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GMDH Neural Network-Based Enhanced Data-Driven Adaptive Control Design for Unknown Nonlinear Systems in the Presence of Quantized Data

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Abstract
This research paper presents a new approach to controlling unknown nonlinear systems using the Group Method of Data Handling (GMDH) neural network. The proposed Enhanced Data-Driven Quantized Model-Free Adaptive Control structure addresses the challenge of data quantization in Data-Driven Control Systems, which results from data loss and affects the performance of the Model-Free Adaptive Control (MFAC). In this study, the output quantized data is fed to the GMDH block, which derives a model to estimate the system's actual output based on the predictive feature of the network. The controller generates the input control signal based on the estimated output data. The stability analysis of the proposed control structure has been investigated through the Lyapunov theory. The proposed structure has been tested and compared against the traditional MFAC controller through simulation. The results illustrate the proposed approach's advantages in overcoming data quantization challenges in Data-Driven Control methods.

Keywords
Data-Driven Control, MFAC method, GMDH Neural Network, Data Quantization.

1. Introduction

Over the years, Model-Based Control (MBC) has been used to design control systems for various applications. In [1], the authors have implemented Dynamic Sliding Mode Control for Nonlinear Systems using a Sliding Mode Observer. In another instance, [2] focused on the stability of closed-loop hybrid dynamical systems with the Model-Predictive Control Approach. However, the increasing use of networked systems and the reliance on data interpretation have added complexity to industrial systems. Due to this complexity, it is not easy to extract a mathematical model that is precise and accurate in describing the dynamics of such systems. These challenges have led to developing a novel control approach called Data-Driven Control (DDC). This approach fundamentally alters the concept of control structures, as it solely relies on the data gathered from the plant and requires zero knowledge of the system's dynamics to generate the control signal.

In recent years, extensive developments have been made in DDC approaches based on time series analysis and learning-based algorithms [3]. Some of these approaches include Data-driven Optimal Control [4], Unfalsified Control [5], Iterative Feedback Tuning (IFT) [6], Model-Free Adaptive Control (MFAC) [7], and Virtual Reference Feedback Tuning [8]. DDC approaches are classified into three types: Online methods, such as Model-Free Adaptive Control (MFAC) [9]; Offline approaches, such as Virtual Reference Feedback Tuning [8]; and Hybrid methods, such as Iterative Learning Control [10]. Among the mentioned methods, MFAC is efficient for time-varying, nonlinear discrete-time systems.

Model-Free Adaptive Control (MFAC) was initially introduced by Hou in 1994 [3]. The primary idea behind this approach is to estimate the nonlinear discrete-time system using an equivalent linearized model. The linearization process involves using a concept known as Pseudo-Partial Derivative (PPD), which is estimated based on the input and output data from the control plant. The MFAC method is divided into three types based on the linearization approach used within the controller structure: Compact Form (CF), Partial Form (PF), and Full Form (FF) [3].

Compared to other linearization methods for nonlinear functions, the PPD approach used by the MFAC has several advantages. For instance, it does not require a time delay on the controlled system and can easily extend to MIMO systems [3]. Due to its superiority in controlling complex systems, the MFAC method can be applied to discrete-time and high-dimensional unknown nonlinear systems [11].

In recent years, Networked Controlled Systems have gained immense popularity due to the numerous benefits of digital communication [12]. These systems connect various elements through a digital network, simplifying installation and reducing costs associated with maintenance and wiring. However, deploying this approach can be challenging due to the communication load generated by the transmitted data. High-resolution data samples can cause problems in conditions with limited bandwidth or transmission noise, such as underwater or in tunnels [13]. To tackle this issue, data quantization can reduce the amount of transferred data without compromising the intended message. This method involves removing some data levels to reduce the resolution of the data sets, making it feasible to send them on mediums with limited bandwidth [14]. Numerous case studies have successfully used this technique, including [15, 16].

Numerous scholars worldwide have been drawn to the benefits of NCSs and have tried to address the problem of quantized control in their efforts. One example is [17], where the authors tackled the issue of quantized feedback stabilization for linear time-invariant systems in continuous and discrete-time scenarios. This method yields global asymptotic stability when applied to linear time-invariant feedback-stabilizable systems. However, the contributions of this paper are limited to linear systems. One can refer to the study [18] to address quantized control in nonlinear systems. In this paper, the input quantization for nonlinear systems is investigated. It concerns the global asymptotic stabilization of continuous-time systems subject to data quantization. Nonetheless, the main drawback of this work is that it only works in situations with full knowledge of the system's dynamics. The mentioned papers are presented as examples to address the issue of quantized control and methods for ensuring its convergence and stability. Some papers dealing with Networked Control Systems (NCSs) and quantized control are provided as follows: The problems of quantized stabilization and control for NCSs are provided in [19]. This study outlines the problem of quantized stabilization and control for NCSs by modeling an NCS system to address both delay and quantization levels. Using the Lyapunov function and Linear Matrix Inequalities (LMI) provides sufficient conditions for stabilizing this system. However, this work is model-based, and the presence of LMI equations makes the stabilization process complex. [16] combines the quantized control of NCSs with data-driven controls. This article explores the stability of the Model-Free Adaptive Control (MFAC) in systems employing quantized data in two separate signal quantization techniques. Although the quantization method utilized in these articles spans the whole period of the experiment, data quantization results in the loss of some of the system's usable and practical data, which might affect the control action of the data-driven controller. To the best of the authors' knowledge, only some references have paid attention to the issues resulting from data quantization in the performance of data-driven control structures. The primary motivation of this article is to fill this gap.

