsilon\omega^2}{\epsilon\sigma^2}\lambda_1 - \frac{2\Omega}{\epsilon\sigma^2}\lambda_1^3,$$

$$\frac{d\lambda_1}{d\eta} = \lambda_2.$$

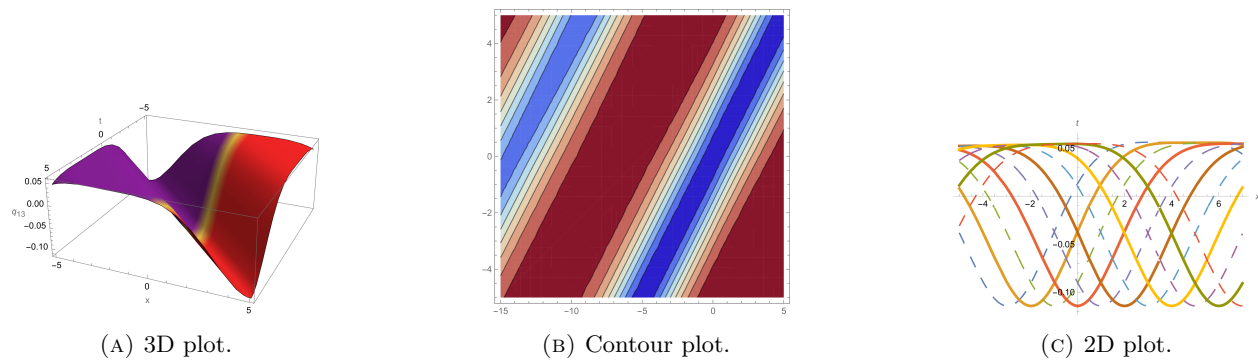


FIGURE 10. The visual representation of rogue wave solution q_{13} , when $\alpha = 0.99, \beta = 0.9, \omega = 0.5, \sigma = 0.5, \tau = 0.5, \gamma = 0.9, = -10, \Omega = 5$.

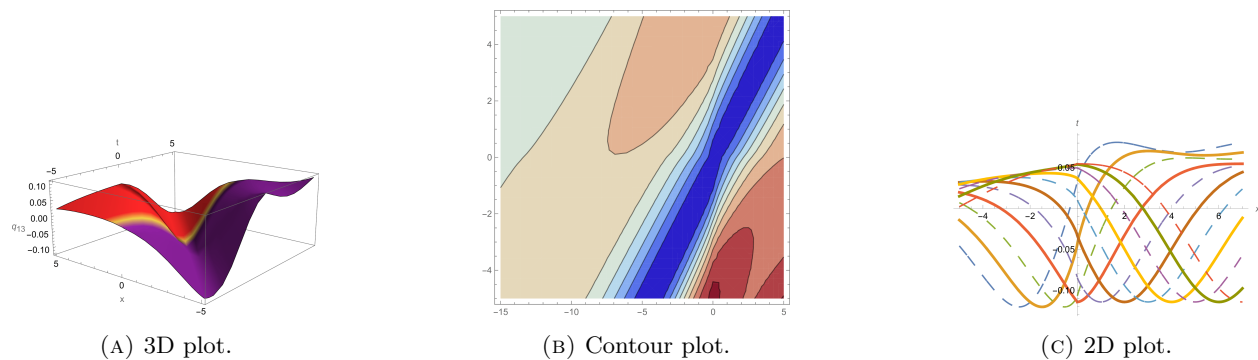


FIGURE 11. Effect of fractional parameter on the solution q_{13} , when $\alpha = 0.89$.

