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Control of fractional-order chaotic systems under perturbations

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Abstract

In this paper, an appropriate fractional-integer integral sliding mode method for the control of fractional-order chaotic systems with perturbations such as uncertainties and external disturbances is addressed. When the upper bound of the perturbations is determined, a sliding mode controller is presented. Also, when the upper bound of the perturbations is unknown, an adaptive sliding mode control is designed. Analysis of the stability of the sliding mode surface is presented using the Lyapunov stability theory. Eventually, the results were carried out for the control of the complex fractional order chaotic T system.

Keywords. Fractional-order system, Chaotic system, Sliding mode control, Perturbation.
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1. Introduction

Fractional calculus as a mathematical topic has many applications in other sciences and engineering. In recent years, fractional-order chaotic systems have drawn a lot of attention due to their possible applications in biological systems [3] and secure communication [17]. A major challenge in the theory of chaos is control and synchronization. Many methods have been presented for controlling and synchronizing fractional-order chaotic systems such as projective synchronization [1], adaptive synchronization [19], sliding mode control [18], fuzzy control [15], etc. As fractional calculations progressed, studying chaotic control for fractional-order chaotic systems has become a hot research subject.

One of the noteworthy control methods is the sliding mode control. Robustness and insensitivity to the perturbations are the major features of sliding mode controllers. Perturbations can include uncertainties of parameters, unmodeled dynamics, and external disturbances. Sliding mode control has been studied in some research to control and synchronize chaotic fractional order systems. In [10], sliding-mode synchronization control is presented for uncertain fractional-order chaotic systems with time delay. In [16], a fractional sliding mode control method is given for the synchronization of disturbed fractional-order chaotic systems. In [9], using the fractional order version of Lyapunov stability theory, a fractional-order adaptive sliding mode control method is presented to synchronize fractional-order neural networks. In [18], generalized function projective synchronization of fractional order chaotic systems with disturbances via a fractional integer integral sliding mode control is investigated. In [2], a model-free adaptive sliding mode control is designed to synchronize chaotic fractional-order systems with input saturation. Notably, many existing methods focus on the synchronization of chaotic systems rather than their control. Also, it is notable that many existing chaos control methods are focused on chaotic systems whose state variables are real, their parameters are known and the system has no perturbations.

Also, there exist some chaotic systems with complex variables such as the hyperchaotic Lorenz complex system [12] and the hyperchaotic Lu complex system [13]. By increasing the content of transmitting signals and enhancing their security, fractional-order complex chaotic systems can provide higher security for cryptography [11]. Therefore, the study of dynamics, chaos control, and synchronization for complex nonlinear systems of fractional order is an interesting

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research topic. For example, in [7], a fractional order complex chaotic system is introduced and its synchronization is investigated.

Motivated by the above, considering that the control of fractional-order chaotic systems has been discussed in fewer articles, in this paper, an adaptive fractional-integer integral sliding mode control law and adaptive parameter update law are introduced to control and stabilize fractional-order chaotic systems with uncertainties of parameters and disturbances. In this scheme, no prior notice of the bounds of uncertainties and random disturbances is required. In numerical results, due to the importance of complex systems, we considered the fractional-order complex T system [8]. The results show that this system can be controlled under a suitable controller and the presented controllers can successfully eliminate chaos.

The structure of the continuation of the article is as follows. Some preliminaries of fractional order systems are given in section 2. In section 3, the formulation of the problem is presented. In section 4, the fractional-integer integral sliding mode surface and the adaptive sliding mode controller are provided to make the system asymptotically stable. In section 5, numerical results are presented. The letter is concluded in section 6.

2. Preliminaries

In this section, the required definitions and theorems of fractional order systems are stated.

Definition 2.1. ([4]). Suppose $q \in \mathbb{R}^+$. The Riemann-Liouville fractional integral operator of order q defined on $L^1[a,b]$ by

$$J_a^q f(x) := \frac{1}{\Gamma(q)} \int_a^x (x-t)^{q-1} f(t)dt, \quad a \le x \le b,$$

where Γ is the Euler's Gamma function.

Definition 2.2. ([4]). Let $q \ge 0$. The Caputo differential operator of order q is

$$\begin{aligned} {}^C D_a^q f &:= J_a^{m-q} D^m f \\ &= \frac{1}{\Gamma(m-q)} \int_a^x (x-t)^{m-q-1} D^m f(t)dt, \end{aligned}$$

where $D^m f \in L^1[a,b]$ and $m := \lceil q \rceil = \min\{z \in \mathbb{Z} : z \ge q\}$.