This research aims to solve the problems caused by data quantization in the performance of data-driven control structures by incorporating the Group Method of Data Handling (GMDH) neural network into the control loop. The quantization process often results in missing data, which is approximated and sent to the MFAC through the GMDH neural network. The controller would then use the approximated data to generate the control signal. This approach ensures that the controller does not detect errors in the received data and that the control signal functions flawlessly. The GMDH neural network is an excellent tool for prediction problems, especially with nonlinear systems. [20-24] provide more information about this topic. Additionally, another GMDH network has been trained to estimate the value of the PPD. This estimated value is then replaced with the recursive equations in the conventional MFAC structure, making the calculation process more accessible for the controller. The contributions to this paper are listed below:

I. A new control structure is built on the MFAC based on the GMDH neural network and output data quantization.
II. The lack of access to the missed output data due to the quantization is fulfilled using the GMDH NN in the control loop.

The following article is structured as follows: In Section 2, the problem definition is provided. In Section 3, one can find a discussion about the suggested EDD-MFAC structure for nonlinear systems, including the prerequisites for its convergence. In Section 4, we present simulations of some practical systems to demonstrate the effectiveness of the proposed control structure. Finally, the paper is concluded in Section 5.

2. Problem Definition

2.1. Problem Considerations

Consider the nonlinear system below:

$$y(c+1) = g\left(y(c), \dots, y(c-n_y), u(c), \dots, u(c-n_u)\right) \quad (1)$$

Where $y(c) = [y_1(c), \dots, y_t(c)]'$ is the system output, $u(c) = [u_1(c), \dots, u_t(c)]'$ is the control input, n_y, n_u are the unknown orders, and $g(.)$ is an unknown nonlinear function.

For facilitation, the following assumptions are assumed to be satisfied.

Assumption 1: The partial derivation of $g(.)$ with respect to control input $u(c), \dots, u(c-n_u)$ is continuous.

Assumption 2: System (1) is generalized Lipschitz, i.e., the following condition is considered to be met:

$$\|\Delta \boldsymbol{y}(C+1)\|_2 \le L\|\Delta \boldsymbol{u}(C)\|_2; \forall\, c$$

Where $\|L\|_2 > 0$, $\Delta u_i(\tau) = u_i(\tau) - u_i(\tau-1)$, and $\Delta y_i(\tau) = y_i(\tau) - y_i(\tau-1)$; $i = 1, \dots, \tau$.

Notice 1: Assumption 1 is a general condition in controller design. Furthermore, assumption 2 specifies that the rate of change in the system's output is related to the rate of change in the system's input. In other words, if the rate of change in the system's input is bounded, the rate of change in the system's output is finite, too. Both assumptions 1 and 2 are feasible, and many practical systems meet both criteria [25]. If assumptions 1 and 2 are met, system (1) can be linearized through an equivalent dynamical linearization model.

Theorem1 [25]: Given that the nonlinear system (1) meets assumptions 1, 2, and $\|L\|_2 \neq 0$, there exists a time-varying Pseudo–Partial Derivative (PPD) parameter, such that system (1) can be translated to the following linear form:

$$\Delta y(c+1) = P(c)\Delta u(c) \quad (2)$$

$$P(c) = \begin{bmatrix} p_{11}(c) & p_{12}(c) & \cdots & p_{1t}(c) \\ p_{21}(c) & p_{22}(c) & \cdots & p_{2t}(c) \\ \vdots & \vdots & \vdots & \vdots \\ p_{\tau 1}(c) & p_{\tau 2}(c) & \cdots & p_{\tau\tau}(c) \end{bmatrix},$$

$$p_{ij}(c) = \frac{\Delta y_i(c+1)}{\Delta u_j(c)}$$

And $\|P(c)\| \le \xi$.

Relation (2) describes the ultimate structure of the traditional MFAC structure with the compact form dynamic linearization (CFDL) model. This model modulates an unknown system like (1) as an estimated linear dynamical relation (2). As one can infer from (2), this relation is defined by changes in the system's output and input data. In order to ensure the accurate performance of the MFAC, the exact value of the PPD parameter must be tracked.

2.2. Quantization Formulation

As previously stated, quantization is required to deliver the data in lower resolutions when the system performs in environments with limited bandwidth.

The quantization technique utilized in this study is of the logarithmic format [16], as explained below:

$$A = \{\pm\kappa_i : \kappa_i = \beta^i \kappa_0, i = 0, \pm 1, \pm 2, \dots\} \cup \{0\}, 0 < \beta < 1, \kappa_0 > 0 \quad (3)$$

Where the parameter β is related to the quantization density. The associated quantizer $d(.)$ is defined as:

$$d(v) = \begin{cases} \kappa_i & \frac{1}{1+\sigma}\kappa_i < v \le \frac{1}{1-\sigma}\kappa_i \\ 0 & v = 0 \\ -d(-v) & v < 0 \end{cases} \quad (4)$$

Which $\sigma = \frac{1-\beta}{1+\beta}$. The quantizer described in (4) is found to be symmetric, which indicates that $d(-v) = -d(v)$.

In the following, we delve into the quantized model-free adaptive control structure to investigate the drawbacks caused by data quantization in the performance of such a Data-driven controller scheme.