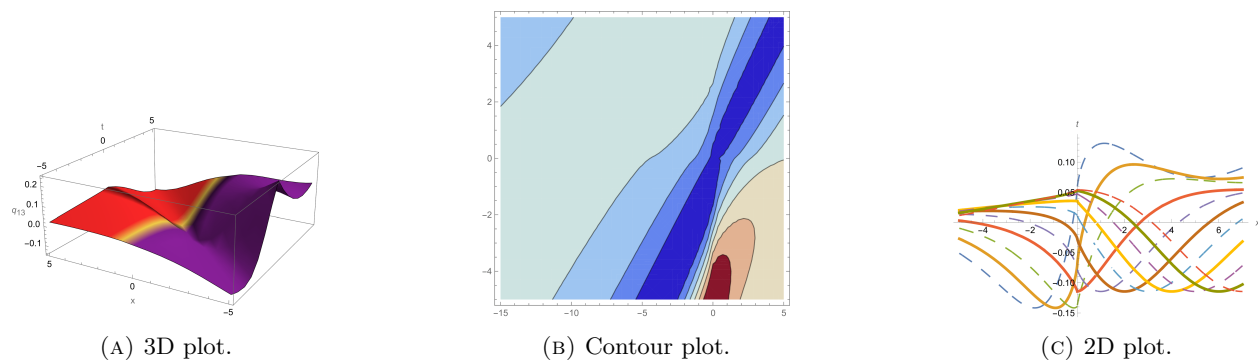


FIGURE 12. Effect of fractional parameter on the solution q_{13} , when $\alpha = 0.79$.

The graphs presented earlier illustrate the sensitive behavior of the dynamic system at $\gamma = 0.2, \epsilon = 0.1, \omega = 0.3, \sigma = 4$ and $\omega = 0.1$. To assess the system's sensitivity, a detailed sensitivity analysis is required. If small changes in the initial conditions lead to significant variations in the system's behavior, it indicates high sensitivity. To showcase this, multiple graphs are generated using a range of initial condition values. These graphs highlight the system's sensitivity to variations in starting conditions.

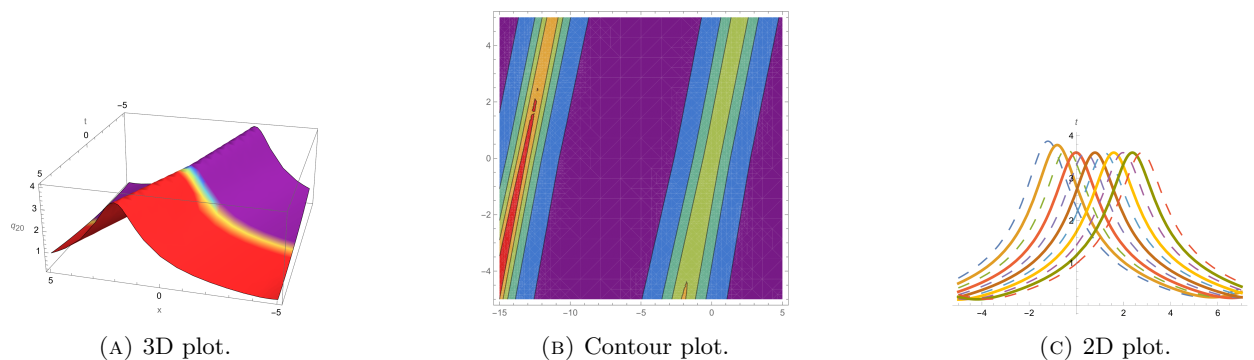


FIGURE 13. The visual representation of rogue wave solution q_{20} , when $\alpha = 0.99, \beta = 0.9, \omega = 0.5, \sigma = -0.5, \tau = 0.2, \gamma = -1, m = -4, n = 5, \Omega = -0.5, \mu = -0.5, \xi = -0.5$.

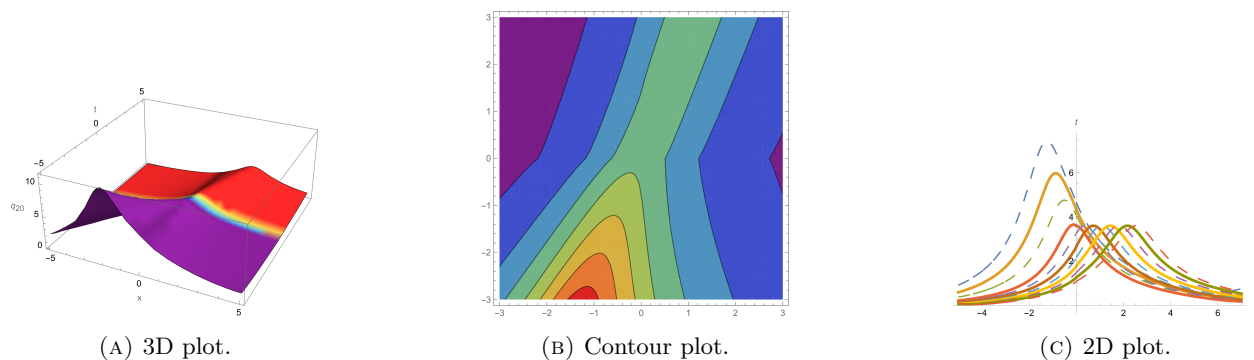


FIGURE 14. Effect of fractional parameter on the solution q_{20} , when $\alpha = 0.90$.