In the case $q \in \mathbb{N}$, we have $m = q$ and ${}^C D_a^q f := J_a^0 D^q f = D^q f$, i.e. we recover the standard definition in the classical case.

Lemma 2.3. *(Barbalat's lemma [6]). Let $f : \mathbb{R}^+ \to \mathbb{R}$ is uniformly continuous for all $t \ge 0$, and $\lim_{t\to\infty} \int_o^t f(\tau)d\tau$ is finite and exists. Then $\lim_{t\to\infty} f(t) = 0$.*

Theorem 2.4. *(Matignon [14]). The autonomous system*

$$ {}^C D^q x(t) = Ax(t), \qquad A \in \mathbb{R}^{n\times n}, \qquad 0 < q \le 1, \tag{2.1}$$

with initial value $x_0 = x(0)$ is

- *asymptotically stable iff $|\arg(eig(A))| > \frac{q\pi}{2}$.*
- *stable iff either it is asymptotically stable, or the eigenvalues satisfying $|\arg(eig(A))| = \frac{q\pi}{2}$ have geometric multiplicity one.*

3. Problem formulation

An n-dimensional chaotic system of the fractional-order under perturbations such as uncertainties and external disturbances can be described by

$$D^q x = Ax + g(x) + \Delta f(x) + d(t), \tag{3.1}$$

where $q = [q_1, q_2, \dots, q_n]^T$ for $0 < q_i < 1$ $(i = 1, 2, \dots, n)$, $x = [x_1, x_2, \dots, x_n]^T \in \mathbb{R}^n$ is the system state vector, $A \in \mathbb{R}^{n\times n}$ is the constant matrix, $g \in \mathbb{R}^n$ is a vector function, $\Delta f(x) \in \mathbb{R}^n$ is the parameter uncertainties, and $d(t) \in \mathbb{R}^n$ is the external disturbances. We show $C(x, t) = \Delta f(x) + d(t)$.

To chaos control in the system (3.1), we add the control input $u(t)$ to the system

$$D^q x = Ax + g(x) + C(x, t) + u(t). \tag{3.2}$$

In this paper, we want to design a suitable fractional order sliding mode controller so that all state variables tend to zero asymptotically, i.e. $\lim_{t\to\infty} ||x_i|| = 0$, for $i = 1, 2, \dots, n$.

4. Main results

In this part, we introduce a new fractional-integer integral-kind sliding surface. Next, we use an appropriate sliding mode controller to the trajectories converge from each initial state to the sliding surface $s(t) = 0$ in the limited time and remain on it evermore. When the sliding motion happens, the states tend asymptotically to zero.

4.1. **Design of sliding surface.** We present a new fractional-integer integral sliding surface

$$s(t) = J^{1-q}x(t) - \int_0^t (A + K)x(\tau)d\tau, \tag{4.1}$$

where $s = [s_1, s_2, \dots, s_n]^T$ and $K \in \mathbb{R}^{n\times n}$ is constant matrix. Then, we have

$$\dot{s}(t) = D^q x(t) - (A + K)x(t). \tag{4.2}$$

When the system is operating in sliding mode, we must have $s(t) = 0$ and $\dot{s}(t) = 0$. Therefore, dynamics on the sliding surface is obtained by:

$$D^q x(t) = (A + K)x(t). \tag{4.3}$$

4.2. **Design of sliding mode controller.** In practice, uncertainties of parameters and disturbances are usually limited. Let $\|C(x, t)\| \le \gamma$.