2.3. Quantized Model-Free Adaptive Control

After quantizing the system's output data using the mentioned technique, the controller algorithm for the MFAC structure is given by:

$$\hat{P}(c) = \hat{P}(c-1) + \frac{\Gamma \Delta u(c-1)}{\mu + \|\Delta u(c-1)\|_2^2}\left(d_\Delta(\Delta y(c)) - \hat{P}(c-1)\Delta u(c-1)\right) \quad (5)$$

$$u(c) = u(c-1) + \frac{\omega \hat{P}(c)}{\lambda + \|\hat{P}(c)\|_2^2}\left(R(c) - d_y(y(c))\right) \quad (6)$$

Where, $\Gamma > \in (0.1), \omega \in (0.1), \mu > 0, \lambda > 0$ are weigh factors.

In order to guarantee the convergence of this algorithm, a reset mechanism has been proposed as follows [16]:

$$\hat{P}(c) = \hat{P}(1), \; if \; \left\|\hat{P}(c)\right\|_2^2 \le \varepsilon, or \|\Delta u(c-1)\|_2^2 \le \varepsilon \quad (7)$$

in which, $\hat{p}(1)$ is the initial value of $\hat{p}(c)$.

As mentioned earlier, the data quantization algorithm leads to the loss of some of the valuable system data, which can negatively affect the performance of the data-driven controller. This article suggests a novel structure based on the GMDH neural network to overcome this issue. Section 3 explains this approach.

3. Proposed EDD-QMFAC Structure

This section presents a concise yet thorough description of the GMDH neural network utilized in this study to establish the foundation of the proposed structure. A clear and detailed explanation of the proposed structure is provided. The section culminates with an in-depth analysis of the stability of the suggested structure, which conclusively demonstrates its practicality and effectiveness.

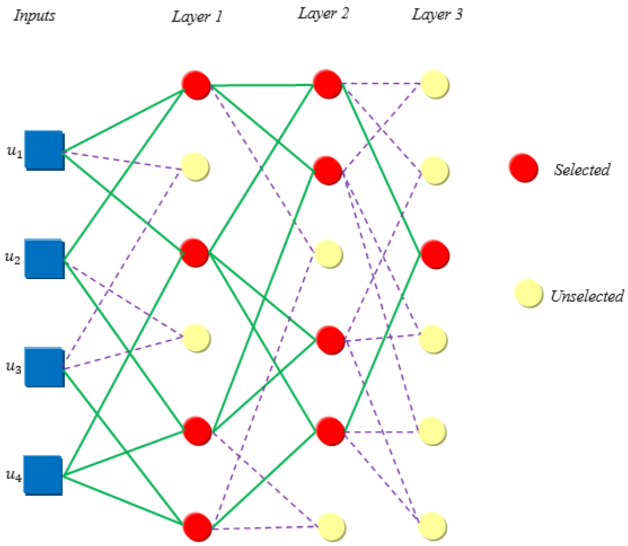


Figure 1. The Structure of GMDH NN

3.1. GMDH Structure

The Group Method of Data Handling (GMDH) is a robust hierarchical free lamination polynomial neural network that surpasses other neural networks in its ability to set the number of nodes and layers objectively. In GMDH, the number of neurons in the first layer corresponds to the number of double permutations of the system inputs, and the number of neurons in each subsequent layer is based on the double permutations of the neurons in the previous layer that satisfy the requirement to advance to the next level. The number of hidden layers in this technique may be pre-specified or adjusted based on an external criteria threshold value [26]. Figure 1 provides a clear depiction of the straightforward structure of the GMDH.

Consider the following nonlinear system to have a better grasp of this algorithm:

$$y = G(\chi_1, \chi_2, \dots, \chi_\tau) \qquad (8)$$

Where $\chi_1, \chi_2, \dots, \chi_\tau$ are input variables, and y is the output variable. To rewrite the connection between the system's inputs and outputs, we must use the Kolmogorov-Gabor form [20].

$$y(\tau) = z_0 + \sum_{i=1}^{\tau} z_i\chi_i + \sum_{i=1}^{\tau}\sum_{j=1}^{\tau} z_{ij}\,\chi_i\chi_j + \sum_{i=1}^{\tau}\sum_{j=1}^{\tau}\sum_{k=1}^{\tau} z\chi_i\chi_j\chi_k + \cdots \qquad (9)$$

Where, $Z = [z_0, z_i, z_j, z_{ij}, z_{ijk}, \dots]$ $\forall (i, j, k = 1,2,3, \dots, \tau)$ is the coefficient matrix.

The output $\hat{y}$ of a neuron with two inputs χ_i, and χ_j could be approximated as follows:

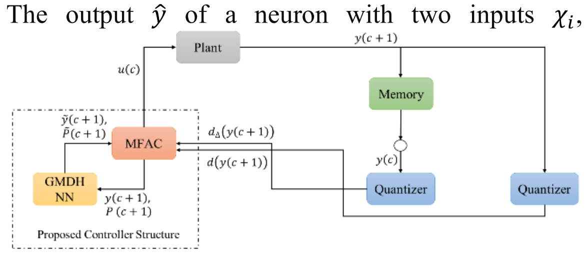


Figure 2. The Block Diagram of the Proposed Control Structure

$$\hat{y} = z_0 + z_1\chi_i + z_2\chi_j + z_3\chi_i^2 + z_4\chi_j^2 + z_5\chi_i\chi_j = Z\chi \qquad (10)$$

Where, χ is defined as below:

$$\chi = \begin{bmatrix} 1 & \chi_i(1) & \chi_j(1) & \chi_i^2(1) & \chi_j^2(1) & \chi_i(1)\,\chi_j(1) \\ 1 & \chi_i(2) & \chi_j(2) & \chi_i^2(2) & \chi_j^2(2) & \chi_i(2)\,\chi_j(2) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & \chi_i(c) & \chi_j(c) & \chi_i^2(c) & \chi_j^2(c) & \chi_i(c)\,\chi_j(c) \end{bmatrix}$$

Therefore, $Z = (\chi^T\chi)^{-1}\chi'\hat{y}$.