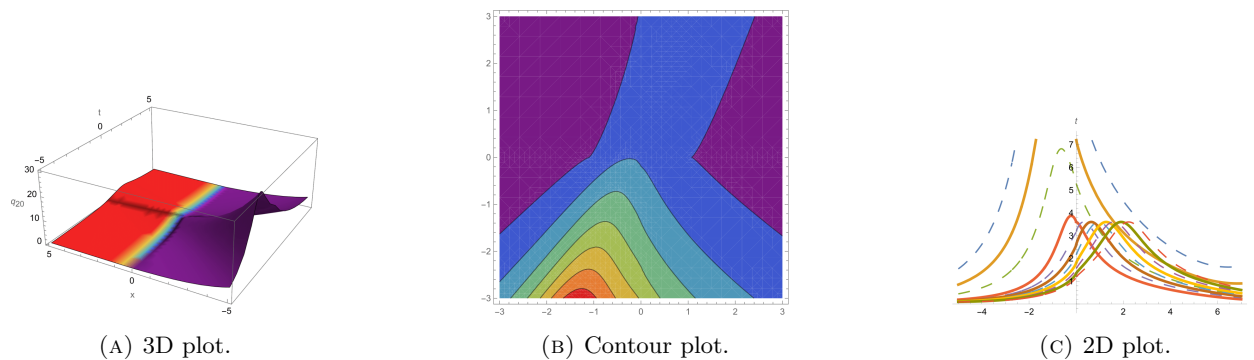


FIGURE 15. Effect of fractional parameter on the solution q_{20} , when $\alpha = 0.80$.

6. MODULATION INSTABILITY ANALYSIS

This section will examine the modulation Instability (MI) of Equation (1.1) when $\alpha = 1$ [3, 21]. Consider the subsequent solution for Equation (1.1)

$$\phi(x, t) = \left(\sqrt{Q_0} + \zeta(x, t) \right) e^{-iQ_0 x}, \quad (6.1)$$



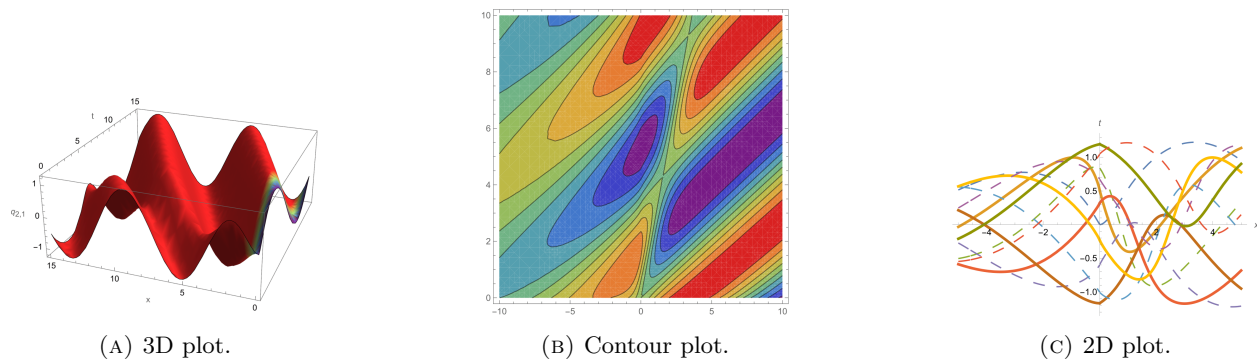


FIGURE 16. The visual representation of rogue wave solution $q_{2,1}$, when $\alpha = 0.9, \beta = 0.1, \omega = 0.5, \sigma = -0.5, \tau = 0.2, \gamma = -1, \delta = 5, \Omega = 5, \mu = 0.5, \nu = 0.5, \lambda = 5$.