Theorem 4.1. *If the sliding mode controller is considered as*

$$u(t) = Kx - g(x) - (rs + \rho\gamma \,\mathrm{sgn}\, s), \tag{4.4}$$

where the constants $r > 0$ and $\rho > 1$, and sgn(.) *denotes sign function, then the states of the system* (3.2) *will tend to the surface $s(t) = 0$ in the finite time $t_f \le \frac{1}{\gamma(\rho-1)}\|s(0)\|$.*

Proof. Select the Lyapunov function

$$V(t) = \frac{1}{2}\|s(t)\|^2 = \frac{1}{2}s(t)^T s(t).$$

By (3.2) and (4.2), and the controller (4.4), we have

$$\begin{aligned}
\dot{V} &= s^T \dot{s} = s^T (D^q x(t) - (A + K)x(t)) \\
&= s^T (Ax + g + C + u - (A + K)x) \\
&= s^T (Ax + g + C + Kx - g(x) - (rs + \rho\gamma \,\mathrm{sgn}\, s) - (A + K)x) \\
&= s^T (C - rs - \rho\gamma \,\mathrm{sgn}\, s) \\
&\le -r\|s\|^2 + \gamma(1 - \rho)\|s\| \\
&\le \gamma(1 - \rho)\|s\| \le 0.
\end{aligned} \tag{4.5}$$

Integrating (4.5) from zero to t, we have

$$\int_0^t \gamma(\rho - 1)\|s\|d\tau \le -\int_0^t \dot{V}(\tau)d\tau = V(0) - V(t).$$

Since $\dot{V} \leq 0$, $V(0) - V(t)$ is finite and positive. Hence, $\int_0^t \gamma(\rho-1)\|s\| d\tau$ exists and is finite. Thus, according to Barbalat's lemma 2.3, $\lim_{t\to\infty} \gamma(\rho-1)\|s\| = 0$. So $s(t) \to 0$ when $t \to \infty$ because $\gamma(\rho-1) > 0$.

Therefore, all states of the system (3.2) asymptotically tend to the sliding surface $s(t) = 0$. Next, we will show that the convergence to $s(t) = 0$ occurs in finite time. For this, from (4.5), we have

$$\dot{V} = \frac{d(\frac{1}{2}\|s\|^2)}{dt} \leq \gamma(1-\rho)\|s\|,$$

which implies that

$$dt \leq -\frac{d(\frac{1}{2}\|s\|^2)}{\gamma(\rho-1)\|s\|} = -\frac{d\|s\|}{\gamma(\rho-1))}. \tag{4.6}$$

Integrating both sides of (4.6) from 0 to reaching time t_f and letting $s(t_f) = 0$ gives

$$\begin{aligned} t_f &\leq -\int_{s(0)}^{s(t_f)} \frac{d\|s\|}{\gamma(\rho-1)} = -\frac{1}{\gamma(\rho-1)}\|s\|\Big|_{s(0)}^{s(t_f)} \\ &= \frac{1}{\gamma(\rho-1)}\|s(0)\|. \end{aligned}$$

Hence, the states trajectories of the error system (3.2) will converge to the sliding surface $s(t) = 0$ in a finite time. □

Now, we assume the upper limit of disturbances is unknown and use an adaptive controller to guesstimate it. Suppose γ is estimated by $\bar{\gamma}$.

Theorem 4.2. *The states of the system* (3.2) *will converge to the sliding surface* $s(t) = 0$ *whenever*

$$\begin{aligned} u(t) &= Kx - g(x) - (rs + \rho\bar{\gamma}(t)\operatorname{sgn} s), \\ \dot{\bar{\gamma}}(t) &= \mu\rho s^T \operatorname{sgn} s, \end{aligned} \tag{4.7}$$

where the constants $r > 0$, $\rho > 1$ *and* $\mu > 0$ *is the gain coefficient of adaptive control.*

Proof. Select the Lyapunov function

$$V(t) = \frac{1}{2}\|s(t)\|^2 + \frac{1}{2\mu}(\bar{\gamma} - \gamma)^2.$$

Using Equations (3.2) and (4.2), and the control law (4.7), we have

$$\begin{aligned} \dot{V} &= s^T\dot{s} + \frac{1}{\mu}(\bar{\gamma}-\gamma)\dot{\bar{\gamma}} \\ &= s^T(D^q x - (A+K)x) + (\bar{\gamma}-\gamma)\rho s^T \operatorname{sgn} s \\ &= s^T(Ax + g + C + u - (A+K)x) + (\bar{\gamma}-\gamma)\rho s^T \operatorname{sgn} s \\ &= s^T(g - Kx + C + u) + (\bar{\gamma}-\gamma)\rho s^T \operatorname{sgn} s \\ &= s^T(g - Kx + C + Kx - g - (rs + \rho\bar{\gamma}\operatorname{sgn} s)) + (\bar{\gamma}-\gamma)\rho s^T \operatorname{sgn} s \\ &= s^T(C - rs - \rho\bar{\gamma}\operatorname{sgn} s) + (\bar{\gamma}-\gamma)\rho s^T \operatorname{sgn} s \\ &\leq \gamma\|s\| - r\|s\|^2 - \rho\gamma s^T \operatorname{sgn} s \\ &= (1-\rho)\gamma\|s\| - r\|s\|^2 < 0. \end{aligned}$$