3.2. Enhanced Data-Driven Quantized Model-Free Adaptive Control

The data quantization technique reduces the resolution of the data by removing some data levels to send data across a medium with limited bandwidth. However, this technique can affect the QMFAC's performance, which relies on the system's data. To overcome these challenges, a new structure based on the predictive function of the GMDH neural network is proposed.

The output quantized data in the proposed control structure is initially sent to the GMDH NN block. The GMDH then derives a model for predicting the plant's output data, which is then sent to the MFAC block. The MFAC generates the appropriate control signal, thus enhancing the QMFAC's performance and eliminating the challenges of data quantization. Figure 2 illustrates this structure.

The accuracy of the $\hat{p}(c)$, is contingent on the precision of the recursive relation (5), which can be prone to errors when dealing with large data systems. However, to address this issue, the suggested structure has implemented an additional GMDH neural network to predict this parameter more accurately.

Figure 2 demonstrates the positive impact of integrating the GMDH NN in the control loop, resulting in a more precise and accurate control signal. The simulation results confirm that utilizing the GMDH neural network's capabilities has resolved the challenges of data quantization in data-driven controllers, such as Model-Free Adaptive control.

Assumption 3: We assumed that the GMDH NN has been accurately tuned for parameters such as the number of neurons and layers.

3.3. Stability Analysis

The sector-bound approach has been implemented in this paper to accurately address the quantization error. It is worth noting that a sector-bound expression is presented as an illustration [27]. An illustration of a sector-bound expression is $q(\mathcal{L}) = (\Delta(\mathcal{L}) + 1)\mathcal{L}$, where $|\Delta(\mathcal{L})| < \sigma$ for a determined signal $\mathcal{L}$ and the quantizer $q(\mathcal{L})$. The ensuing equations are obtained, utilizing the sector-bound approach, and with the denotation of $d_\Delta(.), d_{\mathrm{y}}(.), d_{\mathrm{e}}(.)$ as quantization densities:

$$d_\Delta\big(\Delta y(c)\big) = \Xi_\Delta \Delta y(c) + \Delta y(c) \tag{11}$$
$$d_{\mathrm{y}}\big(y(c)\big) = \Xi_y y(c) + y(c) \tag{12}$$
$$d_{\mathrm{e}}\big(e(c)\big) = \Xi_{\mathrm{e}} e(c) + e(c) \tag{13}$$

Where, $|\Xi_\Delta| \le \sigma_\Delta, |\Xi_{\mathrm{y}}| \le \sigma_{\mathrm{y}}, |\Xi_{\mathrm{e}}| \le \sigma_{\mathrm{e}}$.

The BIBO stability analysis of the PPD estimation and its convergence has been investigate on our previous work [28]. Now, from (12) and (6), for analyzing the bound of the tracking error, we have:

$$u(c) = u(c-1) + \frac{\omega \hat{P}(c)}{\lambda + \|\hat{P}(c)\|_2^2}\Big(R(c) - \Xi_y y(c) + y(c)\Big)$$

$$= u(c-1) + \frac{\omega \hat{P}(c)}{\lambda + \|\hat{P}(c)\|_2^2}\Big(e(c) - \Xi_y y(c)\Big) \tag{14}$$

By substituting (14) in (2), the following is obtained:

$$e(c+1) = e(c)\left(1 - \frac{\omega \hat{P}(c) P(c)}{\lambda + \hat{P}^2(c)}\big(1 + \Xi_y\big)\right) + \Xi_y R(c)\left(\frac{\omega \hat{P}(c) P(c)}{\lambda + \hat{P}^2(c)}\right)$$

$$= v(c)\Xi_y R(c) + \Big(1 - v(c)\big(1 + \Xi_y\big)\Big) e(c) \tag{15}$$

Where, $v(c) = \frac{\omega \hat{P}(c) P(c)}{\lambda + \hat{P}^2(c)}$. From (15), we obtain:

$$\|e(c+1)\|_2 < \big\|1 - v(c)\big(1 + \Xi_y\big)\big\|_2 \|e(c)\|_2 + \sigma_{\mathrm{y}} R(c) \tag{16}$$

Hence, $\hat{P}(c)$ is bounded; if the ω, λ chosen in a way that $\frac{(\omega b)^2}{4} < \lambda$ is satisfied, then $0 < v(c) < 1$. Furthermore, since $\sigma = \frac{1-\beta}{1+\beta}$, then $0 < \sigma_{\mathrm{y}} < 1$, and $\|\Xi_y\|_2 < 1$. There exists a constant like K such that: $\big\|1 - v(c)\big(1 + \Xi_y\big)\big\|_2 \le \mathrm{K} < 1$. From (16) we attain:

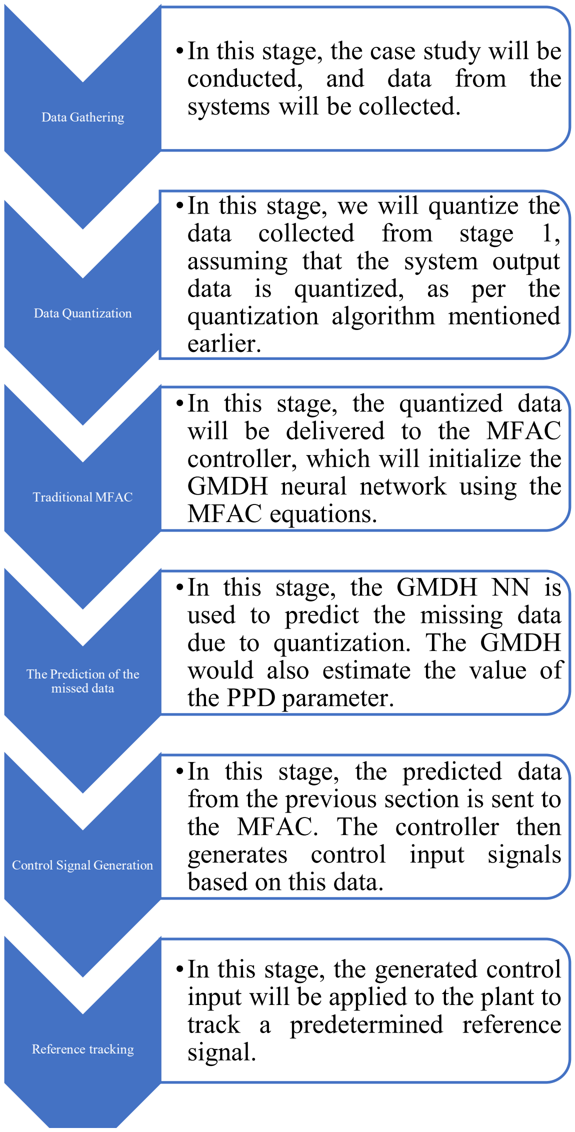
$$\lim_{k\to\infty} \|e(c)\|_2 \le \frac{\sigma_{\mathrm{y}} R(c)}{\mathrm{K}} \qquad \blacksquare$$

It has been proven that the tracking error is uniformly bounded, and this bound is related to both the quantization level and the asddesired signal.

3.4. The Flow Chart of The Proposed Method

In this section, we have provided a flow chart to help readers better understand our proposed method. The following chart clearly shows the steps required for the proposed method discussed in this article.

Table 1. Flow Chart of Proposed Method



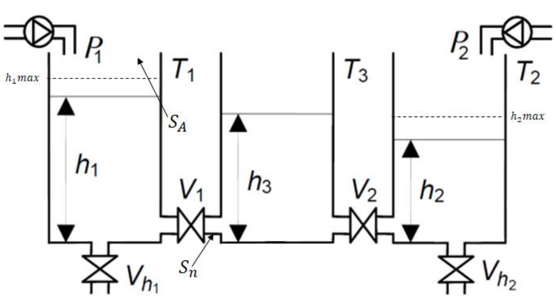


Figure 3. Three-Connected Tanks

4. Simulation

The effectiveness of the proposed controller was demonstrated through the application of the controller on two practical systems. The systems were explicitly chosen to showcase the versatility and adaptability of the controller in real-world scenarios. The results obtained from the testing proved that the proposed controller could effectively control complex systems and make significant improvements in their performance. These findings highlight the potential impact of this research and its ability to help solve real-world problems.

4.1. The Connected Three Tank System

The three-tank system is a widely recognized nonlinear system characterized by a complex three-dimensional structure [29]. The equations of this system is provided below:

$$S_A\frac{dh_1}{dt} = O_1(c) - O_{13}(c) - O_{10}$$

$$S_A\frac{dh_3}{dt} = O_{13}(c) - O_{32}(c)$$

$$S_A\frac{dh_2}{dt} = O_2(c) - O_{32}(c) - O_{20}$$

$$O_{13}(c) = v_1S_n sign\big(h_1(c) - h_3(c)\big)\sqrt{(2g|h_1(c) - h_3(c)|)}$$

$$O_{32}(c) = v_2S_n sign\big(h_3(c) - h_2(c)\big)\sqrt{(2g|h_3(c) - h_2(c)|)}$$

$$O_{20}(c) = vh_2S_n sign\big(h_2(c)\big)\sqrt{(2g|h_2(c)|)}$$

$$O_{10}(c) = vh_1S_n sign\big(h_1(c)\big)\sqrt{(2g|h_1(c)|)}$$

Table 2 gives the parameters of the three-connected tanks used in this simulation.