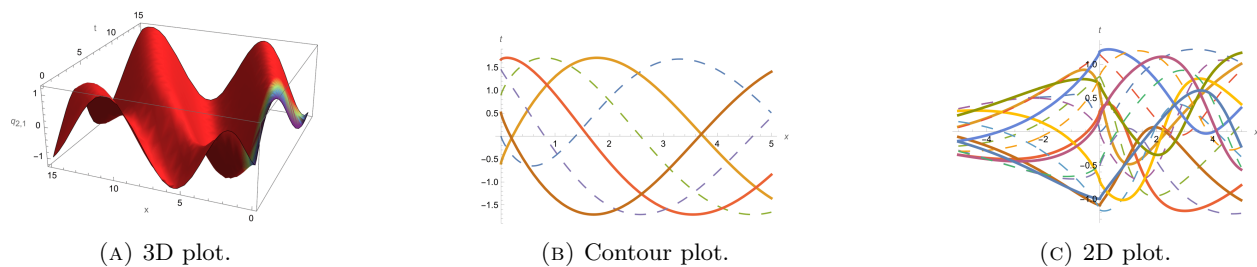


FIGURE 17. Effect of fractional parameter on the solution $q_{2,1}$, when $\alpha = 0.80$.

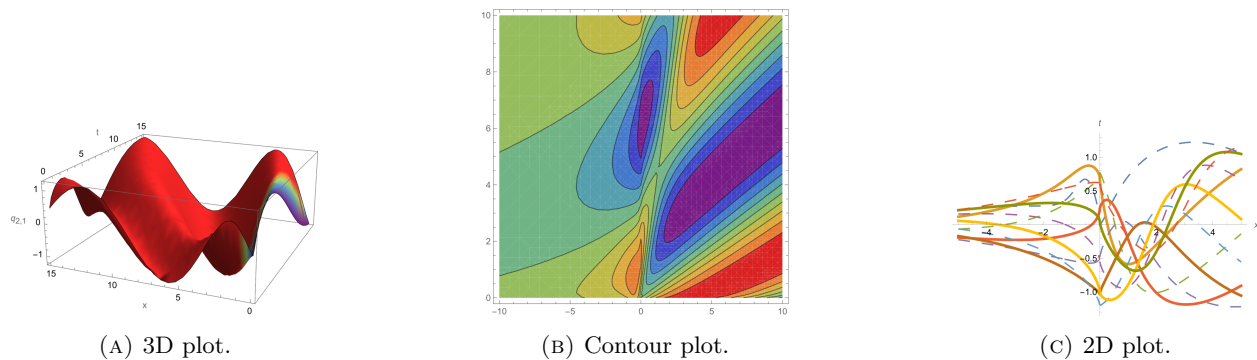


FIGURE 18. Effect of fractional parameter on the solution $q_{2,1}$, when $\alpha = 0.70$.

where the incident power is represented by P_0 . We use the idea of linear stability analysis to examine the evolution of the perturbation $\zeta(x, t)$. By inserting Equation (6.1) into Equation (1.1) and linearizing the outcome in $\zeta(x, t)$, we obtain

$$i\zeta_t + \frac{\varepsilon}{2}\zeta_{xx} + \Omega Q_0(\zeta + \zeta^*) = 0, \quad (6.2)$$

where ε is the dispersion frequency coefficient and ζ^* is the conjugate function. Now, assume the solutions for Equation (6.2) take the following form

$$\zeta(x, t) = \kappa e^{i(\alpha x - \beta t)} + \lambda e^{-i(\alpha x - \beta t)}, \quad (6.3)$$