Using Barbalat's Lemma 2.3, similar to the proof of Theorem 4.1, we can conclude $s(t) \to 0$ when $t \to \infty$, that means that all paths of the system (3.2) asymptotically tend to the sliding surface $s(t) = 0$. □

We have checked that the states of each initial value converge to the sliding surface. The subsequent theorem shows that all states on the sliding surface asymptotically converge to zero and therefore system (3.2) is asymptotically stable.

Theorem 4.3. *The system (3.2) is asymptotically stable iff*

$$|\arg(eig(A+K)| > \frac{q\pi}{2}. \tag{4.8}$$

Proof. Since dynamics on the sliding surface is as (4.3), we conclude by Theorem 2.4, the system (4.3) is asymptotically stable if and only if $|\arg(eig(A+K)| > \frac{q\pi}{2}$. Therefore, by choosing the appropriate matrix K that satisfies condition (4.8), the dynamics of the system (3.2) becomes asymptotically stable. □

5. Numerical results

In this section, we examine chaos control in fractional-order complex T system [8]. For the numerical solutions, we used the Adams-type predictor-corrector method with the time step size 0.001 [5].

The fractional-order complex T system is as follows:

$$\begin{aligned} D^q y_1 &= a(y_2 - y_1), \\ D^q y_2 &= (b-a)y_1 - a y_1 y_3, \\ D^q y_3 &= \frac{1}{2}(\bar{y}_1 y_2 + y_1 \bar{y}_2) - c y_3, \end{aligned} \tag{5.1}$$

where $y = [y_1, y_2, y_3]^T$ is the state vector, $y_1 = x_1 + ix_2$, $y_2 = x_3 + ix_4$ are complex variables, and $y_3 = x_5$ is real variable. Also, a, b, and c are system parameters. Considering the linearity of the Caputo derivative operator and separating complex variables into two real and imaginary parts, the system (5.1) can be written as follows:

$$\begin{aligned} D^q x_1 &= a(x_3 - x_1), \\ D^q x_2 &= a(x_4 - x_2), \\ D^q x_3 &= (b-a)x_1 - a x_1 x_5, \\ D^q x_4 &= (b-a)x_2 - a x_2 x_5, \\ D^q x_5 &= x_1 x_3 + x_2 x_4 - c x_5. \end{aligned} \tag{5.2}$$

The system (5.2) has equilibrium points as

$$E_0 = (0,0,0,0,0), \qquad E_\theta = (r\cos\theta, r\sin\theta, r\cos\theta, r\sin\theta, \frac{b-a}{a}),$$

where $r = \sqrt{c(b-a)/a}$ and $\theta \in [0, 2\pi]$. Equilibrium point E_θ exists when $c(b-a)/a > 0$. The system (5.2) is chaotic for $a = 2.1$, $b = 30$, $c = 0.6$ and $q = 0.99$ [8].

To control chaos, we consider the system (5.2) as follows:

$$\begin{aligned} D^q x_1 &= a(x_3 - x_1) + C_1 + u_1, \\ D^q x_2 &= a(x_4 - x_2) + C_2 + u_2, \\ D^q x_3 &= (b-a)x_1 - a x_1 x_5 + C_3 + u_3, \\ D^q x_4 &= (b-a)x_2 - a x_2 x_5 + C_4 + u_4, \\ D^q x_5 &= x_1 x_3 + x_2 x_4 - c x_5 + C_5 + u_5, \end{aligned} \tag{5.3}$$

where u_i, $i = 1, \dots, 5$ are control inputs and $C_i = \Delta f_i + d_i$, $i = 1, \dots, 5$ are parameter uncertainties and external noise disturbances as

$$C = \begin{pmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \end{pmatrix} = \begin{pmatrix} 0.2\sin(\pi x_5) + 0.2\cos(2t) \\ 0.2\cos(\pi x_3/2) + 0.02\sin(t) \\ 0.03\cos(2\pi x_2) + 0.2\cos(t/2) \\ 0.1\sin(\pi x_1) + 0.2\cos(t) \\ 0.2\sin(\pi x_4) + 0.02\sin(3t) \end{pmatrix}.$$