Table 1 The Parameters of the Three-Connected Tank System

Description	Parameters
Liquid level of tank ith (m)	h_i
Supplying flow rate of pump ith (m^3/s)	$p_i = u_i$
Outflow coefficient between tank 1 and tank 3	v_1=0.22
Outflow coefficient between tank 2 and tank 3	v_2 =0.27

Outflow coefficient from tank 1 to reservoir	vh_1=0.28
Outflow coefficient from tank 2 to reservoir	vh_2=0.4
Maximum level reading ability in tank 1	h_1max=0.6501
Maximum level reading ability in tank 2	h_2max =0.66
Tank cross-section	S_A=0.0154
Pipe cross-section	S_n=5×10^{-5}
Sampling Time	0.25 sec

To demonstrate the proposed structure's efficacy, a comparison has been conducted between a PID control method [29], the conventional MFAC, and the proposed structure. The simulation initialization process is identical for each controller, and the control objective is to finely regulate the liquid level in Tanks 1 and 2 based on a predetermined reference. As per the research conducted in [29], the PID parameters are as follows:

$$\begin{bmatrix} u_1(c+1) \\ u_2(c+1) \end{bmatrix} = \begin{bmatrix} u_1(c) \\ u_2(c) \end{bmatrix} + \begin{bmatrix} K_{p11} & K_{i11} & K_{d11} & K_{p12} & K_{i12} & K_{d12} \\ K_{p21} & K_{i21} & K_{d21} & K_{p22} & K_{i22} & K_{d22} \end{bmatrix} \times \begin{bmatrix} e_1(c) - e_1(c-1) \\ e_1(c) \\ e_1(c) - 2e_1(c) + e_1(c-2) \\ e_2(c) - e_2(c-1) \\ e_2(c) \\ e_2(c) - 2e_2(c) + e_2(c-2) \end{bmatrix}$$

Where, K_{pij}, $K_{i\,i\,j}$, and $K_{d\,i\,j}$ are determined as,

$$Kp_{11} = 0.00512, Ki_{11} = 0.00063, Kd_{11} = 0.00721;$$
$$Kp_{12} = 0.00131, Ki_{12} = 0.00005, Kd_{12} = 0.00130;$$
$$Kp_{21} = 0.00267, Ki_{21} = 0.00027, Kd_{21} = 0.00242;$$
$$Kp_{22} = 0.00985, Ki_{22} = 0.00101, Kd_{22} = 0.00642.$$

To design the traditional MFAC and the proposed EDD-MFAC, the parameters are set as below:

$$\eta = 0.5, \epsilon = 10^{-4}, \omega = 1, \lambda = 1,$$

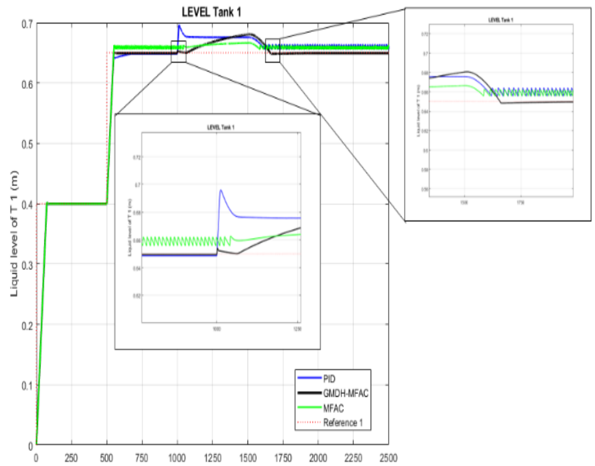


Figure 4 Tracking Performance of Tank 1

$$\mu = 0.0001, \hat{p}(1) = \hat{p}(2) = \hat{p}(3) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Figure 4 shows Tank 1's tracking performance with three controllers: PID, traditional MFAC, and the proposed control structure. Until T = 600 sec, the performance of all three controllers is almost identical. However, after this point, the PID and traditional MFAC controllers show distortions in the system's output due to data quantization. In contrast, the proposed structure prevents this issue and exhibits zero distortion using the actual output data calculated by the GMDH NN.

The liquid level in Tank 1 fluctuated sharply when the reference signal in Tank 2 changed at T = 1000 Secs. However, the proposed structure emerges as the best controller after this point. All of the systems eventually stabilized at T = 1650. However, the output curves of the PID and traditional MFAC controllers have distortions due to data quantization. On the other hand, the output curve of the system controlled by the proposed structure stabilized perfectly without any distortions, indicating that this structure can solve the data quantization challenges on the MFAC.

Figure 5 illustrates Tank 2's tracking performance. It is evident from this figure that the proposed structure significantly improved the MFAC's performance.

Figures 7 and 8 depict the input control signals generated by the PID, the conventional QMFAC, and the suggested control structure. The signals produced by the PID and the QMFAC are step-shaped, indicating that these controllers do not precisely control the system in a quantized data scenario. In contrast, this paper's proposed structure's control signal has a perfectly smooth curve.

The GMDH inputs in this simulation consist of the quantized level height of the liquid in Tanks 1 and 2 and the control input from the previous moments.