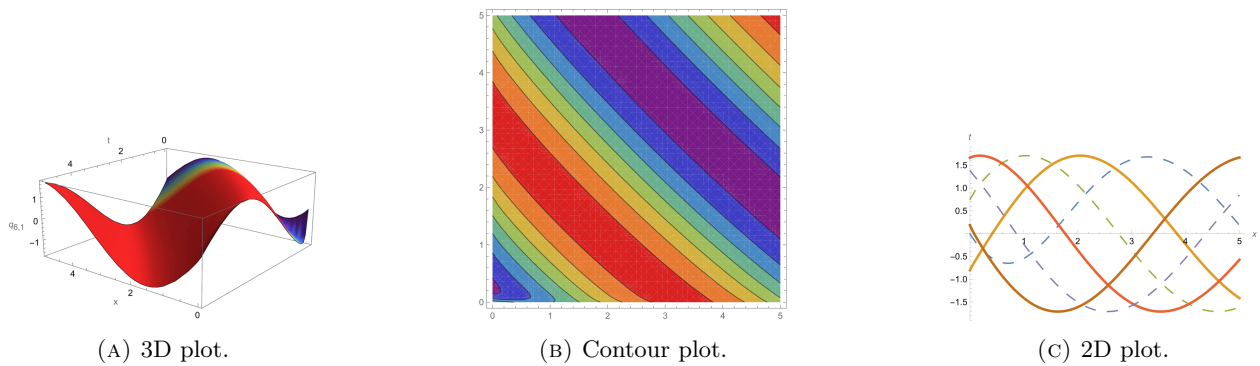


FIGURE 19. The visual representation of rogue wave solution $q_{6,1}$, when $\alpha = 0.9, \beta = 0.05, \omega = 1, \sigma = 0.3, \tau = 3, \gamma = 1, \Omega = 1, \mu = -0.5, \nu = 5, \lambda = -5$.

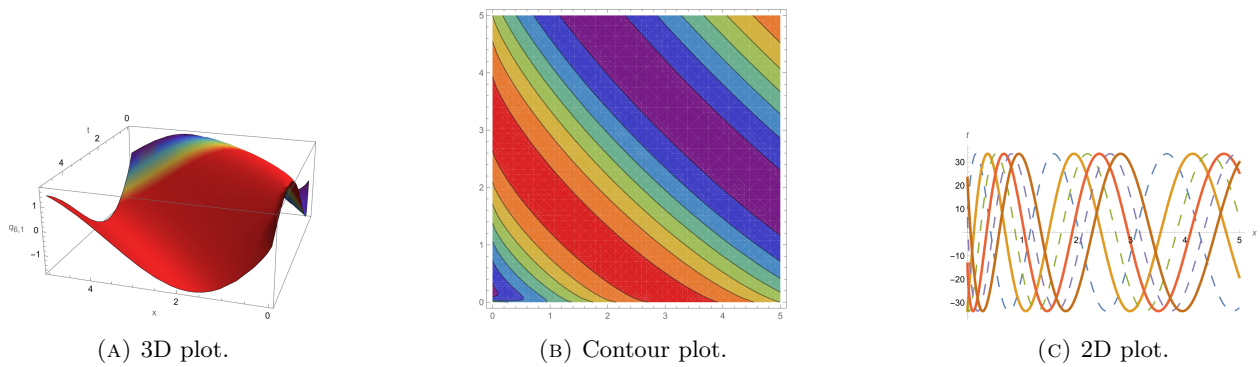


FIGURE 20. Effect of fractional parameter on the solution $q_{6,1}$, when $\alpha = 0.7$.

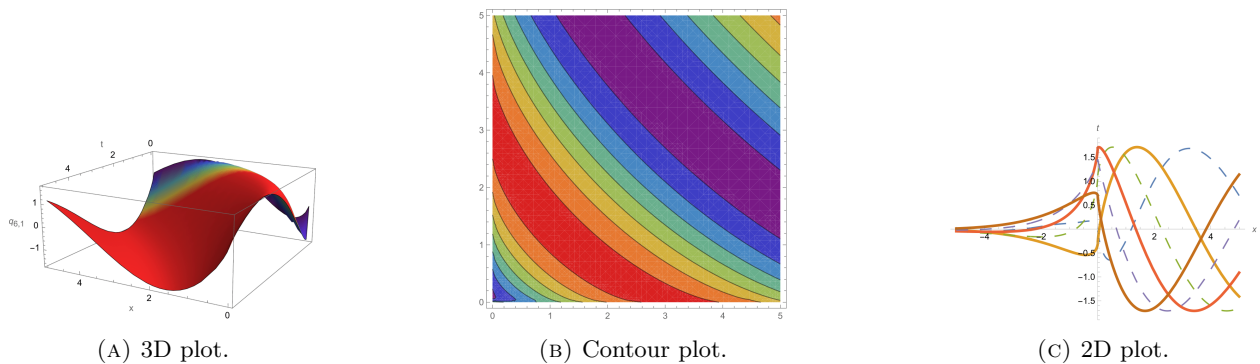


FIGURE 21. Effect of fractional parameter on the solution $q_{6,1}$, when $\alpha = 0.8$.