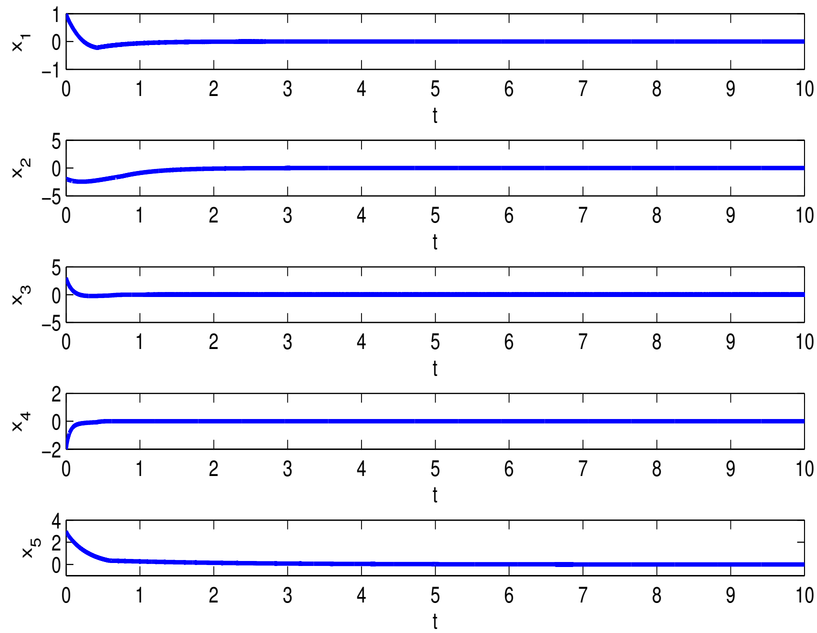

FIGURE 1. Time evolutions of the system (5.3) with the control law (4.4).

We have

$$A = \begin{pmatrix} -a & 0 & a & 0 & 0 \\ 0 & -a & 0 & a & 0 \\ b-a & 0 & 0 & 0 & 0 \\ 0 & b-a & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & c \end{pmatrix}, \quad g(x) = \begin{pmatrix} 0 \\ 0 \\ -ax_1x_5 \\ -ax_2x_5 \\ x_1x_3 + x_2x_4 \end{pmatrix}.$$

If we select

$$K = \begin{pmatrix} 0 & 0 & -a & 0 & 0 \\ 0 & 0 & 0 & -a & 0 \\ a-b & 0 & -10 & 0 & 0 \\ 0 & a-b & 0 & -20 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

we have $|\arg(eig(A+K))| = \pi$ which is more than $\frac{q\pi}{2} = \frac{99\pi}{200}$. Thus, the system (5.3) is asymptotically stable.

We choose the parameters for the controllers as $r = 3$, $\rho = 3$, $\gamma = 0.4$, and $\mu = 0.3$. We consider initial conditions of T systems as

$$(x_1(0), x_2(0), x_3(0), x_4(0), x_5(0)) = (1, -2, 3, -4, 5). \tag{5.4}$$

The simulation results with the control law (4.4) are given in Figures 1 and 2. Also, Figures 3 and 5 show the simulation results with the control law (4.7). We observe that all trajectories and the sliding surface tend to zero in less than $t = 1$ second. That means the proposed control laws are practical in applications.

Remark 5.1. Eliminating disturbances is often unrealistic in real-world systems. This analysis provides insight into the controller's behavior under ideal conditions. Now we want to investigate the effect of removing disturbances on the system performance and controllers.