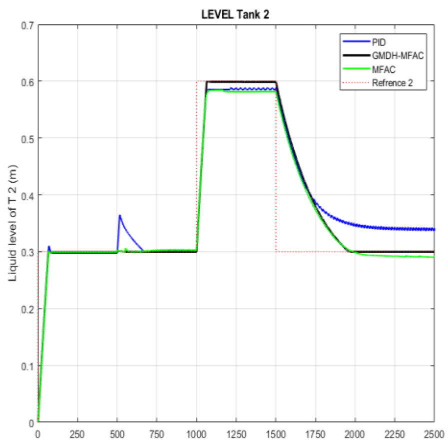


Figure 5. Tracking Performance of Tank 2

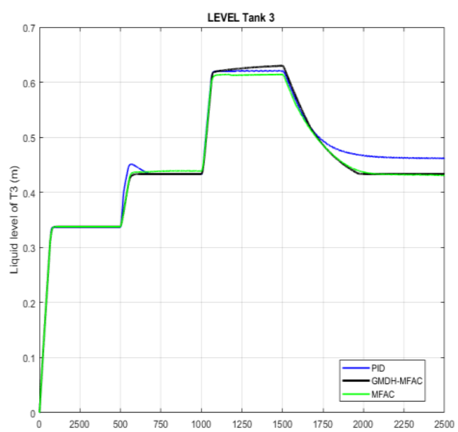


Figure 6. Tracking Performance of Tank 3

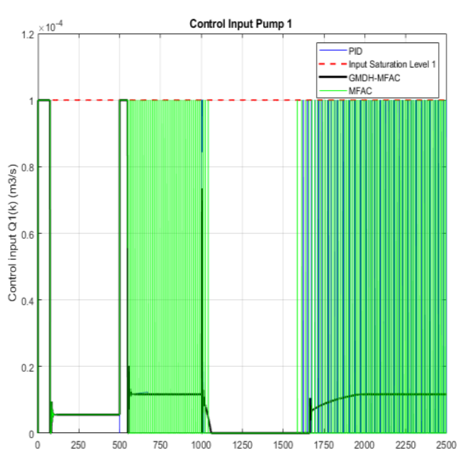


Figure 7. Control Input Signal for Tank 1

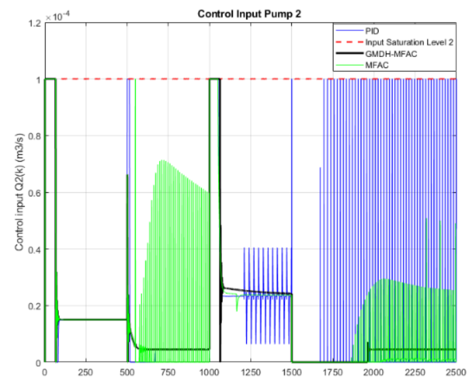


Figure 8. Control Input Signal for Tank 2

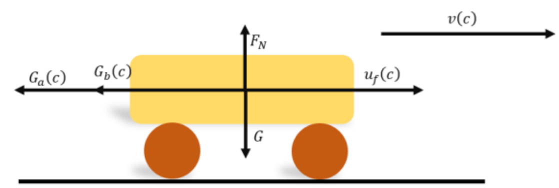


Figure 9. Subway Train Systems

4.2. The Subway Train System

Modern-day subway trains are controlled by sophisticated digital systems [30, 31]. Based on the train's discrete-time concept, these digital control algorithms create exclusively discrete-time control signals. Figure 9 depicts a general point-mass model of a subway train, where, F_N denotes the support force, $v(c)$ the subway train speed, $u_f(c)$ the traction braking force, and $G_a(c)$ and $G_b(c)$ the supplementary and basic resistance values. These supplementary resistances comprise ramp, curve, and tunnel resistance. The motion dynamic model of a subway train is presented below based on this point-mass model:

$$v(c+1) = v(c) + \frac{£}{M_t}\left(u_f(c) - G_a(c) - G_b(c)\right)$$

Where £, is the sample interval, and M_t, represents the subway mass. Table 1 shows the simulation parameters, where α, is the slope of the subway train path. This table illustrates that several parameters in this simulation are considered to be time-varying.

In Table 3 the parameters of the subway train system which are used in this simulation are given.

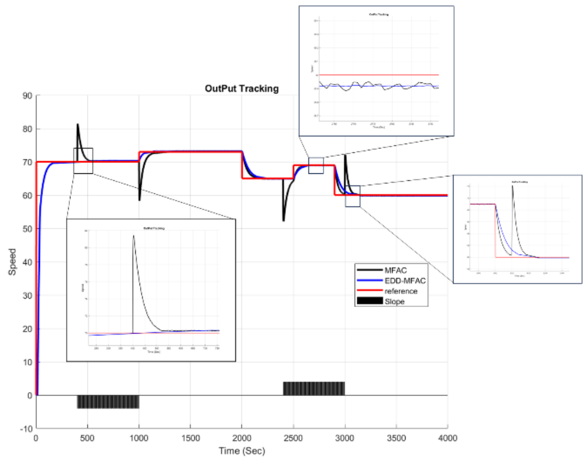


Figure 10. Tracking performance of the subway system

Table 2 The simulation parameters

Variable	Value
M_t	h_i
G	$p_i = u_i$
G_b	v_1=0.22
Ga_a	v_2 =0.27
$V_{\max}\,(\frac{km}{h})$	80
Sampling Time	0.2 sec

In this section, we compare the performance of the proposed technique with the QMFAC method of [40] in train control. The suggested EDD-QMFAC has the same initial settings as the QMFAC in [40], which are:

$$u_f(1) = 0.001, u_f(2) = 1.342, \hat{F}_s(1) = 0.137$$

Figure 11 highlights the proposed control structure's superior performance compared to the traditional MFAC in the train problem. The main objective is to adjust the train's speed and ensure that it follows a predetermined reference signal, regardless of the changes in the path and time. The proposed controller's predictive feature is evaluated by considering two types of slopes that may occur in the train's path. The positive slope type would decrease the train's speed if it ran at the same speed with the same forces, while the negative path slope would increase its speed. The GMDH inputs in this simulation include the road slope, the train's quantized speed, and the preceding moment control signal.