where β is the perturbation's frequency and α is its wave number. The subsequent dispersion relation is obtained by inserting Equation (6.3) into Equation (6.2), separating the coefficients of $e^{i(\alpha x - \beta t)}$ and $e^{-i(\alpha x - \beta t)}$, and determining the determinant of the coefficient matrix:

$$-\beta^2 + \frac{\alpha^4 \varepsilon^2}{4} - \alpha^3 \varepsilon Q_0, \quad (6.4)$$

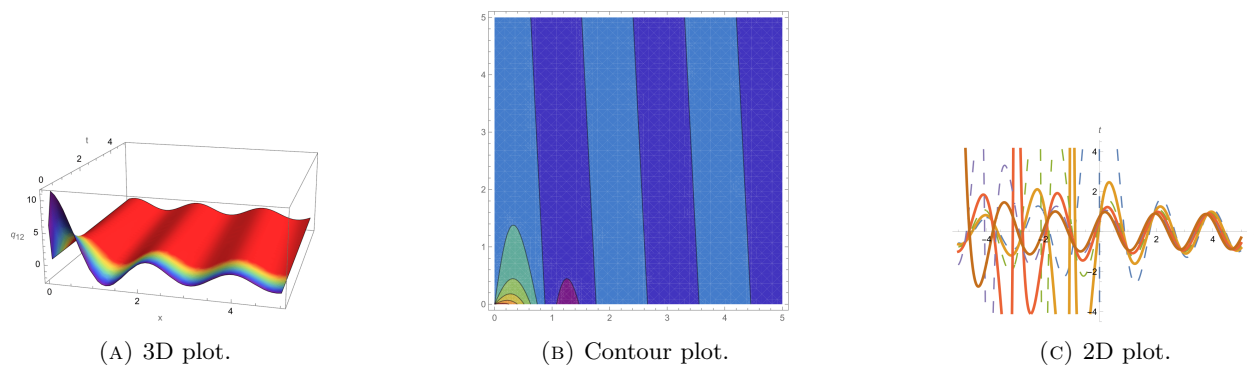


FIGURE 22. The visual representation of rogue wave solution q_{12} , when $\alpha = 0.99, \beta = 0.5, \omega = 4, \sigma = 0.3, \tau = 0.3, \gamma = 0.2, \Omega = -5, K = -2, \nu = -2, \lambda = -2$.

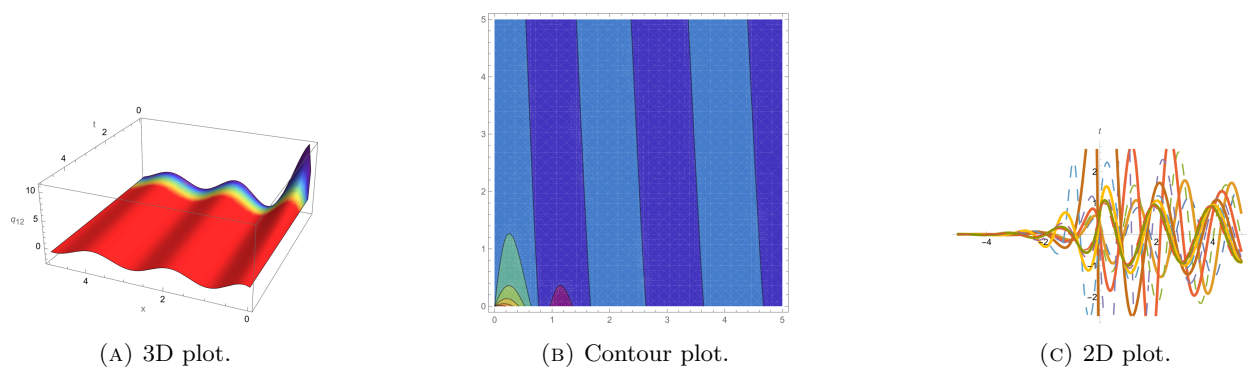


FIGURE 23. Effect of fractional parameter on the solution q_{12} , when $\alpha = 0.89$.