(1) Impact on system dynamics:

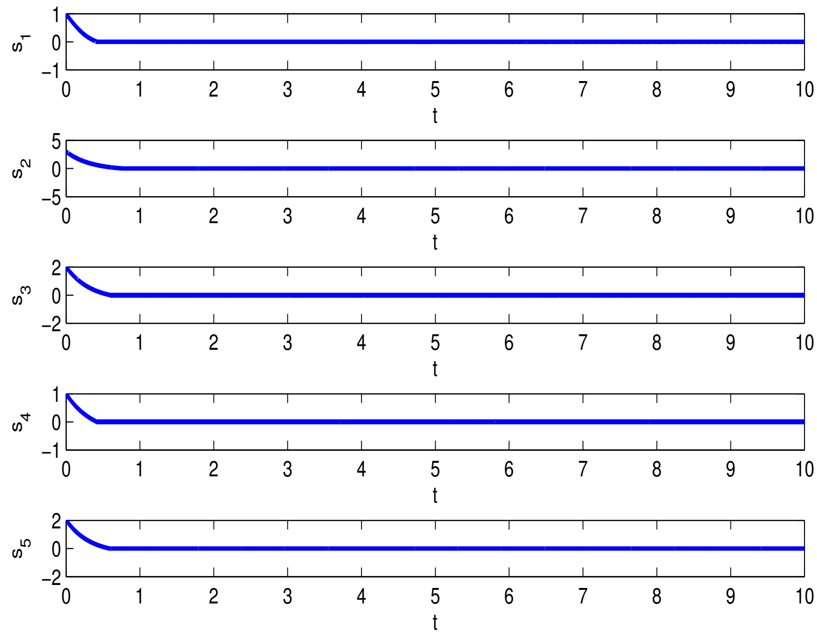

FIGURE 2. Simulation results for the sliding surface with the control law (4.4).

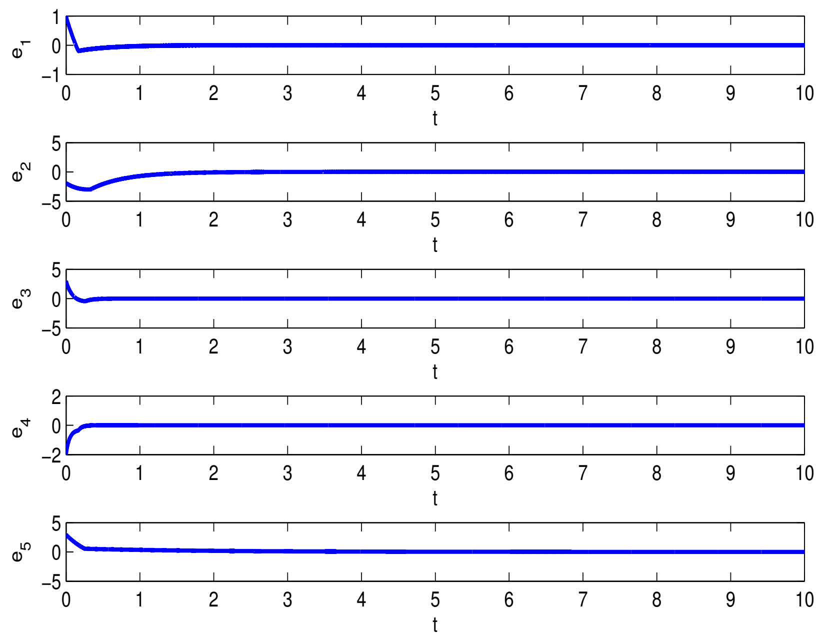

FIGURE 3. Time evolutions of the system (5.3) with the control law (4.7).

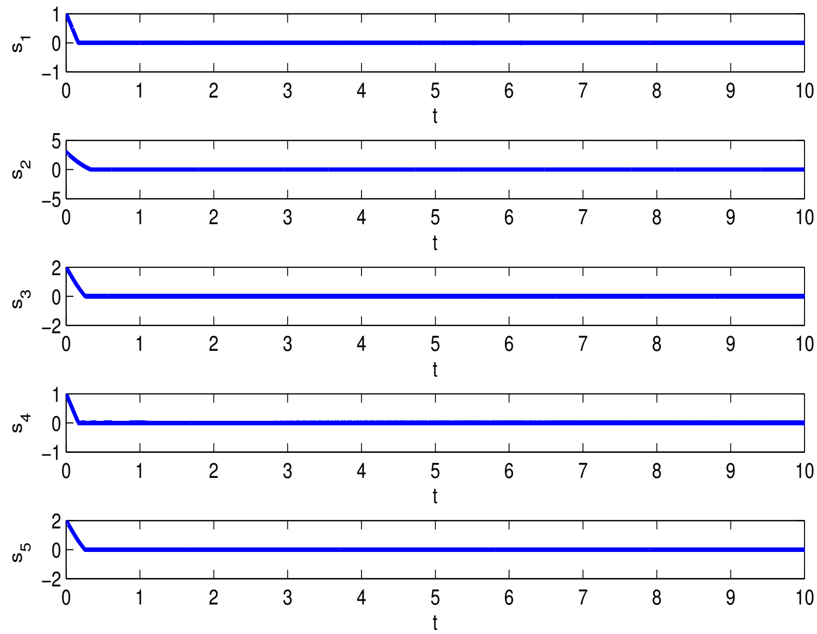

FIGURE 4. Simulation results for the sliding surface with the control law (4.7).