Figure 11 demonstrates that the train's speed is always kept within the reference signal range during the trial period. Whenever the slope is positive, the control signal immediately triggers the train's engine to generate the necessary force that overcomes the slope,

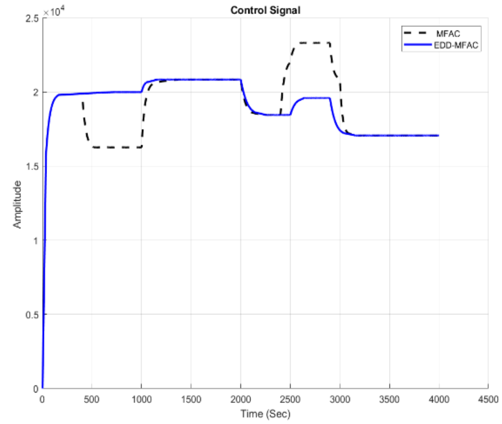


Figure 11. Control Signal Generated by MFAC for the Subway System

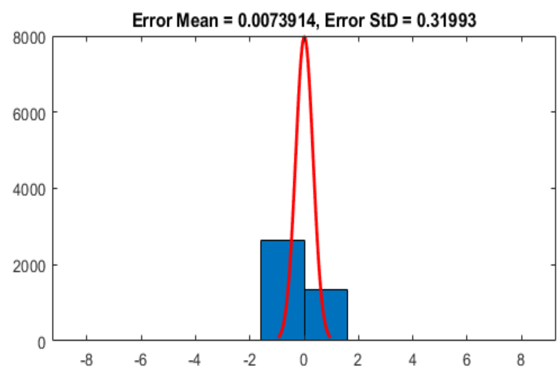


Figure 12. Error Diagram

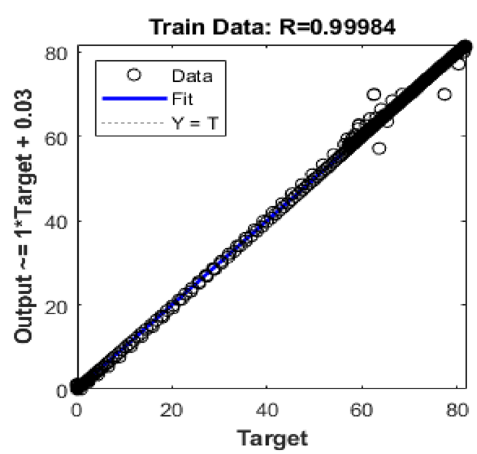


Figure 13. The Training Data

thus increasing the train's power and maintaining its speed within the predetermined range. Consequently, the system's performance is not affected by any train path changes.

On the other hand, when the slope is negative, the controller immediately predicts that the train will be affected by it and generates a control input that causes the engine to decrease its power. This ensures that the train's speed remains within the reference signal range. The proposed controller boasts a predictive feature that enables the system to stay within the range and accurately track the reference signal.

In contrast, traditional MFAC exhibits peaks in response to path slopes, making it difficult for the controller to predict and manage the slope. However, the proposed structure can effortlessly predict the slope and generate the necessary control signal to compensate for these slope changes. As depicted in Figure 11, the proposed controller can track the reference signal perfectly, with no peaks whatsoever.

Figure 11 also demonstrates that the proposed controller outperforms the conventional QMFAC in the quantized data scenario. The suggested structure is much more efficient in correcting the distortions generated by the data quantization algorithm. As illustrated in Fig.10, the traditional MFAC exhibits significant distortions throughout the period that substantially increase the steady-state error. On the other hand, the proposed structure efficiently eliminates these distortions and can manage the tracking problem with superior accuracy, inevitably reducing the error.

Figure 12 presents the control signal generated by traditional MFAC and EDD_MFAC. As observed, the signal generated by the EDD-MFAC structure is more precise and accurately represents the predetermined reference signal.

Moving on to Figure 13, it demonstrates how the GMDH neural network significantly reduces errors. Figure 13 shows the data converging to a 45° slope, indicating that the GMDH has effectively trained the data set. Despite missing some of the system's data during the quantization periods, the GMDH neural network predicts the output data precisely.

Table 4 highlights the proposed structure's superiority over the traditional MFAC. The data shows a significant difference in IAE, ISE, and steady-state error, providing undeniable evidence that the proposed structure outperforms the conventional controller.

Table 4. proposed structure's superiority over the traditional MFAC

Index	MFAC	EDD-MFAC
E_{ss}	3.92	3.08
IAE Error	2.29×10^6	1.44×10^6
ISE Error	79.99	79.99
PPD	7.18	9.14

5. Conclusion

This paper studies the Enhanced Quantized Model-Free Adaptive Control (EDD-MFAC) structure proposed for controlling an unknown class of nonlinear systems in the presence of data quantization. It introduces the standard quantized model-free adaptive controller and then discusses the challenges it poses while dealing with data quantization. To overcome the limitations of the traditional MFAC, the proposed structure in this study integrates the GMDH NN in the control loop. The GMDH neural network estimates the actual output data from the plant's quantized output data. The estimated output data is then used in the MFAC block to generate the control input signal instead of the quantized data. This approach eliminates the data quantization issues and reduces computational errors, making it more efficient than the traditional MFAC. The paper explains the GMDH neural network and its application in the proposed EDD-MFAC structure. The simulation results demonstrate that the suggested structure effectively eliminates distortions and interruptions caused by the quantized data, making it an ideal solution for controlling nonlinear systems in real-world applications. The proposed method can be applied to control both linear and nonlinear systems and, in both MIMO, and SISO scenarios as evidenced by the case studies investigated by this paper.

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