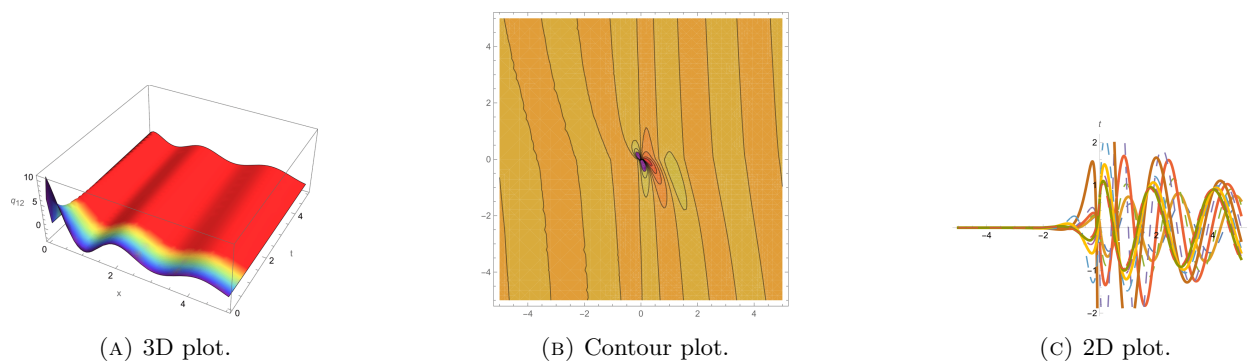


FIGURE 24. Effect of fractional parameter on the solution q_{12} , when $\alpha = 0.79$.

Equation (6.4) provides the following solutions for β :

$$\beta = \pm \frac{1}{2} \alpha^{\frac{3}{2}} \sqrt{\varepsilon} \sqrt{\alpha \varepsilon - 4Q_0}. \quad (6.5)$$



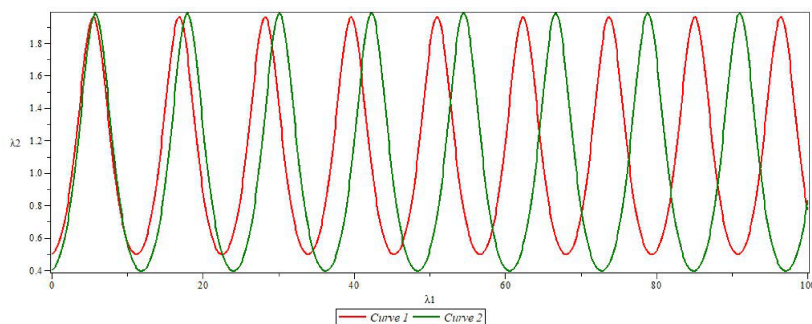


FIGURE 25. The sensitivity analysis is presented for curves 1 and 2 at the points (0.5, 0.03) and (0.4, 0.03), respectively.

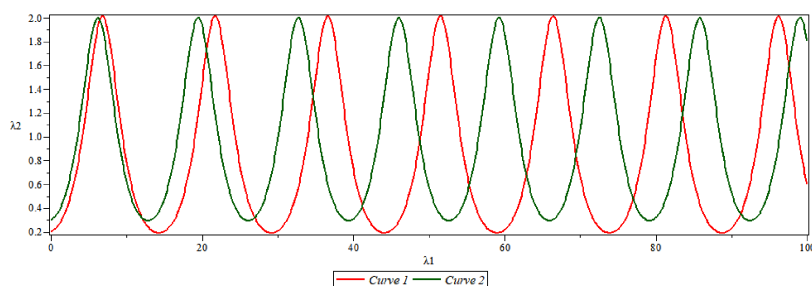


FIGURE 26. The sensitivity analysis is presented for curves 1 and 2 at the points (0.2, 0.03) and (0.3, 0.03), respectively.