Removing the disturbances terms $f(x)$ and $d(t)$ (represented by $C(x,t)$ significantly simplifies the system dynamics to $D^q x = Ax+g(x)+u(t)$ with $u(t) = Kx-g(x)-(rs+\rho\,\mathrm{sgn}\,s)$ and alters the controller's behavior. This eliminates the need for the controller to compensate for uncertainties and external disturbances. The simplified system is deterministic and inherently less complex, making control design and analysis easier.

(2) Impact on stability and convergence:

While the proposed controller guarantees asymptotic stability in the presence of perturbations, removing $C(x,t)$ is expected to improve convergence speed. The system's response will become more predictable and less noise-sensitive because the controller does not need to compensate for unpredictable external effects continuously. Our results and simulations confirm this hypothesis. According to the Theorem (4.1), by removing the disturbances $C(x,t)$ from Equation (3.2), the states of this system will tend to the surface $s(t) = 0$ in the finite time $t_f \le \frac{1}{\rho}\|s(0)\|$ with $\rho > 0$. Hence, the appropriate choice of parameters ρ and γ has a great impact on the controller's performance. Considering given the values $\rho = 3$ and $\gamma = 0.4$ in the numerical simulation, we have $\frac{1}{\gamma(\rho-1)} > \frac{1}{\rho}$. Therefore, as shown in Figure 5, by removing disturbances, the system variables tend to $s(t) = 0$ faster. So, we have a faster convergence rate toward the equilibrium point $(x = 0)$.

6. Conclusions

In this article, control of fractional-order chaotic systems under perturbations such as uncertainties and external disturbances is investigated. For this purpose, a novel fractional-integer integral sliding surface and appropriate adaptive fractional-order sliding mode control laws are proposed to make sure that fractional-order chaotic systems with disturbances can be asymptotically stable. Simulation results demonstrate the efficiency and robustness of the designed controllers for the fractional-order complex chaotic T system.

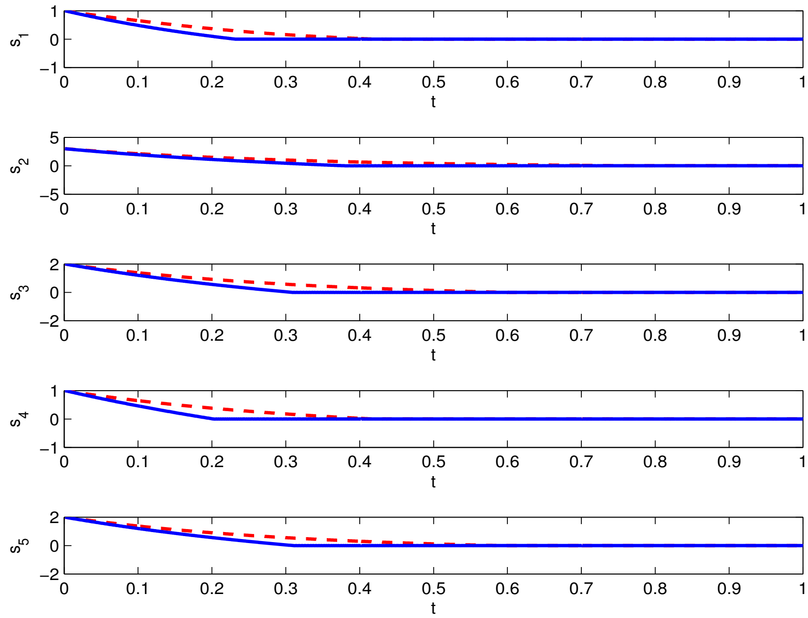

FIGURE 5. Simulation results for the sliding surface with the control law (4.4) for the system (5.3). Dashed lines are for the system with disturbances and solid lines are for the system (5.3) without disturbances.

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