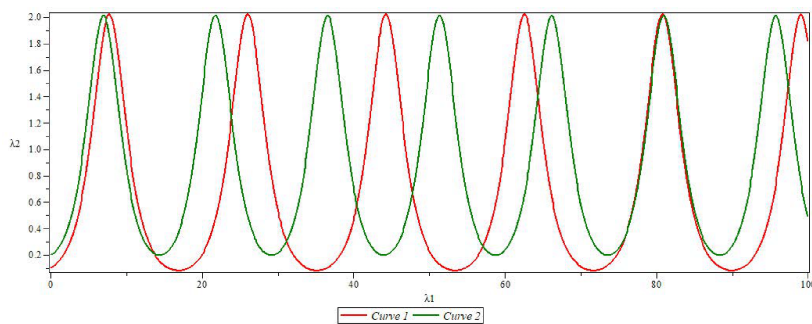


FIGURE 27. The sensitivity analysis is presented for curves 1 and 2 at the points (0.1, 0.03) and (0.2, 0.04), respectively.

When β has an imaginary part, the stability of the steady state is established by Equation (6.5). Thus, the requirement for MI to emerge from Equation (6.5) is when either

$$\alpha\varepsilon - 4Q_0 > 0, \varepsilon < 0, \quad (6.6)$$

or

$$\alpha\varepsilon - 4Q_0 < 0, \varepsilon > 0, \quad (6.7)$$



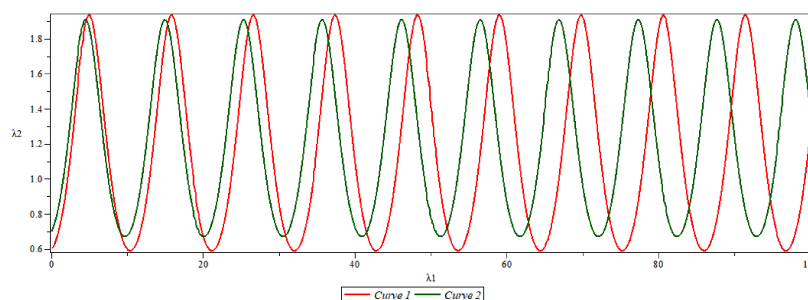


FIGURE 28. The sensitivity analysis is presented for curves 1 and 2 at the points (0.5, 0.01) and (0.6, 0.04), respectively.

Finally, the $H(\alpha)$ gain spectrum is computed as

$$H(\alpha) = 2\text{Im}(\beta) = \mp \alpha^{\frac{2}{3}} \sqrt{\varepsilon} \sqrt{\alpha \varepsilon - 4Q_0}. \quad (6.8)$$

7. CONCLUSIONS

Determining the exact value of options is difficult due to the complexity of the Ivancevic option pricing model. Therefore, in order to understand the complexities of such problems, accurate approximation approaches are required. These techniques are important to the advancement of scientific knowledge in the financial markets and greatly influence current advancements in the area. In this research, we utilized the NAEM and EDAM to investigate analytical solutions of IOPM in the sense of M-truncated derivative. We also examined MI analysis for the stationary solution of Equation (1.1). Many solitonic structures including, bright, dark, periodic and many other are obtained for distinct values of parameters. Notably, graphs move to the left or right and their amplitudes change when the fractional order is changed. Different amplitudes and phase components can be seen when comparing two profiles. This thorough analysis establishes the suggested approach as a dependable and adaptable solution, maintaining the intrinsic physical characteristics of practical processes in the option pricing domain. The equation's modulation instability using linear stability approach that demonstrates the accuracy and stability of all combined solutions have also been examined. The sensitive analysis certifies that the model is sensitive in perception of initial conditions. In conclusion, the primary findings of our work emphasize a fractional nonlinear option pricing model, which boosts the validity and flexibility in simulating market dynamics. The NAEM and the NEAM both efficiently produce unique solitonic wave structures, and the graphical representations help to clarify parameter conformity with observed data and pulse propagation. As far as we are aware, the results presented in this work are unique and have never been published before. Lastly, this work's analysis will definitely be useful in determining comparable solutions to the other physically significant nonlinear equations.

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DECLARATIONS

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